

GAUTENG DEPARTMENT OF EDUCATION



JOHANNESBURG NORTH DISTRICT

2022 GRADE 12 CONTROL TEST



**MATHEMATICS
TERM1**

MARKS : 100
TIME : 2 hours

This questions paper consist of 14 pages

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **8 questions**.
2. Answer **ALL** the questions.
3. Clearly show **ALL** calculations, diagrams, graphs, etc. which was used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. Where necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.
7. Diagrams are **NOT** necessarily drawn to scale.
- 8. Tear off page 11 till page 14 . AND SUBMIT these pages with your answer scripts .**
9. An **information sheet** is on page 10 of the question paper.
10. Number the questions correctly according to the numbering used in the question paper.
11. Write neatly and legibly.

QUESTION 11.1 Solve for x :

1.1.1 $x^2 - 7x + 10 = 0$ (2)

1.1.2 $3x^2 + 2x + 6 = 10$ (correct to two decimal places) (4)

1.1.3 $x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 28 = 0$ (4)

1.1.4 $\sqrt{2-x} = x - 2$ (5)

1.2 Given: $3x^2 + kx - 3x - k = 0$.For which values of k will the equation have real roots? (4)1.3 Solve for x and y :

$3y + x = 5$ and $x^2 + y^2 = 100 + 5y$ (6)

[25]

QUESTION 2

2.1 Consider the sequence: 1; 4; 11; 22; 37;.....

2.1.1 Calculate the n^{th} term. (4)

2.1.2 Which term in the sequence has a value of 407? (4)

2.2 How many terms are there in the following arithmetic sequence.

40; 46; 52; 58;.....202 (3)

[11]

QUESTION 3

3.1 The 4th term of an arithmetic sequence is -3 and the 20th term is -35 .

Determine the common difference and the first term. (5)

3.2 Evaluate : $\sum_{k=1}^{20} 3^{k-2}$ (4)

3.3 The following sequence forms a convergent geometric sequence:

$$\frac{3}{(x-1)^2} + \frac{1}{(x-1)} + \frac{1}{3} + \frac{(x-1)}{9} + \dots$$

3.3.1 Determine the possible values of x . (3)

3.3.2 If $x = 2$, calculate S_{∞} . (2)

[14]

QUESTION 4

4.1 Given $\sin 24^\circ = m$ and $\cos 35^\circ = n$. Determine the following in terms of m or n .

4.1.1 $\tan 66^\circ$ (3)

4.1.2 $\sin 70^\circ$ (3)

4.2 Prove that: $\sin(45^\circ + x) \cdot \sin(45^\circ - x) = \frac{\cos 2x}{2}$ (5)

4.3 Given $\cos(x + 42^\circ) = \sin 2x$. Solve for x if $x \in [-180^\circ, 180^\circ]$. (6)

[17]

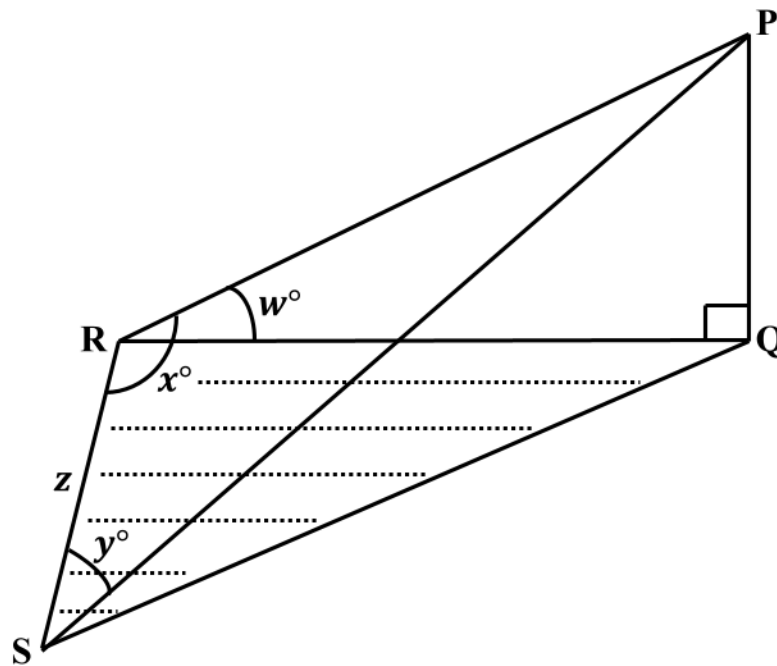
QUESTION 5

A mountain climber wants to determine the height PQ of a mountain. The climber is standing at R on a flat ground. R and S are in the same horizontal plane as the foot of the mountain Q .

From R , he measures the following angles:

- The angle of elevation of the top of the mountain P is w° .
- \widehat{PRS} is x°

He then walks z metres to point S and measures \widehat{RSP} which is y°



5.1 Show that $PQ = \frac{z \sin y \cdot \sin w}{\sin(x + y)}$ (4)

5.2 Determine PQ , if $z = 1000m$, $w = 90^\circ - x$ and $x = y$ (4)

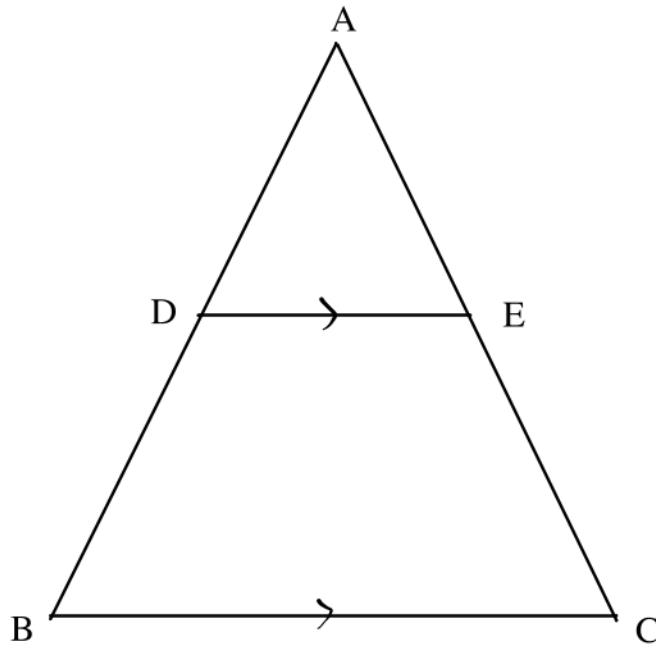
[8]

Give reasons for your statements in Question 6, 7 and 8.

Use the Annexure A provided to answer Question 6, 7 and 8

QUESTION 6

Given $\triangle ABC$ with $DE \parallel BC$ as shown in the figure below:



Prove that: $\frac{AD}{DB} = \frac{AE}{EC}$

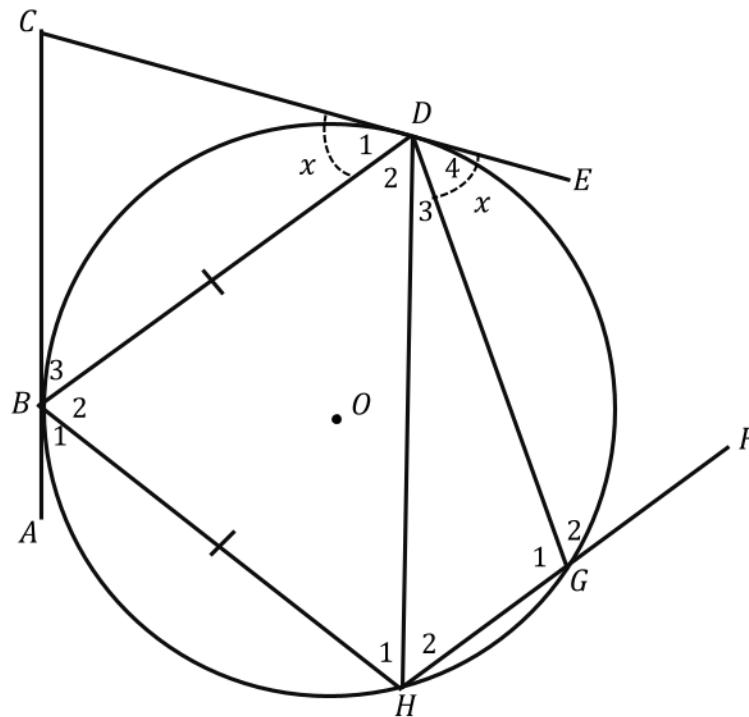
[5]

QUESTION 7

In the diagram below, AC and CE are tangents to the circle with centre O.

B, D, G and H are points on the circumference of the circle.

HG is produced to F. $BD = BH$ $\hat{D}_1 = \hat{D}_4 = x$.



7.1 Find four other angles equal to x . (4)

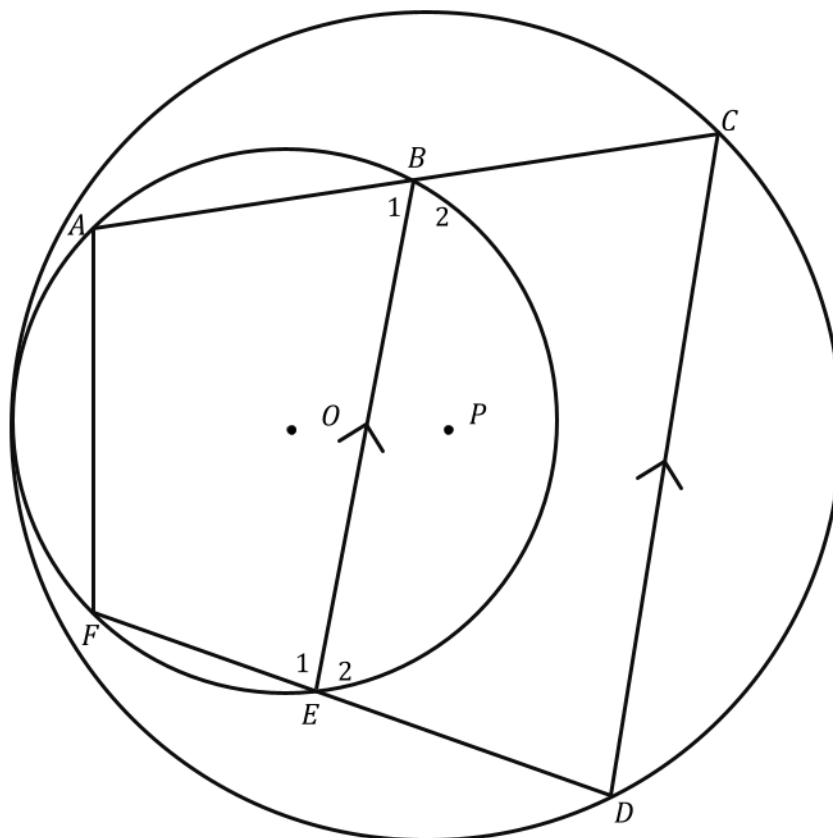
7.2 Hence or otherwise prove that $BD \parallel HG$. (2)

7.3 Show that $\hat{G}_2 = 180^\circ - 2x$. (2)

[8]

QUESTION 8

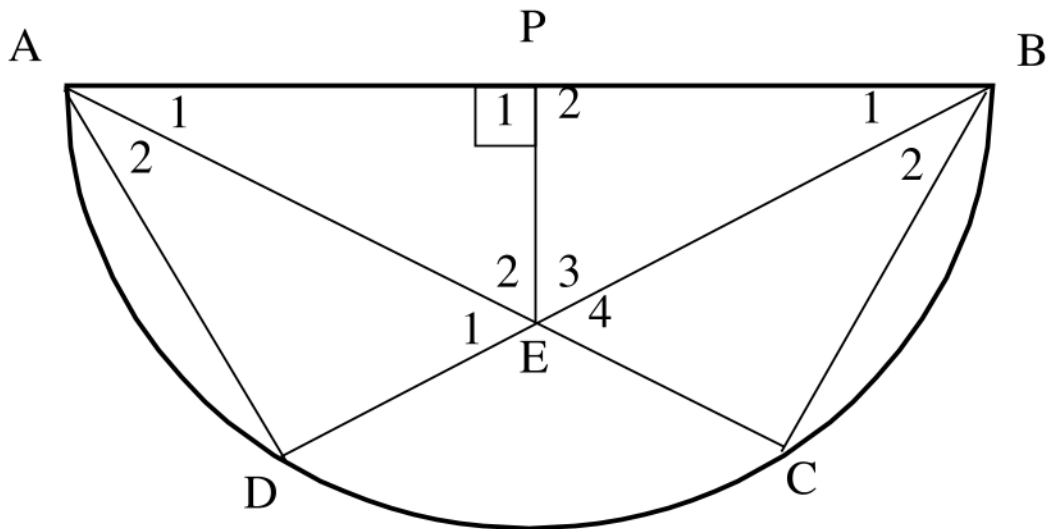
8.1 In the diagram below, AB lies on circle with centre O, and is produced to C which lies on circle with centre P. Similarly, FE is produced to D.. $BE \parallel CD$.



Prove that ACDF is a cyclic quadrilateral.

(4)

8.2 In the diagram below, AB is the diameter of circle with centre P. $EP \perp AB$.



8.2.1 Prove that $\triangle BPE \parallel \triangle BDA$ (4)

8.2.2 Hence, prove that $BE = \frac{PE^2 \cdot BA \cdot BD}{BP}$ (4)

[12]

TOTAL 100

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC :

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$area \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2 \sin^2 \alpha \\ 2 \cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta ; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

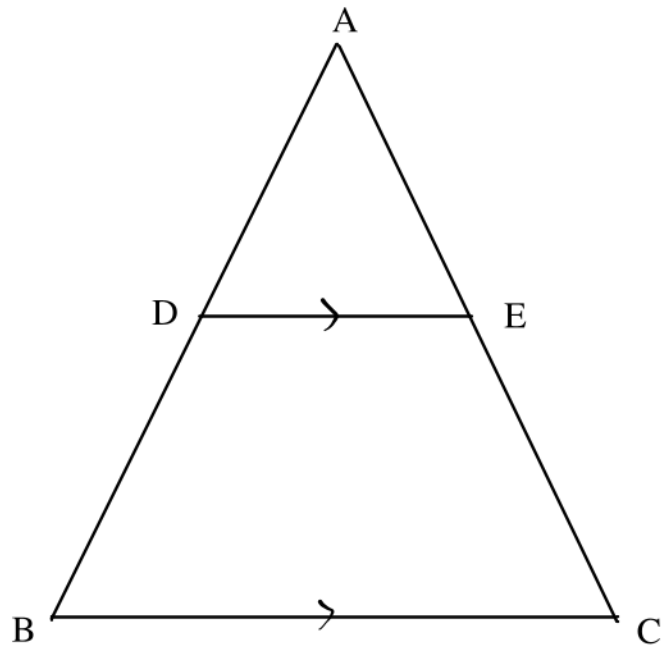
$$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

Name: _____

Class: _____

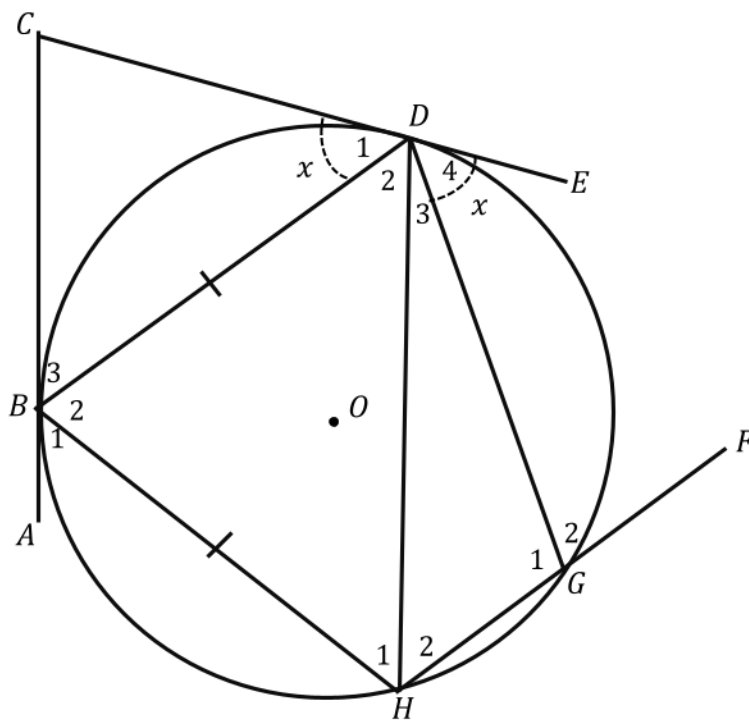
ANNEXTURE A

QUESTION 6



(5)

QUESTION 7



7.1		(4)

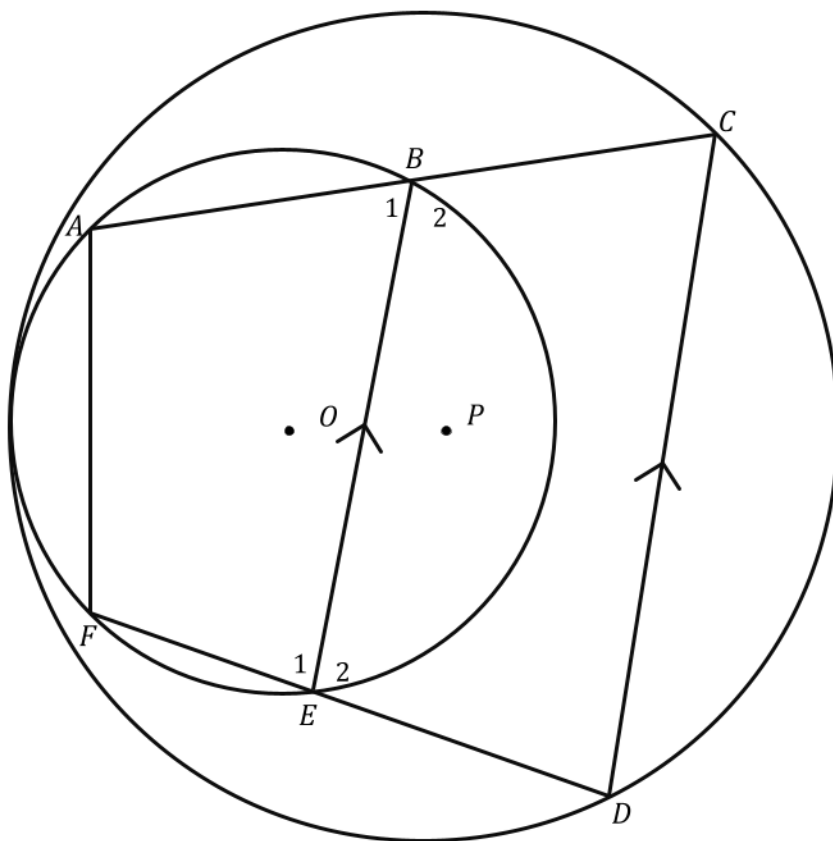
7.2		(2)

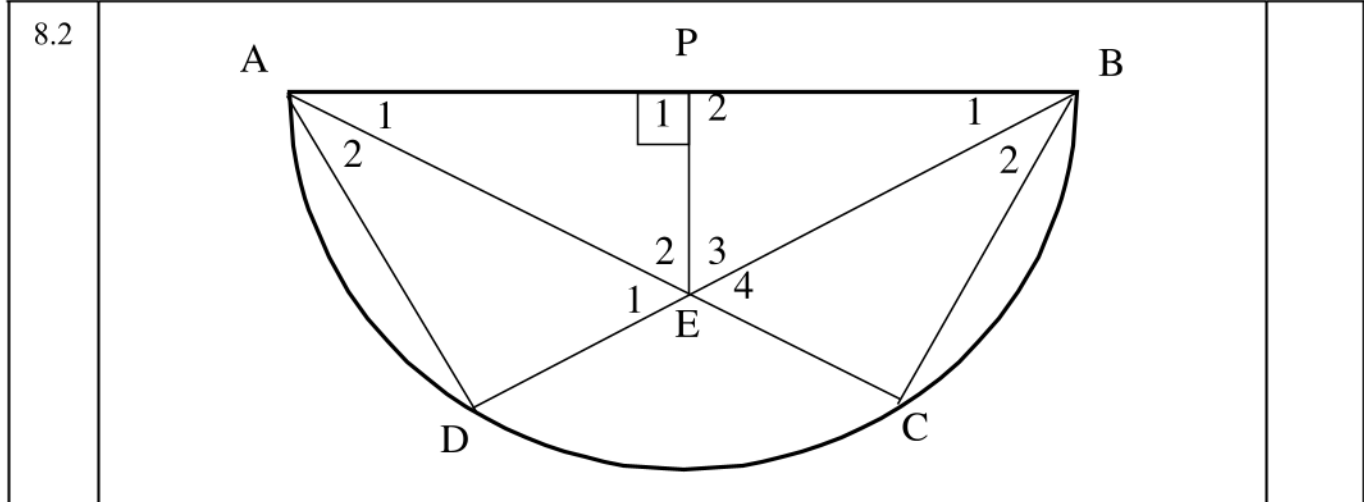
7.3		(2)

QUESTION 8

8.1

(4)





8.2.1		(4)

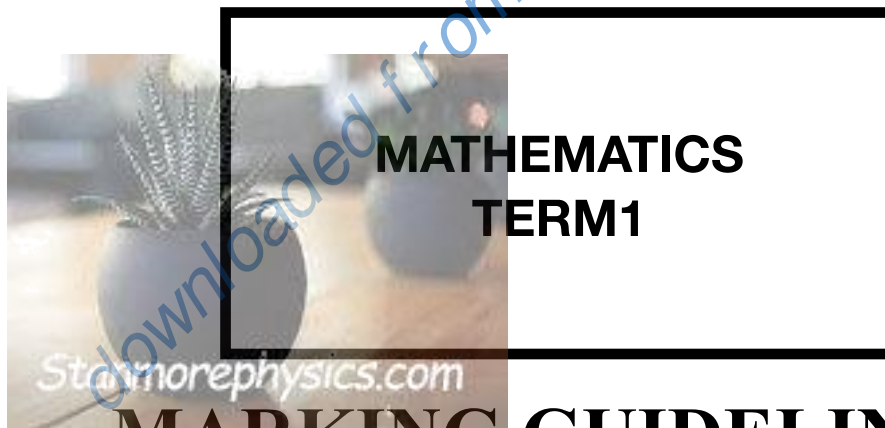
8.2.2		(4)

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2022 GRADE 12 CONTROL TEST



MARKING GUIDELINES

MARKS : 100
TIME : 2 hours

QUESTION 1		
1.1.1	$x^2 - 7x + 10 = 0$ $(x - 5)(x - 2) = 0$ $x = 5$ or $x = 2$	✓ Factors ✓ Ans (2)
1.1.2	$3x^2 + 2x + 6 = 10$ $3x^2 + 2x - 4 = 0$ $x = \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(-4)}}{2(3)}$ $x = 0,87$ or $x = -1,54$	✓ Standard Form ✓ Correct Sub into formula ✓ $x = 0,87$ ✓ $x = -1,54$ (4)
1.1.3	$x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 28 = 0$ $(x^{\frac{1}{4}} - 4)(x^{\frac{1}{4}} + 7) = 0$ $x^{\frac{1}{4}} = 4$ or $x^{\frac{1}{4}} = -7$ $x = 256$ or $x = 2401$ OR $x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 28 = 0$ Let $k^2 = x^{\frac{1}{2}}$ or $k = x^{\frac{1}{4}}$ $\therefore k^2 + 3k - 28 = 0$ $(k - 4)(k + 7) = 0$ $k = 4$ or $k = -7$ $x^{\frac{1}{4}} = 4$ or $x^{\frac{1}{4}} = -7$ $x^{\frac{1}{4} \times 4} = (4)^4$ or $x^{\frac{1}{4} \times 4} = (-7)^4$ $x = 256$ or $x = 2401$	✓ Factors ✓ $x^{\frac{1}{4}} = \dots$ ✓ raising both side to reciprocal ✓ x - values (4) OR ✓ Factors ✓ $x^{\frac{1}{4}} = \dots$ ✓ multiplying by the reciprocal ✓ x - values (4)

1.1.4	$\sqrt{2-x} = x - 2$ $(\sqrt{2-x})^2 = (x-2)^2$ $0 = x^2 - 3x + 2$ $0 = (x-2)(x-1)$ $x = 2 \text{ or } x = 1$ $\therefore x \neq 1$	<ul style="list-style-type: none"> ✓ Squaring ✓ Standard form ✓ Factors ✓ x- values ✓ Selection/ Testing <p style="text-align: right;">(5)</p>
1.2	$3x^2 + kx - 3x - k = 0.$ $3x^2 + x(k-3) - k = 0$ $\Delta \geq 0$ $b^2 - 4ac \geq 0$ $(k-3)^2 - 4(3)(-k) \geq 0$ $k^2 + 6k + 9 \geq 0$ $(k+3)^2 \geq 0$ $\therefore k \geq -3$	<ul style="list-style-type: none"> ✓ $\Delta \geq 0$ ✓ Subbing correctly ✓ Factors ✓ Ans <p style="text-align: right;">(4)</p>

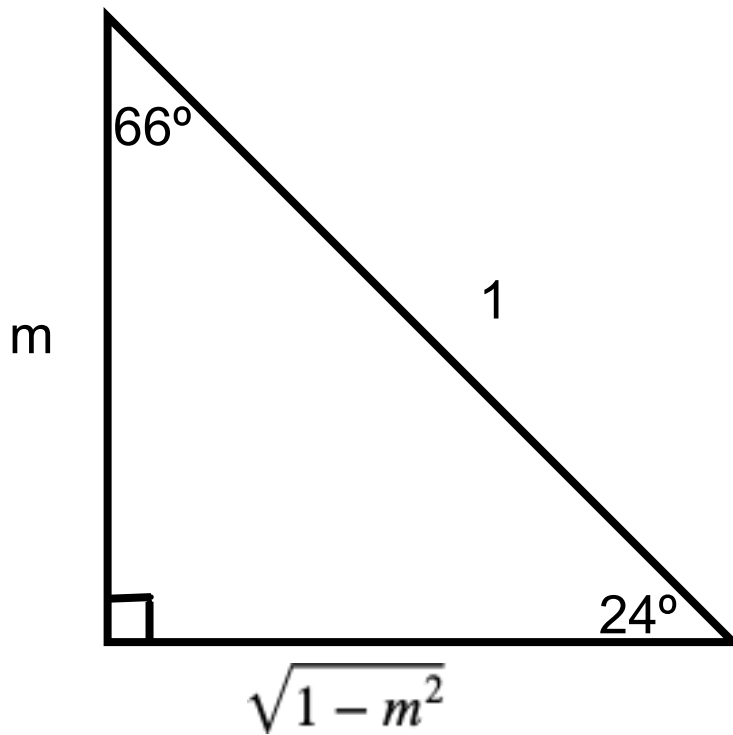
1.3	$3y + x = 5 \dots\dots\dots(1)$ $x^2 + y^2 = 100 + 5y \dots\dots\dots(2)$ $x = 5 - 3y \dots\dots\dots(3)$ Sub (3) into (2) $(5 - 3y)^2 + y^2 = 100 + 5y$ $2y^2 - 7y - 15 = 0$ $(y - 5)(2y + 3) = 0$ $y = 5 \text{ or. } y = \frac{-3}{2}$ $x = -10 \text{ or. } x = \frac{19}{2}$	<ul style="list-style-type: none"> ✓ Equation (3) ✓ Subbing correctly ✓ Standard form ✓ Factors ✓ Y-values ✓ X-values <p style="text-align: right;">(6)</p>
25 MARKS		

QUESTION 2		
2.1.1	$a + b + c = 1 \dots\dots\dots(1)$ $3a + b = 3 \dots\dots\dots(2)$ $2a = 4$ $\therefore a = 2$ $\therefore b = -3$ $\therefore c = 2$ $T_n = 2n^2 - 3n + 2$	✓ Value of a ✓ Value of b ✓ Value of c ✓ General Term (4)
2.1.2	$407 = 2n^2 - 3n + 2$ $0 = 2n^2 - 3n - 405$ $0 = (n - 15)(2n + 27)$ $\therefore n = 15 \quad \text{or} \quad n \neq -\frac{27}{2}$	✓ Equating ✓ Standard Form ✓ Both values of n ✓ Rejection (4)
2.2	$d = 6$ $202 = 40 + 6(n - 1)$ $\therefore n = 28$	✓ $d = 6$ ✓ Sub ✓ Ans (3)
11 MARKS		

QUESTION 3		
3.1	$-3 - 3d = a \dots\dots\dots(1)$ $-35 - 19d = a \dots\dots\dots(2)$ $(1) = (2)$ $-3 - 3d = -35 - 19d$ $\therefore d = -2$ Sub d-value into (1) $a = -3 - (-2)$ $\therefore a = 3$	$\checkmark a = -3 - 3d$ $\checkmark a = -35 - 19d$ \checkmark Equating $\checkmark d = -2$ $\checkmark a = 3$ (5)
3.2	$a = \frac{1}{3}$ $r = 3$ $S_{20} = \frac{\frac{1}{3}(3^{20} - 1)}{3 - 1}$ $S_{20} = 581130733,3$	$\checkmark a = \frac{1}{3}$ $\checkmark r = 3$ \checkmark Sub \checkmark Ans (4)
3.3.1	$\frac{3}{(x-1)^2} + \frac{1}{(x-1)} + \frac{1}{3} + \frac{(x-1)}{9} + \dots$ $r = \frac{x-2}{3}$ $-1 < r < 1$ $-1 < \frac{x-2}{3} < 1$ $-1 < y < 5$	$\checkmark -1 < r < 1$ \checkmark Sub \checkmark Ans (3)
3.3.2	$S_{\infty} = \frac{3}{1 - \frac{1}{3}}$ $S_{\infty} = \frac{9}{2}$	\checkmark Sub \checkmark Ans (2)
14 MARKS		

QUESTION 4

4.1.1

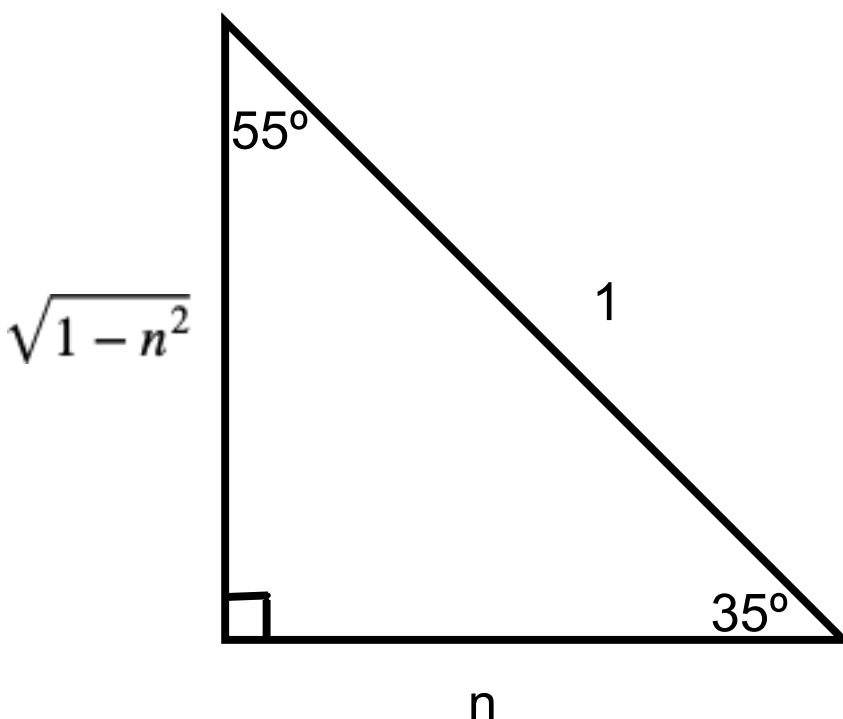


$$\tan 66^\circ = \frac{\sqrt{1 - m^2}}{m}$$

Award Full marks for answer only

- ✓ Sketch
- ✓ $\sqrt{1 - m^2}$
- ✓ Ans

(3)

<p>4.1.2</p>	 <p> $\sqrt{1-n^2}$ n 1 55° 35° </p> <p> $\sin 70^\circ = \sin(2 \times 35^\circ)$ $= 2 \sin 35^\circ \cdot \cos 35^\circ$ $= 2(\sqrt{1-n^2})(n) \quad \text{or} \quad 2n\sqrt{1-n^2}$ </p>	<p> ✓ Sketch ✓ Double angle identity ✓ Ans (3) </p>
<p>4.2</p>	<p> $\sin(45^\circ + x) \cdot \sin(45^\circ - x) = \frac{\cos 2x}{2}$ LHS = $\sin(45^\circ + x) \cdot \sin(45^\circ - x)$ $= (\sin 45^\circ \cdot \cos x + \cos^\circ \cdot \sin x)(\sin 45^\circ \cdot \cos x - \cos^\circ \cdot \sin x)$ $= \sin^2 45^\circ \cdot \cos^2 x - \cos^2 45^\circ \sin^2 x$ $= \left(\frac{1}{\sqrt{2}}\right)^2 \cos^2 x - \left(\frac{1}{\sqrt{2}}\right)^2 \sin^2 x$ $= \frac{1}{2}(\cos^2 x - \sin^2 x)$ $= \frac{1}{2} \cos 2x$ $\therefore \text{LHS} = \text{RHS}$ </p>	<p> ✓ Compound angle expansion ✓ Simplification ✓ Special angles ✓ Double angle identity ✓ $\frac{1}{2} \cos 2x$ (5) </p>

4.3	$\cos(x + 42^\circ) = \sin 2x$ $\cos(x + 42^\circ) = \cos(90^\circ - 2x)$ $x + 42^\circ = 90^\circ - 2x + k \cdot 360^\circ$ $\therefore 3x = 90^\circ - 42^\circ + k \cdot 360^\circ$ $\text{OR } x + 42^\circ = -(90^\circ - 2x) + k \cdot 360^\circ$ $\therefore x + 42^\circ = -90^\circ + 2x + k \cdot 360^\circ$ $\therefore 3x = 48^\circ + k \cdot 360^\circ \quad \text{OR} \quad -x = -132^\circ + k \cdot 360^\circ$ $x = 16^\circ + k \cdot 120^\circ \quad \text{OR} \quad x = 132^\circ - k \cdot 360^\circ$ $x \in \{-104; 16; 132; 136\}$	<p>✓ $\cos(90-2x)$</p> <p>✓ ✓ Both gen solution</p> <p>✓ simplify x=</p> <p>✓ ✓ Two correct x-values</p> <p style="text-align: right;">(6)</p>
17 MARKS		

QUESTION 5		
5.1	<p>In $\triangle PRS$: $R\hat{P}S = 180^\circ - (x + y)$ sum \angle's \triangle</p> $\frac{RP}{\sin y} = \frac{z}{\sin[180^\circ - (x + y)]}$ $RP = \frac{z \sin y}{\sin(x + y)}$ <p>In $\triangle PRQ$: $\frac{PQ}{PR} = \sin w$</p> $PQ = PR \sin w$ $\therefore PQ = \frac{z \sin y \cdot \sin w}{\sin(x + y)}$	<p>✓ $R\hat{P}S$</p> <p>✓ Sine rule</p> <p>✓ Reduction</p> <p>✓ Ans</p> <p style="text-align: right;">(4)</p>
5.2	$PQ = \frac{1000 \sin x \cdot \sin(90^\circ - x)}{\sin(x + x)}$ $= \frac{1000 \sin x \cdot \cos x}{2 \sin x \cdot \cos x}$ $= 500\text{m}$	<p>✓ Substitution</p> <p>✓ Co-ratio</p> <p>✓ Double angle</p> <p>✓ Ans</p> <p style="text-align: right;">(4)</p>
8 MARKS		

QUESTION 6

RTP: $\frac{AD}{DB} = \frac{AE}{EC}$

Construct: an altitude DF & GE in $\triangle ADE$ and join DC & BE

1.
$$\frac{\text{Area}\triangle ADE}{\text{Area}\triangle DEB} = \frac{\frac{1}{2} \times AD \times GE}{\frac{1}{2} \times DB \times GE}$$

$$= \frac{AD}{DB}$$

[\triangle 's share same height]

2.
$$\frac{\text{Area}\triangle DEA}{\text{Area}\triangle DEC} = \frac{\frac{1}{2} \times AE \times DF}{\frac{1}{2} \times EC \times DF}$$

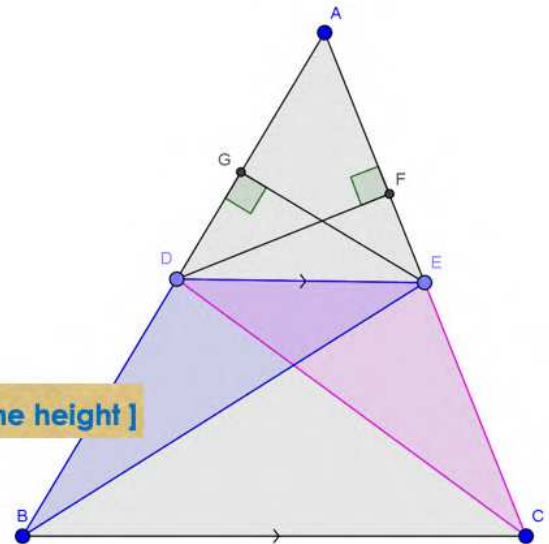
$$= \frac{AE}{EC}$$

[\triangle 's share same height]

3. BUT $\text{Area}\triangle DBE = \text{Area}\triangle DEC$

[\triangle 's lie between parallel lines]

$$\therefore \frac{\text{Area}\triangle ADE}{\text{Area}\triangle DBE} = \frac{\text{Area}\triangle DEA}{\text{Area}\triangle DEC} = \frac{AD}{DB} = \frac{AE}{EC}$$



5 MARKS

QUESTION 7		
7.1	$\hat{B}_3 = x$ (BC=CD; tans from the same point) $\hat{H}_1 = \hat{B}_3 = x$ (tan chord th) or $\hat{H}_1 = \hat{D}_1 = x$ (tan chord th) $\hat{D}_2 = \hat{H}_1 = x$ (\angle 's opp equal sides) $\hat{H}_2 = \hat{D}_4 = x$ (tan chord th)	✓ \hat{B}_3 & reason ✓ \hat{H}_1 & reason ✓ \hat{D}_2 & reason ✓ \hat{H}_2 & reason (4)
7.2	$\hat{H}_2 = \hat{D}_2 = x$ (proved above) $\therefore HG \parallel BD$ (alt \angle 's =)	✓ S ✓ R (2)
7.3	$\hat{B}_2 = 180^\circ - 2x$ (sum of \angle s in Δ) $\hat{G}_2 = \hat{B}_2 = 180^\circ - 2x$ (ext \angle of cyclic quad) Alternative solution $\hat{D}_1 + \hat{D}_2 = 2x$ $\hat{G}_1 = 2x$ (tan chord th) $\hat{G}_1 = 180^\circ - 2x$ (\angle s on a str line)	✓ \hat{B}_2 & reason ✓ \hat{G}_2 & reason OR ✓ \hat{G}_2 & reason ✓ \hat{G}_1 & reason (2)
8 MARKS		

QUESTION 8		
8.1	$\hat{A} = \hat{E}_2$ (ext \angle of cyclic quad) $\hat{D} = 180^\circ - \hat{E}_2$ (co-int \angle 's BE//CD) $\therefore \hat{D} + \hat{A} = 180^\circ$ $\therefore ACDF$ is a cyclic quad (opp \angle 's quad sup) Alternative solution $\hat{D} = \hat{E}_1$ (corres \angle s BE//CD) $\hat{E}_2 = 180^\circ - \hat{E}_1$ (\angle s on a str line) $\hat{A} = 180 - \hat{E}_1$ (opp \angle of cyclic quad $ABEF$) $\therefore \hat{D} + \hat{A} = 180^\circ$ $\therefore ACDF$ is a cyclic quad (opp \angle 's quad sup)	$\checkmark \hat{A} = \hat{E}_2$ $\checkmark \hat{D}$ & reason $\checkmark \hat{D} + A$ \checkmark conclude OR $\checkmark \hat{D} = \hat{E}_1$ $\checkmark \hat{A}$ & reason $\checkmark \hat{D} + A$ \checkmark conclude (4)
8.2.1	$\hat{B} = \hat{B}$ (common \angle) $\hat{D} = 90^\circ$ (\angle in semi circle) $\hat{P}_2 = 90^\circ$ (adj \angle 's on str line) $\therefore \hat{A} = \hat{E}$ (3^{rd} \angle) $\triangle BPE \parallel \triangle BDA$ ($\angle \angle \angle$)	\checkmark S/R \checkmark S/R \checkmark S/R \checkmark S/R (4)

8.2.2	$\frac{BP}{BD} = \frac{PE}{DA} = \frac{BE}{BA} \quad (// \triangle' s)$ $\frac{BP}{BD} = \frac{PE}{DA}$ $\therefore DA = \frac{BP}{BD \cdot PE} \dots\dots\dots(1)$ $\frac{PE}{DA} = \frac{BE}{BA}$ $\therefore DA = \frac{PE \cdot BA}{BE} \dots\dots\dots(2)$ <p>(2) = (1)</p> $\frac{BP}{BD \cdot PE} = \frac{PE \cdot BA}{BE}$ $\therefore BP \cdot BE = PE^2 \cdot BA \cdot BD$ $\therefore BE = \frac{PE^2 \cdot BA \cdot BD}{BP}$	<p>✓ S/R</p> <p>✓ Equation 1</p> <p>✓ Equation 2</p> <p>✓ $BP \cdot BE = PE^2 \cdot BA \cdot BD$</p> <p style="text-align: right;">(4)</p>
12 MARKS		