KWAZULU-NATAL PROVINCE

EDUCATION

REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS

COMMON TEST

MARCH 2024

MARKS: **100**

TIME: **2 hours**

N.B. This question paper consists of 7 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 8 questions.

2. Answer **ALL** questions.

3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

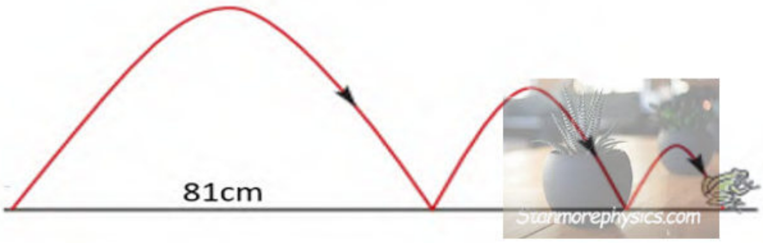
1.1 Consider the arithmetic sequence: 8 ; 15 ; 22 ; …………………….

1.1.1 Determine the 36^{th} term (2)

1.1.2 Calculate the sum of the first 36 terms. (2)

1.1.3 If it is given that $T_{72} + T_{72-m} = 786$, determine the value of *m*. (4)

1.2 A frog is making a series of jumps. With every next jump, he has only enough energy left to jump $\frac{2}{3}$ the distance of his previous jump.



1.2.1 If his first jump is 81cm long, calculate the length of his second jump. (1)

1.2.2 Determine the length of his ninth jump. (2)

1.2.3 If the frog continues to jump in this way, will he be able to catch a trapped insect that is 230 cm away from his starting point? Show all your calculations. (3)

[14]

QUESTION 2

2.1 The given number pattern is a combination of a quadratic sequence and an arithmetic sequence: 16 ; 32 ; 0 ; 28 ; – 12 ; 24 ; – 20 ; 20 ; ……..

2.1.1 Determine the general term of the quadratic sequence. (4)

2.1.2 Determine the general term of the arithmetic sequence. (2)

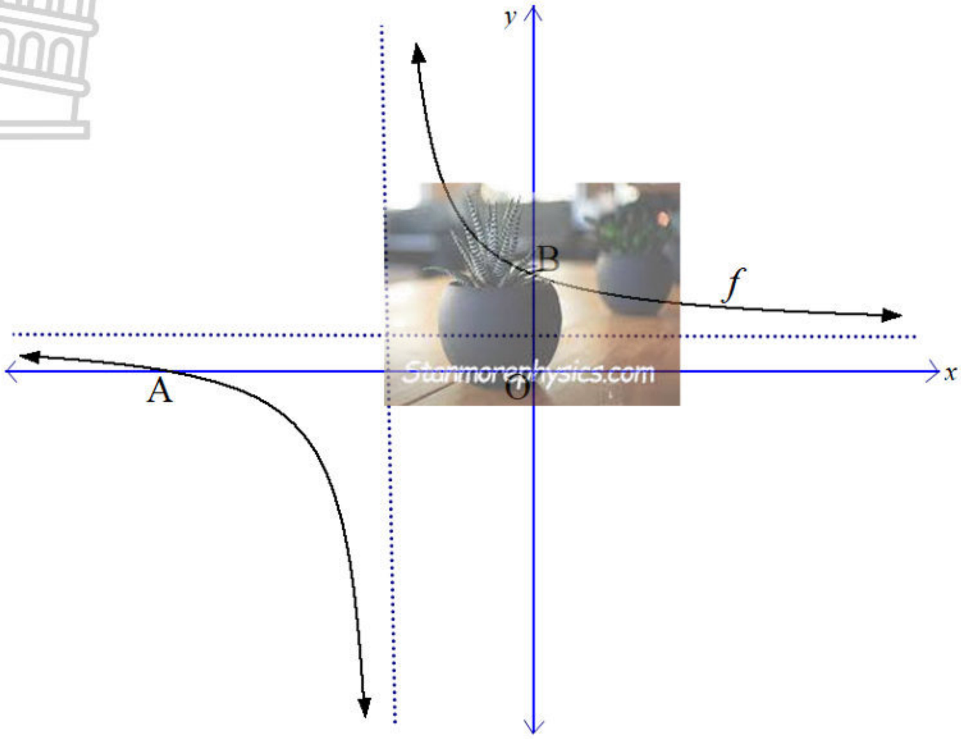
2.1.3 The given number pattern has two consecutive terms that are equal in value. Determine the positions of the two terms. (4)

2.2 Calculate: $\sum_{k=3}^{9} 2(-3)^k$ (4)

[14]

QUESTION 3

The diagram shows the graph of $f(x)=\frac{3}{x+2}+1$



3.1 Write down the equations of the asymptotes of f. (2)

3.2 Determine the coordinates of A. (2)

3.3 Determine the coordinates of B. (2)

3.4 The graph of g is formed by first reflecting f in the y-axis and then translating it upwards by 2 units. Determine the equation of g. (2)

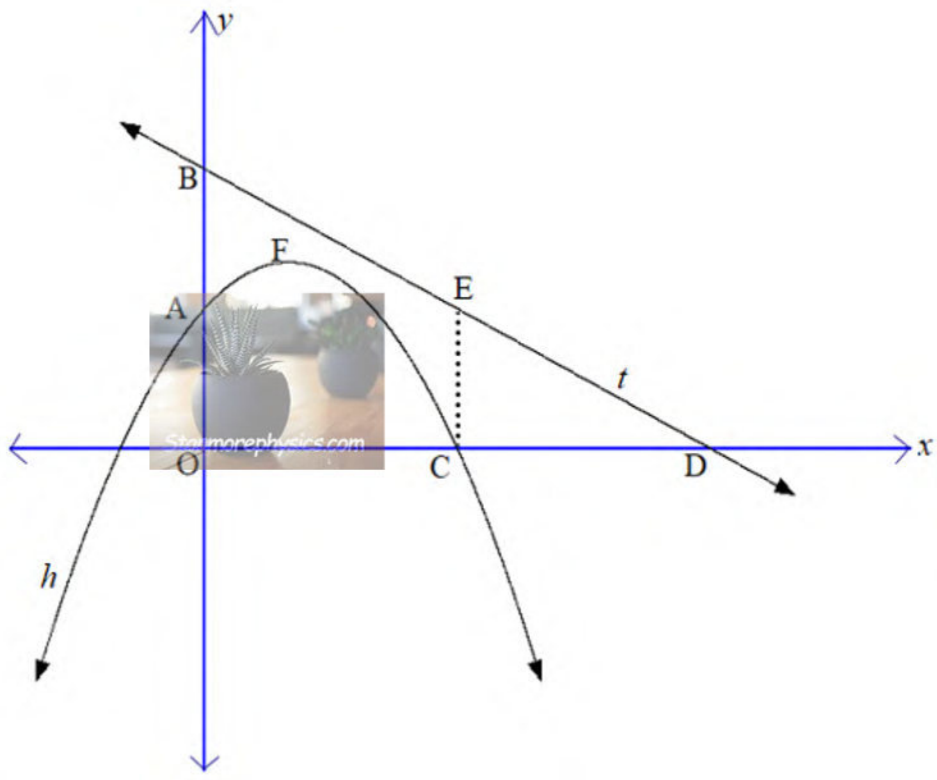
3.5 $y = x + c$ is the equation of one of the axes of symmetry of g. Determine the value of c. (2)

[10]

QUESTION 4

The sketch below shows the graphs of $h(x) = -x^2 + 2x + p$ and $t(x) = mx + 5$.

- D is the x-intercept, and B the y-intercept of t.
- A is the y-intercept of h.
- C is the x-intercept of h.
- F is the turning point of h.
- E is a point on t, such that EC is parallel to the y-axis.
- AB is 2 units and CD is 7 units.



4.1 Show that $p = 3$. (1)

4.2 Determine the coordinates of F. (3)

4.3 Determine the coordinates of C. (3)

4.4 Determine the length of EC. (5)

[12]

QUESTION 5

Given: $f(x)=3x^2$, where $x \geq 0$.

5.1 Determine the equation of f^{-1}. (3)

5.2 On the same set of axes, draw the graphs of f and f^{-1}, showing the intercepts with the axes as well as coordinates of two points on each graph. (4)

5.3 Determine the values of x for which $f(x)=f^{-1}(x)$. (4)

[11]

QUESTION 6

6.1 Given: $\tan x = \frac{3}{4}$, where $x \in [180°; 270°]$.

With the aid of a sketch, and **without the use of a calculator**, calculate:

6.1.1 $\sin x$ (3)

6.1.2 $2 - \sin 2x$ (3)

6.1.3 $\cos^2(90° - x) - 1$ (3)

6.2 Evaluate: $\dfrac{-1 + \cos(180° - \theta) \,.\, \sin(\theta - 90°)}{\cos(-\theta) \,.\, \sin(90° + \theta) \,.\, \tan^2(540° + \theta)}$ (7)

[16]

QUESTION 7

7.1 Given: $\cos(A - B) = \cos A \cos B + \sin A \sin B$

7.1.1 Use the above identity to deduce that $\sin(A+B) = \sin A \cos B + \cos A \sin B$. (3)

7.1.2 Hence determine the general solution of the equation $\sin(2x + 50°) - \sin 15° \cos 48° = \sin 48° \cos 15°$. (4)

7.2 Given: $\dfrac{4 \sin x \cos x}{2\sin^2 x - 1}$

7.2.1 Simplify $\dfrac{4 \sin x \cos x}{2\sin^2 x - 1}$ to a single trigonometric ratio. (3)

7.2.2 For which value(s) of x in the interval $-90° < x < 90°$ will the above expression be undefined? (3)

7.2.3 Without using a calculator, determine the value of $\dfrac{4\sin 15° \cos 15°}{2\sin^2 15° - 1}$. (2)

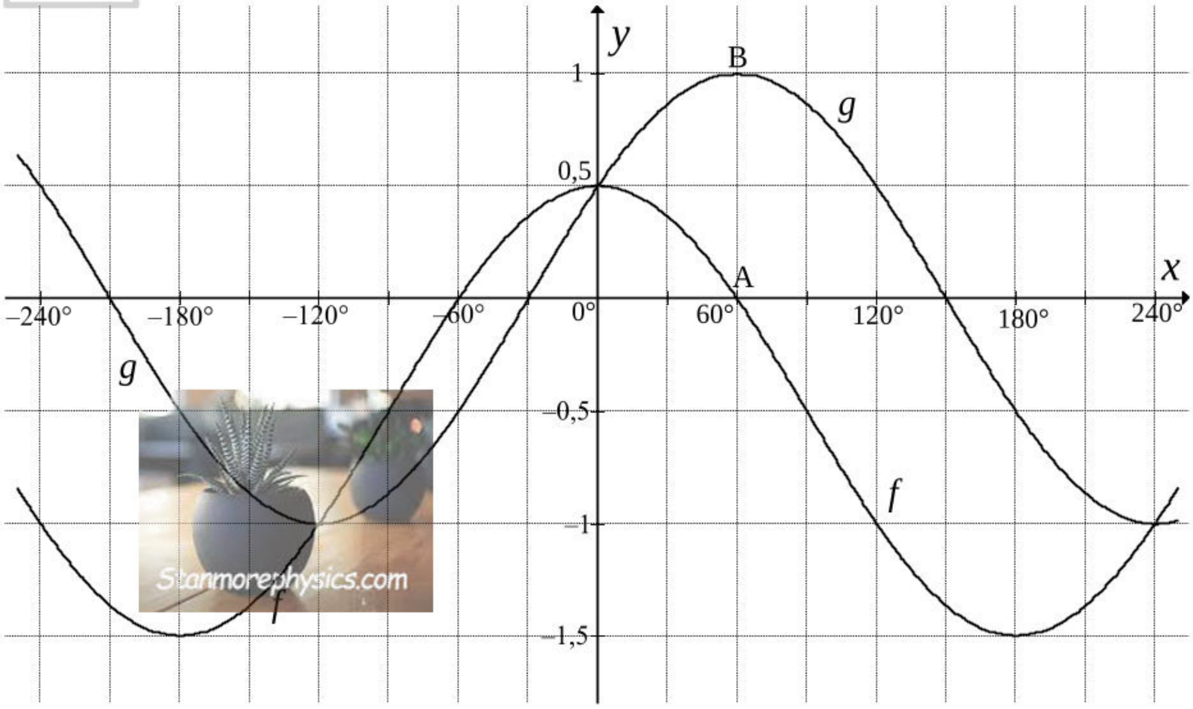
[15]

QUESTION 8

In the diagram below, the graphs of $f(x) = \cos x + m$ and $g(x) = \sin(x+n)$ are drawn on the same set of axes for $x \in [-240° ; 240°]$.

A is an x-intercept of f and has coordinates $(60° ; 0)$.

B is a turning point of g and has coordinates $(60° ; 1)$.



8.1	Determine the values of m and n.	(2)
8.2	Write down the amplitude of f.	(1)
8.3	If $h(x) = g(2x)$, write down the period of h.	(1)
8.4	For which values of x will $f(x).g(x) \leq 0$ in the interval $x \in [0°; 240°]$?	(2)
8.5	Describe the transformations that the graph of g has to undergo to form the graph of p, where $p(x) = -\cos x$.	(2)
		[8]

TOTAL: 100 marks

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\quad \frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta \mathrm{ABC} = \frac{1}{2} ab.\sin \mathrm{C}$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\sin\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

FINAL

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MATHEMATICS

MARKING GUIDELINES

COMMON TEST

MARCH 2024

MARKS: 100

These marking guidelines consist of 10 pages.

QUESTION 1			
1.1.1	8 ; 15 ; 22 ; ……….. $a=8;\quad d=7;\quad n=36$ $T_n=a+(n-1)d$ $T_{36}=8+(36-1)(7)$ $T_{36}=253$	✓A substitution ✓CA answer	(2)
1.1.2	$a=8;\quad d=7;\quad n=36$ $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$ $S_{36}=\frac{36}{2}\left[2(8)+(36-1)(7)\right]$ $S_{36}=4698$ **OR** $S_n=\frac{n}{2}(a+l)$ $S_{36}=\frac{36}{2}(8+253)$ $=4698$	✓CA substitution ✓CA answer **OR** ✓CA substitution ✓CA answer	(2)
1.1.3	$T_n=a+(n-1)d$ $T_{72}=8+(72-1)(7)=505$ $T_{72\text{-}m}=8+(72-m-1)(7)$ $T_{72\text{-}m}+T_{72}=786$ $505+8+497-72-m=786$ $-7m=-224$ $m=32$	✓CA value of T_{72} ✓CA Substitution in $T_{72\text{-}m}$ ✓CA Simplification ✓CA value of m	(4)
1.2.1	$81\times\frac{2}{3}=54cm$ The next jump is $54cm$	✓A answer	(1)
1.2.2	81 ; 54 ; 36 ; ……. $a=81;\quad r=\frac{2}{3};\quad n=9$ $T_n=ar^{n-1}$ $T_9=81\left(\frac{2}{3}\right)^8$ $=\frac{256}{81}=3{,}16\,cm$	✓A substitution ✓A answer	(2)

1.2.3	$S_\infty = \dfrac{a}{1-r}$ $S_\infty = \dfrac{81}{1-\frac{2}{3}}$ $= 243\text{cm}$ $243 \text{ cm} > 230\text{cm}$ Yes he will be able to catch the insect **OR** Showing that he can reach a distance > 230 through **substitution of a value of *n* that is ≥ 8** in the S_n formula.	✓A substitution ✓CA value of S_∞ ✓CA conclusion **OR** ✓A substitution ✓CA value of S_n ✓CA conclusion	(3)
			[14]

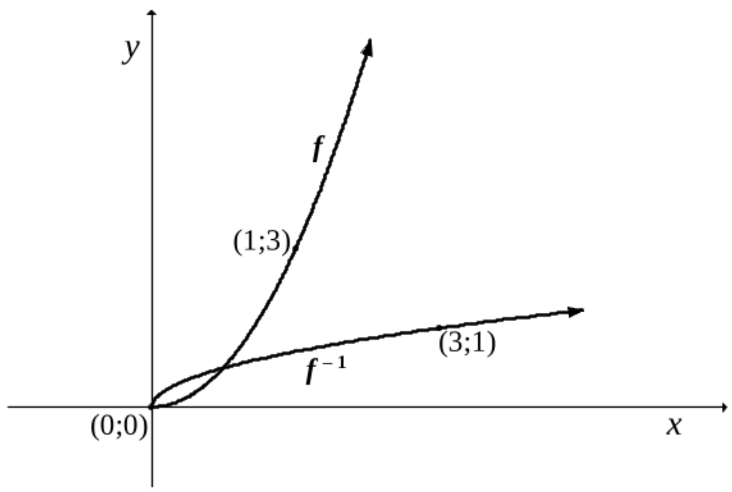
QUESTION 2			
2.1.1	16 ; 0 ; – 12 ; – 20 ; ……… – 16 ; – 12 ; – 8 ; ……. 4 ; 4 ; ………… $2a = 4$ $a = 2$ $-16 = 3a + b$ $-16 = 3(2) + b$ $b = -22$ $16 = 2 - 22 + c$ $c = 36$ $T_n = 2n^2 - 22n + 36$	✓A value of *a* ✓CA value of *b* ✓CA value of *c* ✓CA answer	(4)
2.1.2	32 ; 28 ; 24 ; 20 ; …… $a = 32$ $d = -4$ $T_n = a + (n-1)d$ $T_n = 32 + (n-1)(-4)$ $T_n = -4n + 36$ **Answer only: Full marks**	✓A substitution ✓CA answer	(2)
2.1.3	$2n^2 - 22n + 36 = -4n + 36$ $2n^2 - 18n = 0$ $n^2 - 9n = 0$ $n(n-9) = 0$ $n = 0$ or $n = 9$ $n = 9$ only $\therefore T_{17}$ and T_{18} are the terms	✓CA equating ✓CA factors ✓CA $n = 9$ only ✓CA answer	(4)

2.2	$T_1 = -54;\quad T_2 = 162;\quad T_3 = -486$ $a = -54;\quad r = -3;\quad n = 7$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $S_7 = \frac{-54((-3)^7 - 1)}{-3 - 1}$ $= -29\,538$ **Answer only: Full marks**	✓A $a = -54$ ✓A $r = -3$ ✓CA substitution ✓CA answer	(4)
			[14]

QUESTION 3			
3.1	$x = -2$ $y = 1$	✓A answer ✓A answer	(2)
3.2	$0 = \frac{3}{x+2} + 1$ $-1 = \frac{3}{x+2}$ $-x - 2 = 3$ $x = -5$ A$(-5\,;0)$	✓A substituting $y = 0$ ✓ A value of x	(2)
3.3	$y = \frac{3}{0+2} + 1$ $y = \frac{5}{2}$ B$\left(0\,;\frac{5}{2}\right)$ **Answer only: Full marks**	✓A substituting $x = 0$ ✓ A value of y	(2)
3.4	$g(x) = \frac{3}{-x+2} + 1 + 2$ $g(x) = \frac{-3}{x-2} + 3$ **OR** $g(x) = \frac{3}{-x+2} + 3$ **Answer only: Full marks**	✓A $-x$ ✓A answer **OR** ✓A✓A answer	(2)
3.5	$y = x + c$ $3 = 2 + c$ subst$(2;3)$ $c = 1$	✓A substituting $(2;3)$ ✓CA answer	(2)
			[10]

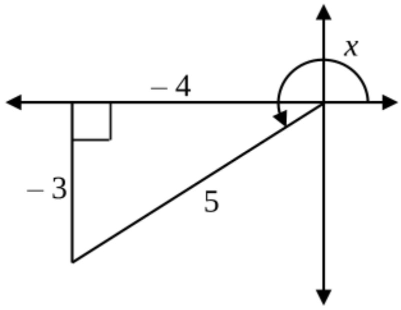
QUESTION 4			
4.1	$y = mx + 5$ B(0 ; 5): y-intercept OB = 5 units OA = OB – AB = 5 – 2 = 3 units A(0 ; 3) $f(x) = -x^2 + 2x + 3$ $p = 3$	✓A 5 – 2	(1)
4.2	$x = \frac{-b}{2a}$ $x = \frac{-2}{2(-1)}$ $= 1$ $y = -(1)^2 + 2(1) + 3 = 4$ F(1; 4) **OR** $f(x) = -\left[x^2 - 2x - 3\right]$ $f(x) = -\left[(x^2 - 2x + 1) - 3 - 1\right]$ $f(x) = -\left[(x-1)^2 - 4\right]$ $f(x) = -(x-1)^2 + 4$ F(1; 4)	✓A substitution ✓CA value of x ✓CA value of y **OR** ✓A completing the square ✓CA value of x ✓CA value of y	(3)
4.3	$f(x) = -x^2 + 2x + 3$ $0 = x^2 - 2x - 3$ $0 = (x-3)(x+1)$ $x = 3$ or -1 C(3 ; 0)	✓A equating to 0 ✓A factors ✓CA answer	(3)

4.4	C(3 ; 0) CD = 7 units OD = 7 + 3 = 10 units D(10 ; 0) $y=mx+5$ **or** $m=\frac{y_2-y_1}{x_2-x_1}$ $0=10m+5$ $\quad=\frac{0-10}{5-0}$ $m=-\frac{1}{2}$ $\quad=-\frac{1}{2}$ $y=-\frac{1}{2}x+5$ **or** $C\hat{D}E=180°-\angle$ of incl of DE At E: $x=3$ $\quad=26,57°$ $\therefore y=-\frac{1}{2}(3)+5=\frac{7}{2}$ $\quad CE=CD\times\tan 26,57°$ $\therefore CE=\frac{7}{2}-0=\frac{7}{2}$ units $\quad=7\times\tan 26,57°=\frac{7}{2}$	✓CA OD = 10 units ✓CA substituting D(10 ; 0) in $y=mx+5$ **OR** substituting in gradient formula ✓CA value of m ✓CA substituting $x=3$ in $y=-\frac{1}{2}x+5$ **OR** ✓CA $CE=CD\times\tan 26,57°$ ✓CA answer	(5)
			[12]

QUESTION 5			
5.1	$f: y=3x^2$ $f^{-1}: x=3y^2$ $y^2=\frac{1}{3}x$ $y=\pm\sqrt{\frac{1}{3}x}$ $y=\sqrt{\frac{1}{3}x}\ ;\ y\geq 0$	✓A swapping x and y ✓A $\pm\sqrt{\frac{1}{3}x}$ ✓A answer $y=\sqrt{\frac{1}{3}x}$	(3)
5.2		✓A ✓A shape of each graph ✓A coordinates of any two points on f ✓A coordinates of any two points on f^{-1}	(4)

5.3	$3x^2 = \sqrt{\frac{1}{3}x}$ $(3x^2)^2 = \left(\sqrt{\frac{1}{3}x}\right)^2$ $9x^4 = \frac{1}{3}x$ $27x^4 - x = 0$ $x(27x^3 - 1) = 0$ $x(3x-1)(9x^2+3x+1) = 0$ **or** $x = 0$ or $27x^3 = 1$ $x = 0$ or $x = \frac{1}{3}$ $x = 0$ $x = \frac{1}{3}$ **OR** $x = \sqrt{\frac{1}{3}x}$ $x^2 = \left(\sqrt{\frac{1}{3}x}\right)^2$ $x^2 = \frac{1}{3}x$ $3x^2 = x$ $3x^2 - x = 0$ $x(3x-1) = 0$ $x = 0$ or $x = \frac{1}{3}$ **OR** $3x^2 = x$ $3x^2 - x = 0$ $x(3x-1) = 0$ $x = 0$ or $x = \frac{1}{3}$	✓CA equating ✓CA squaring both sides ✓A $x = 0$ ✓CA $x = \frac{1}{3}$ **OR** ✓CA equating ✓CA squaring both sides ✓A $x = 0$ ✓CA $x = \frac{1}{3}$ **OR** ✓A equating ✓A factors ✓A $x = 0$ ✓CA $x = \frac{1}{3}$	(4)
			[11]

QUESTION 6

6.1.1	 $r^2 = (-4)^2 + (-3)^2 = 25$ $\therefore r = 5$ $\therefore \sin x = \dfrac{-3}{5}$	✓ A sketch ✓ A $r = 5$ ✓ CA answer	(3)
6.1.2	$2 - \sin 2x$ $= 2 - 2\sin x \cos x$ $= 2 - 2\left(\dfrac{-3}{5}\right)\left(\dfrac{-4}{5}\right)$ $= 2 - \dfrac{24}{25}$ $= \dfrac{26}{25}$	✓ A expansion ✓ CA substitution ✓ CA answer	(3)
6.1.3	$\cos^2(90° - x) - 1$ $= \sin^2 x - 1$ $= \left(\dfrac{-3}{5}\right)^2 - 1$ $= \dfrac{9}{25} - 1$ $= \dfrac{-16}{25}$	✓ A $\sin^2 x$ ✓ CA substitution ✓ CA answer	(3)
6.2	$\dfrac{-1 + \cos(180° - \theta) \,.\, \sin(\theta - 90°)}{\cos(-\theta) \,.\, \sin(90° + \theta) \,.\, \tan^2(540° + \theta)}$ $= \dfrac{-1 + (-\cos\theta) \,.\, -\cos\theta}{\cos\theta \,.\, \cos\theta \,.\tan^2(540° - 360° + \theta)}$ $= \dfrac{\cos^2\theta - 1}{\cos\theta \,.\, \tan^2(180° + \theta)}$ $= \dfrac{-(1 - \cos^2\theta)}{\cos^2\theta \,.\, \tan^2\theta}$ $= \dfrac{-\sin^2\theta}{\cos^2\theta \,.\, \dfrac{\sin^2\theta}{\cos^2\theta}}$ $= -1$	In numerator: ✓ A $-\cos\theta$, ✓ A $-\cos\theta$ In denominator: ✓ A $\cos\theta$, ✓ A $\cos\theta$ ✓ A $\tan^2\theta$ ✓ CA $-\sin^2\theta$ ✓ CA answer after simplification	(7)
			[16]

QUESTION 7

7.1.1	$\sin(A+B)$ $=\cos[90°-(A+B)]$ $=\cos(90°-A-B)$ $=\cos[(90°-A)-B]$ $=\cos(90°-A).\cos B+\sin(90°-A).\sin B$ $=\sin A.\cos B+\cos A.\sin B$	✓ A co-ratio ✓ A re-arrangement ✓ A expansion	(3)
7.1.2	$\sin(2x+50°)-\sin 15°\cos 48°=\sin 48°\cos 15°$ $\sin(2x+50°)=\sin 48°\cos 15°+\sin 15°\cos 48°$ $\sin(2x+50°)=\sin 63°$ $2x+50°=63°+k.360°$ OR $2x+50°=180°-63°+k.360°$ $2x=13°+k.360°$ $2x=67°+k.360°$ $x=6,5°+k.180°$, $k\in Z$ $x=33,5°+k.180°$, $k\in Z$	✓A using compound angle identity ✓ A both solutions ✓CA $x=6,5°+k.180°$ ✓CA $x=33,5°+k.180°$ $k\in Z$: penalty of 1 if not written at least once	(4)
7.2.1	$\frac{4\sin x\cos x}{2\sin^2 x-1}$ $=\frac{2(2\sin x\cos x)}{-(1-2\sin^2 x)}$ $=\frac{2\sin 2x}{-\cos 2x}$ $=-2\tan 2x$	✓ A 2 sin 2x ✓ A – cos x ✓ CA answer	(3)
7.2.2	$2\sin^2 x-1=0$ $\sin x=\pm\frac{1}{\sqrt{2}}$ $x=-45°$ or $x=45°$	✓ A $\pm\frac{1}{\sqrt{2}}$ ✓ CA –45° ; CA ✓ 45°	(3)
7.2.3	$\frac{4\sin 15°\cos 15°}{2\sin^2 15°-1}$ $=-2\tan 30°$ $=-2\left(\frac{1}{\sqrt{3}}\right)$ $=\frac{-2}{\sqrt{3}}$ or $\frac{-2\sqrt{3}}{3}$	✓ A – 2 tan 30° ✓ A answer	(2)
			[15]

QUESTION 8

8.1	$m=-\frac{1}{2}$ $n=30°$	✓ A $m=-\frac{1}{2}$ ✓ A $n=30°$	(2)
8.2	amplitude of f = 1	✓ A answer	(1)
8.3	period of h = 180°	✓ A answer	(1)
8.4	$60° \le x \le 150°$	✓ ✓ A A answer	(2)
8.5	translation of 60° to the left; and reflection in the x-axis	✓ A translation of 60° to the left ✓ A reflection in the x-axis	(2)
			[8]

GRAND TOTAL: 100