basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE
NASIONALE SENIOR SERTIFIKAAT

GRADE 12/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2017

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE:* *150*

These marking guidelines consist of 29 pages.
Hierdie nasienriglyne bestaan uit 28 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

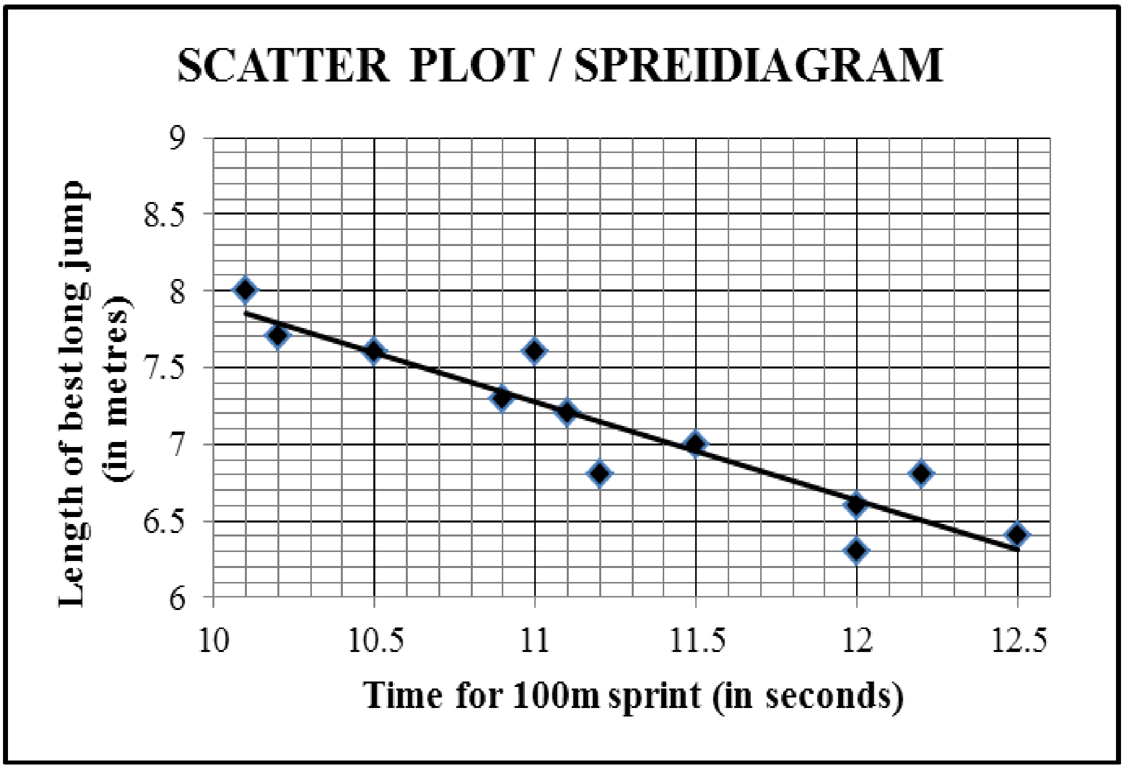
NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason.)**
	'n Punt vir 'n korrekte bewering **('n Punt vir 'n bewering is onafhanklik van die rede.)**
R	**A mark for a correct reason** **(A reason mark may only be awarded if the statement is correct.)**
	'n Punt vir 'n korrekte rede **('n Punt word slegs vir die rede toegeken as die bewering korrek is.)**
S/R	**Award a mark if the statement AND reason are both correct.**
	Ken 'n punt toe as beide die bewering EN rede korrek is.

QUESTION/*VRAAG* 1

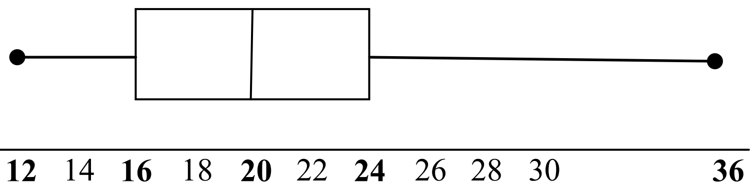
Time for 100 m sprint (in seconds) ***Tyd vir 100 m-naelloop (in sekondes)***	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres) ***Afstand van beste sprong in verspring (in meter)***	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4



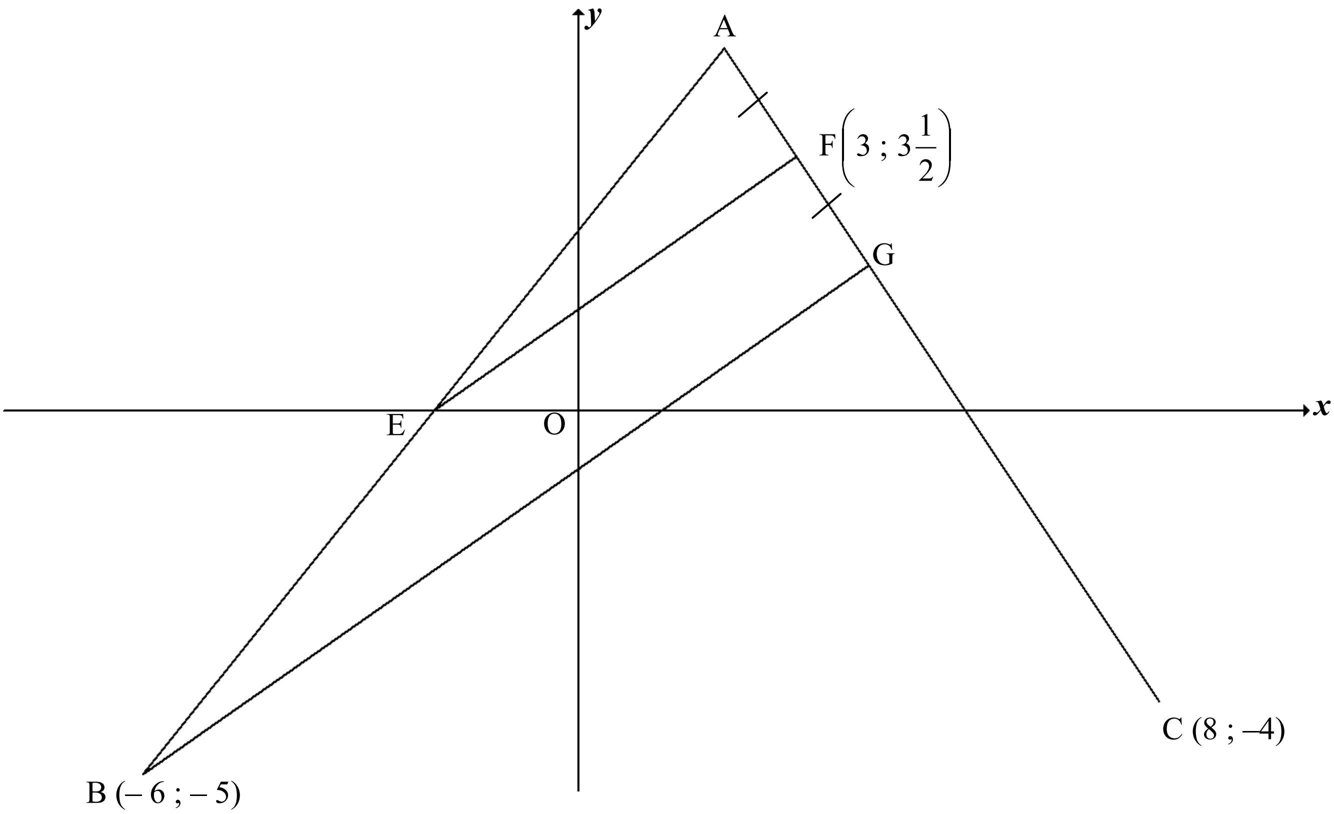
1.1	$a = 14{,}343\ldots = 14{,}34$ $b = -0{,}642\ldots = -0{,}64$	✓✓ value of a ✓ value of b (3)
1.2	$y = 14{,}34 - 0{,}64(11{,}7)$ $= 6{,}85$ **OR/OF** $y = 6{,}83$ (calculator / *sakrekenaar*)	✓ substitution correctly ✓ answer (2) ✓✓answer (2)
1.3	The gradient increases / *Die gradient neem toe* The point (12,3 ; 7,6) lies some distance above the current data. */Die punt (12,3 ; 7,6) lê bokant die huidige data.*	✓ increases/*neem toe* ✓ reasoning in words/ *redenasie in woorde* (2) **[7]**

QUESTION/*VRAAG* 2

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1.1	$\bar{x} = \frac{472}{23}$ $\bar{x} = 20{,}52$ seconds / *sekonde*	✓ $\frac{472}{23}$ ✓ answer (2)
2.1.2	$Q_1 = 16$ $Q_3 = 24$ IQR/*IKO* $= Q_3 - Q_1$ $= 24 - 16 = 8$	✓ Q_1 ✓ Q_3 ✓ answer (3)
2.2	$20{,}52 + 5{,}94 = 26{,}46$ $\therefore > 26{,}46$ $\therefore$ 4 girls/*dogters*	✓ 26,46 ✓ answer (2)
2.3	 	✓ whiskers ending at 12 & 36 ✓ $Q_1 = 16$ & $Q_3 = 24$ (box) ✓ $Q_2 = 20$ (3)
2.4.1	Girls / *Meisies*	✓ answer (1)
2.4.2	Five-number summary of boys: (15 ; 21 ; 23,5 ; 26 ; 38) **None** of the boys / ***Nie een** van die seuns **nie*** 5 girls completed in less than 15 seconds which was the minimum time taken by the boys. *5 meisies voltooi in minder as 15 sekondes, wat die minimumtyd is wat die seuns geneem het.*	✓ answer ✓ reason/*rede* (2) **[13]**

QUESTION/*VRAAG* 3



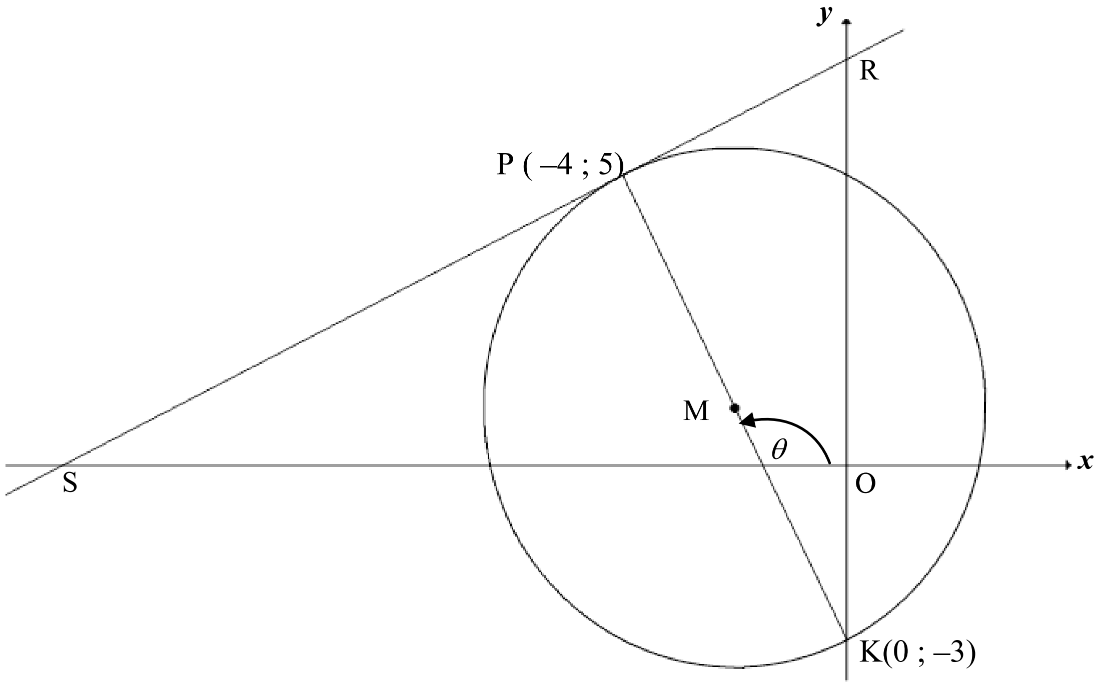
3.1.1	$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{3\frac{1}{2} - (-4)}{3 - 8}$ $= -\frac{3}{2}$ $y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $y = -\frac{3}{2}x + c$ $-4 = -\frac{3}{2}(8) + c$ **OR/*OF*** $(y - (-4)) = -\frac{3}{2}(x - 8)$ $c = 8$ $\quad$ $y + 4 = -\frac{3}{2}x + 12$ $y = -\frac{3}{2}x + 8$ $\quad$ $y = -\frac{3}{2}x + 8$ **OR/*OF***	✓ substitution of (8 ; –4) & $\left(3 ; 3\frac{1}{2}\right)$ ✓ gradient ✓ substitution of *m* and (8 ; –4) ✓ equation of AC (4)

	$m_{FC}=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{(-4)-\left(3\frac{1}{2}\right)}{8-3}$ $=-\dfrac{3}{2}$ $y=mx+c$ $\quad$ $y-y_1=m(x-x_1)$ $3\frac{1}{2}=-\frac{3}{2}(3)+c$ $\quad$ $\left(y-3\frac{1}{2}\right)=-\frac{3}{2}(x-3)$ $c=8$ **OR/*OF*** $\left(y-3\frac{1}{2}\right)=-\frac{3}{2}x+\frac{9}{2}$ $y=-\frac{3}{2}x+8$ $\quad$ $y=-\frac{3}{2}x+8$	✓ substitution of (8 ; –4) & $\left(3;3\frac{1}{2}\right)$ ✓ gradient ✓ substitution of m and $\left(3;3\frac{1}{2}\right)$ ✓ equation of AC (4)
3.1.2	AC: $3x+2y=16$ and BG: $7x-10y=8$ $15x+10y=80$ $\underline{7x-10y=8}$ $22x=88$ $x=4$ $3(4)+2y=16$ $y=2$ $\therefore G(4;2)$ **OR/*OF*** BG: $7x-10y=8$ $\quad \therefore y=\frac{7}{10}x-\frac{8}{10}$ $\therefore \frac{7}{10}x-\frac{8}{10}=-\frac{3}{2}x+8$ [CA from 3.1.1] $\frac{11}{5}x=\frac{44}{5}$ $x=4$ $3(4)+2y=16$ $y=2$ $\therefore G(4;2)$	✓ method /*metode*: solving simultaneously / *los gelyktydig op* ✓ x coordinate ($x>0$) ✓ y coordinate (3) ✓ method: equating *metode: stel vgls gelyk* ✓ x coordinate ($x>0$) ✓ y coordinate (3)
3.2	$\frac{x_A+4}{2}=3$ and $\frac{y_A+2}{2}=3\frac{1}{2}$ $\therefore A(2;5)$ **OR/*OF*** by translation/*deur translasie*: $x_A=3-(4-3)=2$ $y_A=3\frac{1}{2}+(3\frac{1}{2}-2)=5$ $\therefore A(2;5)$	✓ equation ito x ✓ equation ito y (2) ✓ equation ito x ✓ equation ito y (2)

3.3	The coordinates of the midpt of AB / *Die koordinaat van midpt van AB is*: $\left(\frac{2+(-6)}{2};\frac{5+(-5)}{2}\right)=(-2\ ;\ 0)$ **But the *y*-coordinate of E is 0** **∴ E(–2 ; 0) is the midpoint of AB** ∴ EF ‖ BG [midpoint theorem/*middelpuntst* ***OR/OF*** line divides 2 sides of Δ in prop/*lyn verdeel 2 sye van Δ in dies verh*]	✓ subst A & B into midpt formula ✓ *y* coordinate = 0 ✓ E = midpt ✓ Reason (4)
	OR/*OF* The coordinates of the midpt of AB / *Die koordinaat van midpt van AB is*: $\left(\frac{2+(-6)}{2};\frac{5+(-5)}{2}\right)=(-2\ ;\ 0)$ $AE=\sqrt{(-2-2)^2+(0-5)^2}=\sqrt{41}$ $EB=\sqrt{(-2-(-6))^2+(0-(-5))^2}=\sqrt{41}$ ∴ In ΔABG: **AE = EB** and AF = FG ∴ EF ‖ BG [midpoint theorem/*middelpuntst*]	✓ subst A & B into midpt formula ✓ lengths of AE & EB ✓ AE = EB or E = midpt ✓ Reason (4)
	OR/*OF* Equation of AB: $y-(-5)=\left(\frac{5-(-5)}{2-(-6)}\right)(x-(-6))$ $y+5=\frac{10}{8}x+\frac{15}{2}$ $\therefore y=\frac{5}{4}x+\frac{5}{2}$ x-intercept of AB: $0=\frac{5}{4}x+\frac{5}{2}$ $\therefore x=-2$ ∴ E(–2 ; 0) $m_{EF}=\frac{3\frac{1}{2}-0}{3-(-2)}=\frac{7}{10}$ $m_{EF}=m_{BG}=\frac{7}{10}$ ∴ EF ‖ BG BG: $7x-10y=8$ $\therefore\ y=\frac{7}{10}x-\frac{8}{10}$ $\therefore\ m_{BG}=\frac{7}{10}$	✓ equation of AB ✓ coordinates of E ✓ gradient of EF ✓ gradient EF = gradient BG (4)

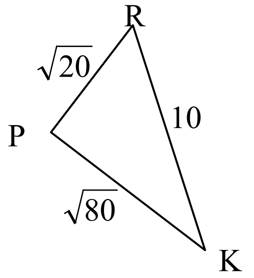
3.4	Midpoint of AC = $\left(5\ ;\ \frac{1}{2}\right)$ $\frac{x_D + (-6)}{2} = 5$ and $\frac{y_D + (-5)}{2} = \frac{1}{2}$ $\therefore$ D(16 ; 6)	✓✓ $\left(5\ ;\ \frac{1}{2}\right)$ ✓ x value ✓ y value (4)
	OR/*OF* by translation/*dmv translasie*: D(16 ; 6)	✓ method finding x ✓ method finding y ✓ x value ✓ y value (4)
	OR/*OF* $m_{BC} = \frac{-5-(-4)}{-6-8} = \frac{1}{14}$ and $m_{AB} = \frac{5-(-5)}{2-(-6)} = \frac{5}{4}$ AD: $y - 5 = \frac{1}{14}(x-2) \Rightarrow y = \frac{1}{14}x + \frac{34}{7}$ CD: $y + 4 = \frac{5}{4}(x-8) \Rightarrow y = \frac{5}{4}x - 14$ $\frac{5}{4}x - 14 = \frac{1}{14}x + \frac{34}{7}$ $\therefore \quad x = 16$ $y = 6$	✓✓ equating ✓ x value ✓ y value (4) **[17]**

QUESTION/*VRAAG* 4



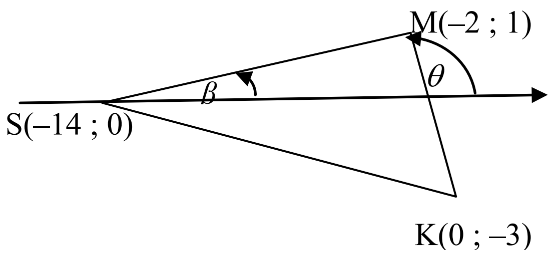
4.1.1	$m_{PK} = \frac{5-(-3)}{-4-0}$ $= -2$ PK $\perp$ SR [radius $\perp$ tangent/*raaklyn*] $\therefore\ m_{PK} \times m_{RS} = -1$ $\therefore m_{RS} = \frac{1}{2}$	✓ substitution P & K into gradient formula ✓ gradient of PK ✓ PK $\perp$ SR **OR** r $\perp$ tangent ✓ answer (4)
4.1.2	$y = \frac{1}{2}x + c$ $5 = \frac{1}{2}(-4) + c$ **OR/*OF*** $(y-5) = \frac{1}{2}(x-(-4))$ $c = 7$ $(y-5) = \frac{1}{2}x + 2$ $y = \frac{1}{2}x + 7$ $y = \frac{1}{2}x + 7$	✓ substitution of *m* and P ✓ equation (2)

4.1.3	$M\left(\frac{-4+0}{2}\ ;\ \frac{5+(-3)}{2}\right)$ $\therefore M(-2\ ;\ 1)$ $r^2=(x_2-x_1)^2+(y_2-y_1)^2$ $r^2=(-2+4)^2+(1-5)^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20\ or\ (\sqrt{20})^2$	✓ x value of M ✓ y value of M ✓ $r^2=20$ ✓ equation (4)
	OR/*OF*	
	$M\left(\frac{-4+0}{2}\ ;\ \frac{5+(-3)}{2}\right)\quad \therefore M(-2\ ;\ 1)$ $(x+2)^2+(y-1)^2=r^2$ $(-4+2)^2+(5-1)^2=r^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20\ or\ (\sqrt{20})^2$	✓✓ M (– 2 ; 1) $r^2=20$ ✓ equation (4)
	OR/*OF*	
	$M\left(\frac{-4+0}{2}\ ;\ \frac{5+(-3)}{2}\right)\quad \therefore M(-2\ ;\ 1)$ $PK=\sqrt{(-4-0)^2+(5-(-3))^2}=\sqrt{80}$ $r=\frac{\sqrt{80}}{2}=\sqrt{20}$ $\therefore (x+2)^2+(y-1)^2=20\ or\ (\sqrt{20})^2$	✓✓ M (– 2 ; 1) $r^2=20$ ✓ equation (4)

4.1.4	$\tan\theta = m_{PK} = -2$ $\therefore\ \theta = 180° - 63{,}43°$ $= 116{,}57°$ $P\hat{K}R = 116{,}57° - 90°$ [ext $\angle$ of ΔMOK] $= 26{,}57°$	✓ $\tan\theta = -2$ ✓ size of θ ✓ answer (3)
	OR/*OF* 	
	In ΔRPK: $PK = \sqrt{(0-(-4))^2 + (-3-5)^2} = \sqrt{80}$ $PR = \sqrt{(-4-0)^2 + (5-7)^2} = \sqrt{20}$ RK = 10	✓ lengths of PK, PR & RK
	$\cos P\hat{K}R = \frac{PK^2 + KR^2 - PR^2}{2.PK.KR} = \frac{(\sqrt{80})^2 + (10)^2 - (\sqrt{20})^2}{2(\sqrt{80})(10)}$ $= \frac{2\sqrt{5}}{5}$ $P\hat{K}R = 26{,}57°$	✓ correct values into cos rule ✓ answer (3)
	OR/*OF* $\sin P\hat{K}R = \frac{\sqrt{20}}{10}$ **OR/*OF*** $\cos P\hat{K}R = \frac{\sqrt{80}}{10}$ $P\hat{K}R = 26{,}57°$ $P\hat{K}R = 26{,}57°$	✓ lengths of sides ✓ ratio ✓ answer (3)
	OR/*OF* $\tan P\hat{K}R = \frac{\sqrt{20}}{\sqrt{80}}$ $P\hat{K}R = 26{,}57°$	✓ lengths of sides ✓ ratio ✓ answer (3)

4.1.5	RS ∥ tangent at K(0 ; −3) $\therefore m_{\text{PS}} = m_{\text{tang}} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$ **OR/*OF*** $m_{PK} = \frac{1-5}{-2+4} = -2$ $m_{PK} \times m_{\tan g} = -1$ [radius ⊥ tangent/*raaklyn*] $\therefore m_{\tan g} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$	✓ gradient ✓ equation (2) ✓ gradient ✓ equation (2)
4.2	$t \in (-3 \; ; \; 7)$ **OR/*OF*** $-3 < t < 7$	✓ −3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3) ✓ −3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3)
4.3	RS: $y = \frac{1}{2}x + 7$ ∴ S(–14 ; 0) $\text{SP} = \sqrt{(-14-(-4))^2 + (0-5)^2} = \sqrt{100+25} = \sqrt{125}$ Area ΔSMK $= \frac{1}{2}$. MK . SP $= \frac{1}{2}(\sqrt{20})(\sqrt{125})$ = 25 square units	✓ coordinates of S ✓ length of SP ✓ correct base & height into Area rule ✓ correct substitution ✓ answer (5)

OR/OF



Let β = inclination of SM/ *inklinasie van SM*

RS: $y = \frac{1}{2}x + 7 \quad \therefore$ S(–14 ; 0) — ✓ coordinates of S

$SM = \sqrt{(-14-(-2))^2 + (0-1)^2} = \sqrt{145}$ — ✓ length of SM

$\tan\beta = \frac{1-0}{-2-(-14)} = \frac{1}{12} \quad \therefore \beta = 4{,}76°$

$\therefore S\hat{M}K = 116{,}57° - 4{,}76°$ [ext ∠ of Δ]

$= 111{,}81°$ — ✓ size of/*grootte v* $S\hat{M}K$

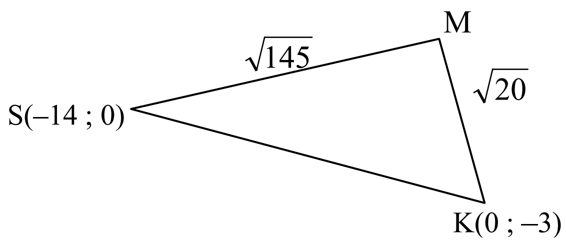
$\text{Area } \Delta SMK = \frac{1}{2}(SM)(MK).\sin S\hat{M}K$

$= \frac{1}{2}(\sqrt{145})(\sqrt{20}).\sin 111{,}81°$ — ✓ correct substitution into area rule

$= 24{,}9985 = 25$ square units — ✓ answer

(5)

OR/*OF*



RS: $y = \frac{1}{2}x + 7 \quad \therefore$ S(–14 ; 0) — ✓ coordinates of S

$SK = \sqrt{(-14-0)^2 + (0+3)^2} = \sqrt{205}$ — ✓ length of SK

$\cos S\hat{M}K = \frac{(\sqrt{145})^2 + (\sqrt{20})^2 - (\sqrt{205})^2}{2(\sqrt{145})(\sqrt{20})} = -\frac{2\sqrt{29}}{29}$

$S\hat{M}K = 111{,}80°$ — ✓ size of /*grootte v* $S\hat{M}K$

$\text{Area } \Delta SMK = \frac{1}{2}(SM)(MK).\sin S\hat{M}K$

$= \frac{1}{2}(\sqrt{145})(\sqrt{20}).\sin 111{,}81°$ — ✓ correct substitution into area rule

$= 24{,}9985 = 25$ square units — ✓ answer

(5)

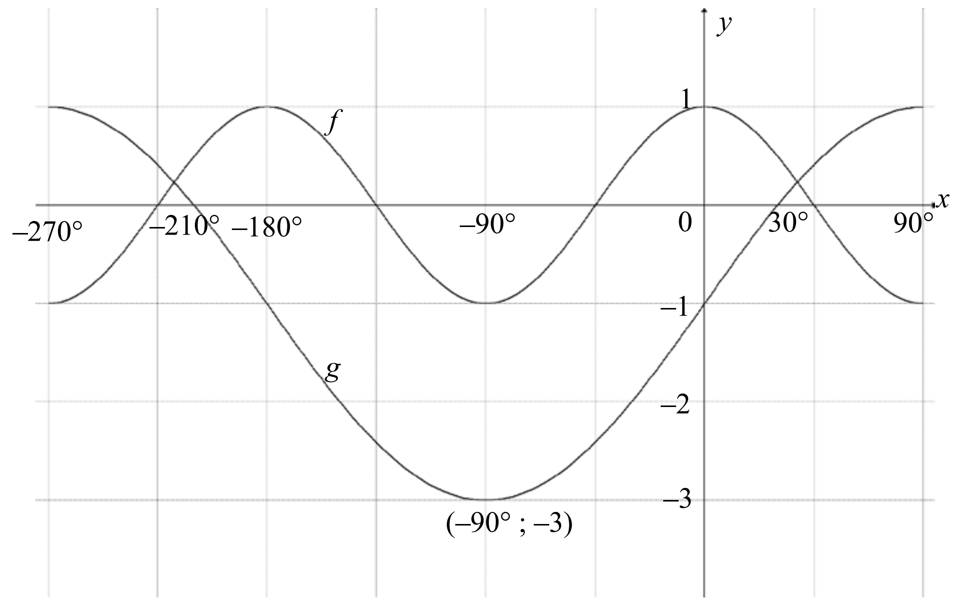
	OR/*OF* Produce KS to T RS: $y=\frac{1}{2}x+7 \quad \therefore$ S(–14 ; 0) $SK=\sqrt{(-14-0)^2+(0+3)^2}=\sqrt{205}$ $SM=\sqrt{(-14-(-2))^2+(0-1)^2}=\sqrt{145}$ $m_{SK}=-\frac{3}{14} \Rightarrow T\hat{S}O=167{,}91°$ $m_{SM}=\frac{1}{12} \Rightarrow M\hat{S}O=4{,}76°$ $M\hat{S}K=180°-167{,}91°+4{,}76°=16{,}85°$ Area $\Delta SMK=\frac{1}{2}(SM)(SK).\sin M\hat{S}K$ $=\frac{1}{2}(\sqrt{145})(\sqrt{205}).\sin 16{,}85°$ $=24{,}9985=25$ square units	✓ coordinates of S ✓ length of SK & SM ✓ size of /*grootte v* $M\hat{S}K$ ✓ correct substitution into area rule ✓ answer (5)

QUESTION/*VRAAG* 5

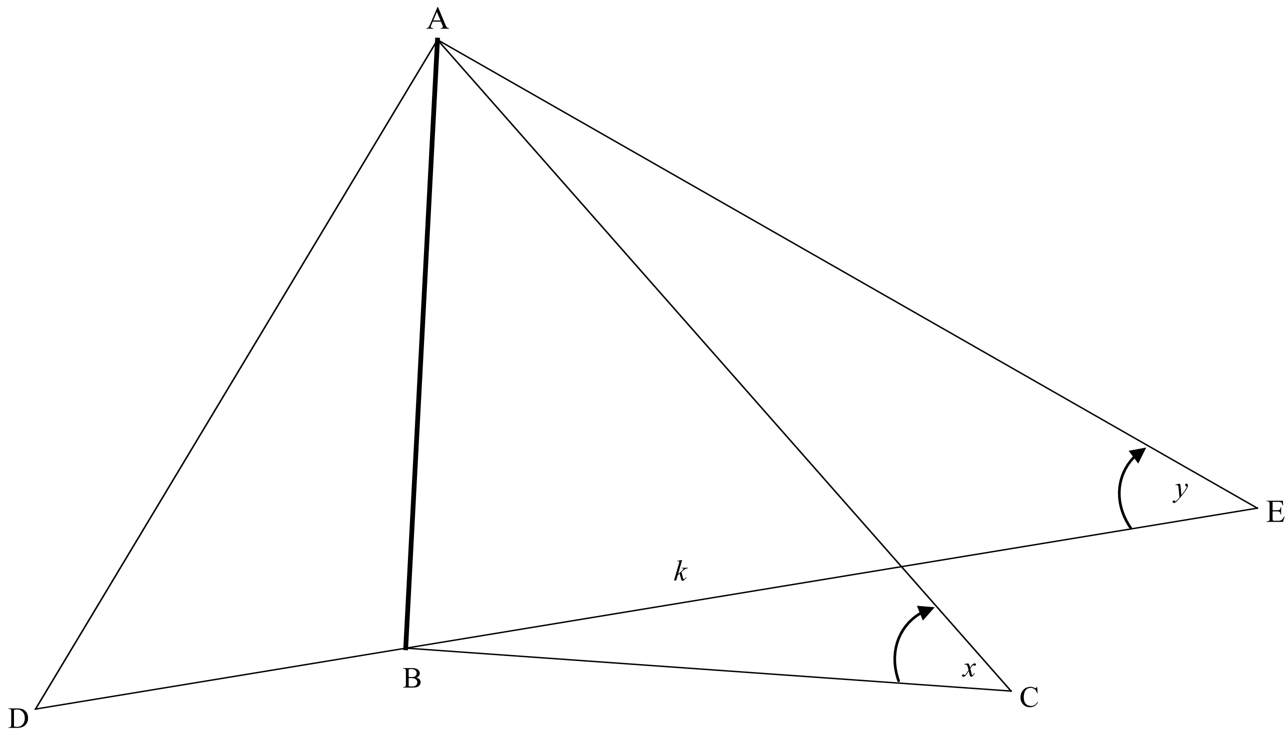
5.1	$\frac{\sin(A-360°).\cos(90°+A)}{\cos(90°-A).\tan(-A)}$ $=\frac{\sin A(-\sin A)}{\sin A(-\tan A)}$ $=\frac{\sin A}{\left(\frac{\sin A}{\cos A}\right)}$ $=\cos A$	✓ sin A ✓–sin A ✓ sin A ✓ –tan A ✓ $\tan A=\frac{\sin A}{\cos A}$ ✓ answer (6)
5.2.1	$t^2=(\sqrt{34})^2-(3)^2$ $\therefore t=-5$	✓ substitution ✓ answer (2)
5.2.2	$\tan\beta=\frac{-5}{3}$	✓ correct ratio (1)
5.2.3	$\cos 2\beta = 2\cos^2\beta-1$ $=2\left(\frac{3}{\sqrt{34}}\right)^2-1$ $=2\left(\frac{9}{34}\right)-1$ $=-\frac{16}{34}$ OR $-\frac{8}{17}$	✓ compound formula ✓ substitution ✓ simplification ✓ answer (4)
	OR/*OF* $\cos 2\beta=1-2\sin^2\beta$ $=1-2\left(-\frac{5}{\sqrt{34}}\right)^2$ $=1-2\left(\frac{25}{34}\right)$ $=-\frac{16}{34}$ OR $-\frac{8}{17}$	✓ compound formula ✓ substitution ✓ simplification ✓ answer (4)
	OR/*OF* $\cos 2\beta=\cos^2\beta-\sin^2\beta$ $=\left(\frac{3}{\sqrt{34}}\right)^2-\left(-\frac{5}{\sqrt{34}}\right)^2$ $=\frac{9}{34}-\frac{25}{34}$ $=-\frac{16}{34}$ OR $-\frac{8}{17}$	✓ compound formula ✓ substitution ✓ simplification ✓ answer (4)

5.3.1	LHS = sin(A + B) – sin(A – B) = sin A.cos B + cos A.sin B – (sin A. cos B – cos A. sin B) = sin A.cos B + cos A.sin B – sin A. cos B + cos A. sin B = 2cos A.sin B = RHS	✓ compound formula ✓ compound formula (2)
5.3.2	$\sin 77° - \sin 43° = \sin(60° + 17°) - \sin(60° - 17°)$ $= 2\cos 60°.\sin 17°$ $= 2 \times \frac{1}{2} \times \sin 17°$ $= \sin 17°$ **OR/*OF*** $\sin 77° - \sin 43° = \sin(60° + 17°) - \sin(60° - 17°)$ $= (\sin 60° \cos 17° + \cos 60° \sin 17°) -$ $(\sin 60° \cos 17° - \cos 60° \sin 17°)$ $= \frac{\sqrt{3}}{2}\cos 17° + \frac{1}{2}\sin 17° - \frac{\sqrt{3}}{2}\cos 17° + \frac{1}{2}\sin 17°$ $= \sin 17°$	✓ 60° + 17° ✓ 60° – 17° ✓ simplify ✓ $\frac{1}{2}$ (4) ✓ 60° + 17° ✓ 60° – 17° ✓ expansion ✓ $\frac{1}{2}$ (4) **[19]**

QUESTION/*VRAAG* 6

6.1		✓ (–90° ; –3) ✓ (0 ; –1) ✓ x – intercepts: –210° & 30° ✓ shape (4)
6.2	$\cos 2x = 2\sin x - 1$ $1 - 2\sin^2 x = 2\sin x - 1$ $2\sin^2 x + 2\sin x - 2 = 0$ $\sin^2 x + \sin x - 1 = 0$ $\sin x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$ $\sin x = \dfrac{-1+\sqrt{5}}{2}$, since $\sin x = \dfrac{-1-\sqrt{5}}{2} < -1$ has no solution	✓ $\cos 2x = 1 - 2\sin^2 x$ ✓ standard form ✓ using quadratic formula ✓ substitution into quadratic formula (4)
6.3	$\sin x = \dfrac{-1+\sqrt{5}}{2} = 0{,}618...$ Reference $\angle = 38{,}17°$ $\therefore x = 38{,}17° + k.360°$ or $x = 141{,}83° + k.360°$; $k \in Z$ $\therefore x = 38{,}17°$ or $-218{,}17°$ $y = 0{,}24$ $\therefore$ Points of intersection/*snypunte*: $(38{,}17° ; 0{,}24)$ and $(-218{,}17° ; 0{,}24)$	✓ 38,17° ✓ 141,83° ✓ –218,17° ✓ 0,24 (4) **[12]**

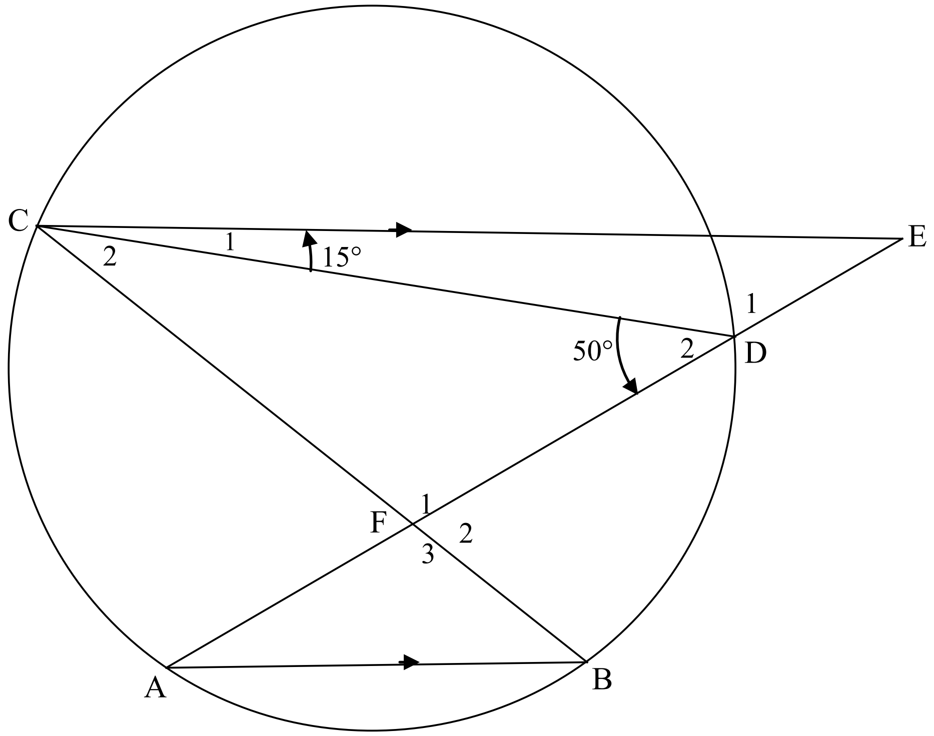
QUESTION/*VRAAG* 7



7.1	$A\hat{B}C = 90°$	✓ answer (1)
7.2	In Δ ABE: $\frac{AB}{BE} = \tan y$ $AB = k \tan y$ In Δ ABC**:** $\frac{AB}{AC} = \sin x$ $AC = \frac{AB}{\sin x}$ $= \frac{k \tan y}{\sin x}$	✓ correct ratio ✓ value AB ✓ correct ratio ✓ AC as subject and substitution (4)

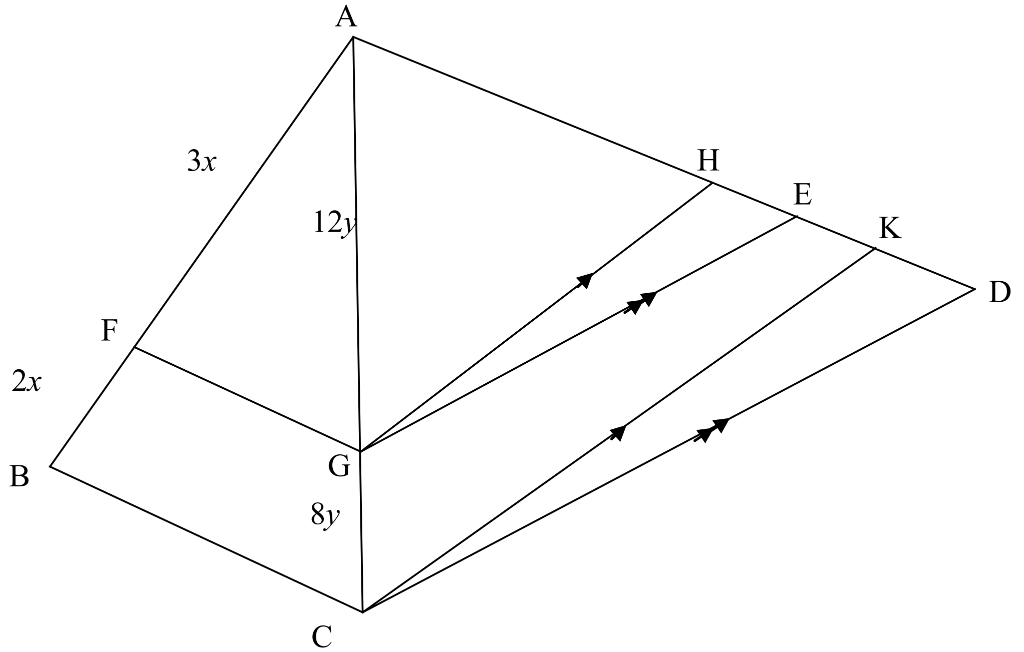
7.3	$\hat{ADC} = \hat{ACD} = \frac{180° - 2x}{2} = 90° - x$ $\frac{DC}{\sin 2x} = \frac{AC}{\sin(90° - x)}$ $\frac{DC}{2\sin x \cos x} = \frac{AC}{\cos x}$ $DC = \frac{AC(2\sin x \cos x)}{\cos x}$ $= \frac{k \tan y}{\sin x} . \frac{2\sin x \cos x}{\cos x}$ $= 2k \tan y$	✓ $90° - x$ ✓subst into sine rule ✓ $2\sin x \cos x$ ✓ $\cos x$ ✓ substitution (5)
	OR/*OF* $DC^2 = AD^2 + AC^2 - 2AD.AC\cos 2x$ $= AC^2 + AC^2 - 2AC^2 \cos 2x$ $= 2AC^2(1 - \cos 2x)$ $= 2AC^2(1 - 1 + \sin^2 x)$ $= 4AC^2 \sin^2 x$ $DC = 2AC.\sin x$ $= 2\left(\frac{k.\tan y}{\sin x}\right).\sin x$ $= 2k.\tan y$	✓substitution into cos rule ✓ factorisation ✓ $1 - 2\sin^2 x$ ✓ DC ito AC and sin x ✓ substitution (5)
	OR/*OF* $DC^2 = AD^2 + AC^2 - 2AD.AC\cos 2x$ $= 2\left(\frac{k \tan y}{\sin x}\right)^2 - 2\left(\frac{k \tan y}{\sin x}\right)^2 \cos 2x$ $= \frac{2k^2 \tan^2 y}{\sin^2 x} - \frac{2k^2 \tan^2 y}{\sin^2 x}(1 - 2\sin^2 x)$ $= \frac{2k^2 \tan^2 y}{\sin^2 x} - \frac{2k^2 \tan^2 y}{\sin^2 x} + 4k^2 \tan^2 y$ $DC = \sqrt{4k^2 \tan^2 y}$ $= 2k \tan y$	✓correct cos rule ✓ substitution ✓ $1 - 2\sin^2 x$ ✓ squaring and multiplication ✓ $\sqrt{4k^2 \tan^2 y}$ (5) **[10]**

QUESTION/*VRAAG* 8



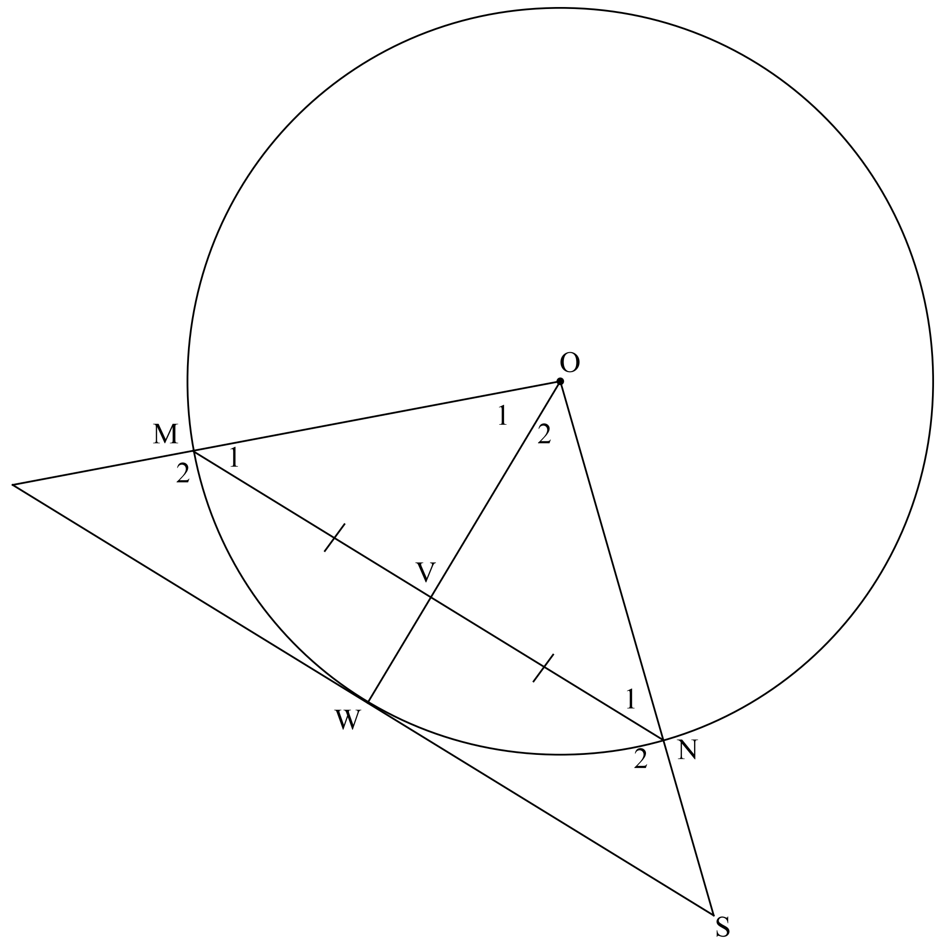
8.1.1	$\hat{E} = 50° - 15° = 35°$ [ext ∠ of Δ/*buite ∠ van Δ*] $\hat{A} = 35°$ [alt ∠s / *verwiss ∠e*; CE ‖ AB] **OR/*OF*** $\hat{E} = 180° - (130° + 15°) = 35°$ [str line; ∠s of Δ/*rt lyn;∠e van Δ*] $\hat{A} = 35°$ [alt ∠s / *verwiss ∠e*; CE ‖ AB] **OR/*OF*** $\hat{B} = 50°$ [∠s in same segment/*∠e in dieselfde segment*] $\hat{C}_2 + 15° = 50°$ [alt ∠s / *verwiss ∠e*; CE ‖ AB] $\therefore \hat{C}_2 = 35°$ $\hat{A} = 35°$ [∠s in same segment/*∠e in dieselfde segment*]	✓ S ✓S ✓ R (3) ✓ S ✓S ✓ R (3) ✓ S ✓S ✓ R (3)
8.1.2	$\hat{C}_2 = 35°$ [∠s in same segment/*∠e in dieselfde segment*]	✓ S ✓ R (2)
8.2	$\hat{C}_2 = \hat{E}$ [from 8.1.1 and 8.1.2] ∴ CF is a tangent to the circle [converse tan chord theorem] *∴ CF is 'n raaklyn aan die sirkel [omgekeerde raakl koordst]*	✓ S ✓ R (2) **[7]**

QUESTION/*VRAAG* 9



9.1.1	$\frac{AF}{BF}=\frac{3x}{2x}=\frac{3}{2}$ & $\frac{AG}{CG}=\frac{12y}{8y}=\frac{3}{2}$ $\therefore \frac{AF}{BF}=\frac{AG}{CG}$ $\therefore$ FG ‖ BC [conv prop th/*omg eweredigh st.* **OR** line divides 2 sides of Δ in prop/*lyn verdeel 2 sye v Δ in dies verh*]	✓ $\frac{AF}{BF}=\frac{AG}{CG}$ ✓R (2)
9.1.2	$\frac{AG}{GC}=\frac{AH}{HK}$ [prop theorem/*eweredigh st*; <u>GH ‖ CK</u> **OR** line ‖ to 1 side of Δ/*lyn ‖ 1 sy van Δ*] $\frac{AG}{GC}=\frac{AE}{ED}$ [prop theorem/*eweredigh st*; <u>GE ‖ CD</u>] $\therefore \frac{AH}{HK}=\frac{AE}{ED}$	✓S ✓ R ✓ S (3)
9.2	$\frac{AE}{ED}=\frac{3}{2}$ and $\frac{AH}{HK}=\frac{3}{2}$ $\frac{AE}{12}=\frac{3}{2}$ and $\frac{15}{HK}=\frac{3}{2}$ $\therefore$ AE = 18 and HK = 10 $\therefore$ HE = AE – AH = 18 – 15 = 3 $\therefore$ EK = HK – HE = 10 – 3 = 7 **OR/*OF*** AD = 30 KD = AD – AH – HK = 30 – 15 – 10 = 5 EK = ED – KD = 12 – 5 = 7	✓ use of ratios ✓ AE = 18 ✓ HK = 10 ✓ HE = 3 **or** KD = 5 ✓ EK = 7 (5) **[10]**

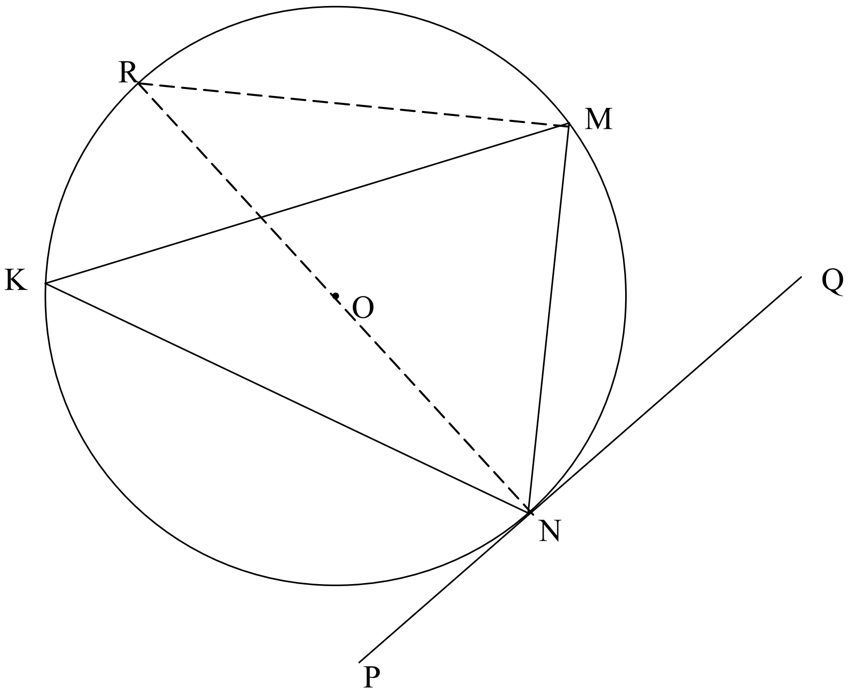
QUESTION/*VRAAG* 10



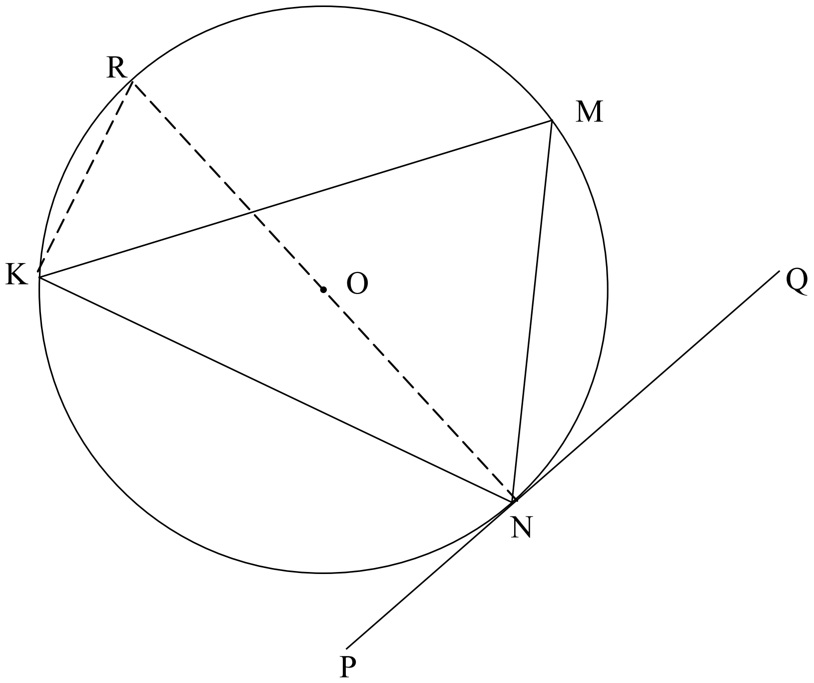
10.1	Line from centre to midpoint of chord/ *lyn vanaf midpt na midpt van koord*	✓ R (1)
10.2.1	$\hat{OWT} = \hat{OWS} = 90°$ [radius ⊥ tangent/*raaklyn*] $\therefore$ MN ∥ TS [corresp ∠s =/*ooreenkomstige ∠e* = **OR** co-int ∠s 180°/*ko-binne∠e 180°* **OR** alternate ∠s/*verwiss ∠e*]	✓ R ✓ R (2)
10.2.2	$\hat{M}_1 = \hat{N}_1$ [∠s opp = sides/ *∠e teenoor = sye*] $\hat{M}_1 = \hat{T}$ [corresp ∠s/*ooreenk ∠e*; MN ∥ TS] $\therefore \hat{N}_1 = \hat{T}$ $\therefore$ TMNS is a cyclic quadrilateral [conv: ext∠ cyclic quad] TMNS *is 'n koordevierhoek* [*omgek: buite∠ kdvh*] **OR/*OF*** $\hat{M}_1 = \hat{N}_1$ [∠s opp = sides/*∠e teenoor = sye*] $\hat{N}_1 = \hat{S}$ [corresp ∠s/*ooreenk ∠e*; MN ∥ TS] $\therefore \hat{S} = \hat{M}_1$ $\therefore$ TMNS is a cyclic quadrilateral [conv: ext∠ cyclic quad] TMNS *is 'n koordevierhoek* [*omgek: buite∠ kdvh*]	✓ S ✓ S ✓ S ✓ R (4) ✓ S ✓ S ✓ S ✓ R (4)

10.2.3	In ΔOVN and ΔOWS $\hat{O}_2 = \hat{O}_2$ [common/*gemeenskaplik*] $O\hat{V}N = O\hat{W}S = 90°$ [from 10.1] $O\hat{N}V = O\hat{S}W$ [sum ∠s Δ/*som ∠e Δ*] $\therefore \Delta OVN \mid\mid\mid \Delta OWS$ [∠, ∠, ∠] $\therefore \frac{VN}{WS} = \frac{ON}{OS}$ But $VN = \frac{1}{2}MN$ [given] $\therefore \frac{\frac{1}{2}MN}{WS} = \frac{ON}{OS}$ $\therefore OS.MN = 2ON.\ WS$	✓ S; S; S **OR** S; S; R ✓ΔOVN ||| ΔOWS ✓ $\frac{VN}{WS} = \frac{ON}{OS}$ ✓ $VN = \frac{1}{2}MN$ ✓ substitution (5)
	OR/*OF* In ΔOVM and ΔOWS $O\hat{V}M = O\hat{W}S = 90°$ [from 10.1] $O\hat{M}V = O\hat{S}W$ [sum ∠s Δ/*som ∠e Δ*] $\therefore \Delta OVM \mid\mid\mid \Delta OWS$ [∠, ∠, ∠] $\therefore \frac{OM}{OS} = \frac{VM}{WS}$ But $VN = \frac{1}{2}MN$ [given] $\therefore \frac{\frac{1}{2}MN}{WS} = \frac{OM}{OS}$ $\therefore OS.MN = 2ON.\ WS$ [VM = VN]	✓ S; S; S **OR** S; S; R ✓ΔOVM ||| ΔOWS ✓ $\frac{OM}{OS} = \frac{VM}{WS}$ ✓ $VN = \frac{1}{2}MN$ ✓ substitution (5)
	OR/*OF* If any other 2 Δs are used, first need to prove that TW = WS by proving ΔOWT ≡ ΔOWS In ΔOVM and ΔOWT $\hat{O}_1 = \hat{O}_1$ [common/*gemeenskaplik*] $O\hat{V}M = O\hat{W}T = 90°$ [from 10.1] $O\hat{M}V = O\hat{T}W$ [sum ∠s Δ/*som ∠e Δ*] $\therefore \Delta OVM \mid\mid\mid \Delta OWT$ [∠, ∠, ∠] $\therefore \frac{VM}{WT} = \frac{OM}{OT}$ But $VN = VM = \frac{1}{2}MN$ [given] and WT = WS and OT = OS [ΔOWT ≡ ΔOWS] $\therefore \frac{\frac{1}{2}MN}{WS} = \frac{ON}{OS}$ $\therefore OS.MN = 2ON.\ WS$	✓✓ similarity ✓✓ congruency ✓ $VN = VM = \frac{1}{2}MN$ (5) **[12]**

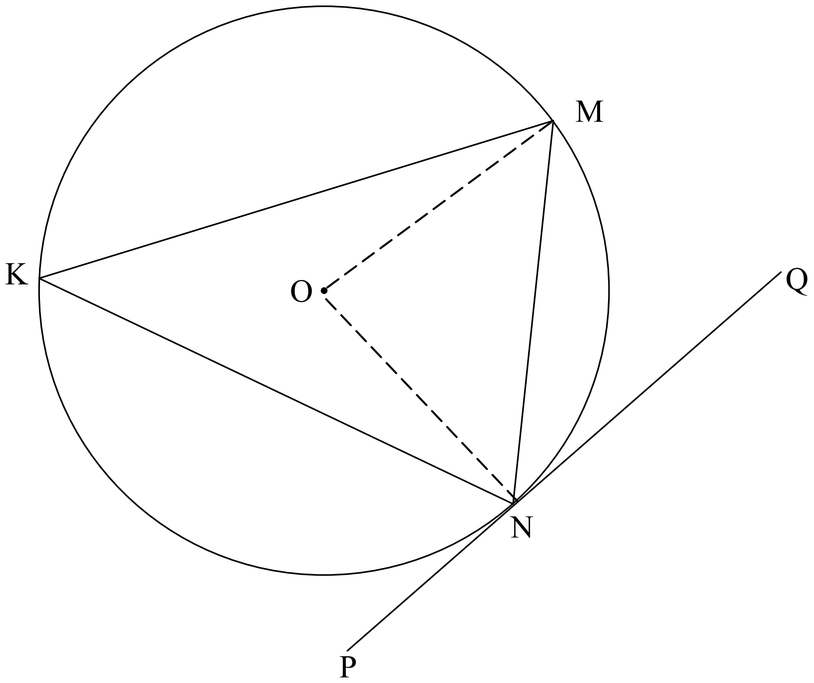
QUESTION/*VRAAG* 11



11.1	Construction: Draw diameter NR and draw RM *Konstruksie: Trek middellyn NR en verbind RM* $O\hat{N}M + M\hat{N}Q = 90°$ [radius ⊥ tangent/*raaklyn*] $N\hat{M}R = 90°$ [∠ in semi circle/*semi-sirkel*] $\therefore M\hat{R}N = 180° - (90° + 90° - M\hat{N}Q)$ [sum ∠s Δ] $= M\hat{N}Q$ but $M\hat{R}N = M\hat{K}N$ [∠s same segment/*∠e dieselfde segment*] $\therefore M\hat{N}Q = \hat{K}$ **OR/*OF***	✓ construction ✓ S /R ✓ S/ R ✓ S ✓ S / R (5)

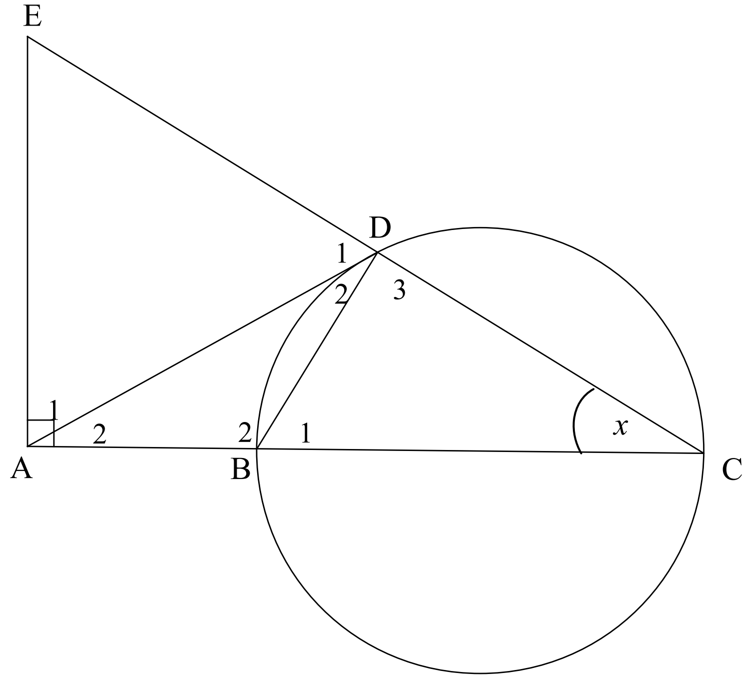


11.1	Construction: Draw diameter NR and draw RK *Konstruksie: Trek middellyn NR en verbind RK* $M\hat{N}Q + R\hat{N}M = 90°$ [radius ⊥ tangent/*raaklyn*] $N\hat{K}R = 90°$ [∠ in semicircle/*semi-sirkel*] $\therefore M\hat{K}N = 90° - R\hat{K}M$ $= 90° - R\hat{N}M$ [∠s same segment/*∠e dieselfde segment*] $\therefore M\hat{N}Q = \hat{K}$	✓ construction ✓ S /R ✓ S/ R ✓ S ✓ S / R (5)



11.1	Construction: Draw radii ON and OM *Konstruksie: Trek radiusse ON en OM* $\hat{MON} = 2\hat{K}$ [∠ at centre = 2∠ at circumf/*midpts∠ = 2 omtreks∠*] $\hat{ONM} + \hat{OMN} = 180° - 2\hat{K}$ [∠s of Δ/*∠e van Δ*] $\hat{ONM} = \hat{OMN} = \frac{180° - 2\hat{K}}{2} = 90° - \hat{K}$ [∠s opp = sides/*∠e teenoor = sye*] $\hat{ONQ} = 90°$ [radius⊥tangent/*radius⊥raaklyn*] $\therefore \hat{MNQ} = \hat{K}$	✓ construction ✓ S /R ✓ S ✓ S/ R ✓ S / R (5)

11.2



11.2.1(a)	Angle in a semi circle/ *Hoek in halfsirkel*	✓ R (1)
11.2.1(b)	Exterior ∠of quad = opp interior ∠/ *Buite ∠ van vierh = teenoorst binne ∠* **OR/*OF*** Opp ∠s of quad supplementary/ *Teenoorst ∠e van vierh is supplementêr*	✓ R (1)
11.2.1(c)	tangent chord theorem/ *raaklyn koord stelling*	✓ R (1)
11.2.2(a)	In ΔAEC $\hat{E} = 180° - (90° + x)$ [sum ∠s Δ] $= 90° - x$ $\hat{D}_1 = 180° - (90° + x)$ [∠s on a straight line] $= \hat{E} = 90° - x$ $\therefore$ AD = AE [sides opp = ∠s/ *sye teenoor = ∠e*]	✓ S ✓ S ✓ R (3)
11.2.2(b)	In ΔADB and ΔACD $\hat{A}_2 = \hat{A}_2$ [common] $\hat{D}_2 = \hat{C}$ [proven] $\hat{B}_2 = \hat{D}_2 + \hat{D}_3$ [sum $\angle^e$ Δ] $\therefore$ΔADB ||| ΔACD **OR/*OF*** In ΔADB and ΔACD $\hat{A}_2 = \hat{A}_2$ [common] $\hat{D}_2 = \hat{C}$ [proven] $\therefore$ΔADB ||| ΔACD [∠, ∠, ∠]	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ R (3)

11.2.3(a)	$\frac{AD}{AC}=\frac{AB}{AD}$ [||| Δs] $AD^2 = AC \, . \, AB$ $= 3r \times r$ $= 3r^2$	✓ ratio ✓ substitution (2)
11.2.3(b)	$AD = AE = \sqrt{3}r$ [from 11.2.2(a) &11.2.3(a)] $AB = r$ and $BC = 2r$ $\therefore AC = 3r$ In ΔACE: $\tan \hat{E} = \frac{AC}{AE}$ $= \frac{3r}{\sqrt{3}r} = \sqrt{3}$ $\therefore \hat{E} = 60°$ $\therefore \hat{D}_1 = 60°$ [from 11.2.2(a)] $\therefore \hat{A}_1 = 60°$ [∠s of Δ = 180°] ∴ ΔADE is equilateral/*is gelyksydig* **OR/*OF*** $\frac{AD}{AC}=\frac{DB}{CD}$ [||| Δs] $\frac{\sqrt{3}r}{3r}=\frac{DB}{CD}$ $\tan x = \frac{1}{\sqrt{3}}$ ∴ In ΔBDC: $x = 30°$ $\therefore \hat{E} = 60°$ $\therefore \hat{D}_1 = 60°$ [from 11.2.2(a)] $\therefore \hat{A}_1 = 60°$ [∠s of Δ = 180°] ∴ ΔADE is equilateral/*is gelyksydig* **OR/*OF*** $\frac{AD}{AC}=\frac{DB}{CD}$ [||| Δs] $\frac{\sqrt{3}r}{3r}=\frac{DB}{CD}$ $\therefore BD = \frac{CD}{\sqrt{3}}$ $DC^2 = BC^2 - DB^2$ $= 4r^2 - \frac{CD^2}{3}$ $3DC^2 = 12r^2 - CD^2$ $4CD^2 = 12r^2$ $DC = \sqrt{3}r$	✓ AC ito r ✓ trig ratio ✓ simplification ✓ all 3 ∠s = 60° (4) ✓ $\frac{\sqrt{3}r}{3r}=\frac{DB}{CD}$ ✓ $\frac{1}{\sqrt{3}} = \tan x$ ✓ $x = 30°$ ✓ all 3 ∠s = 60° (4) ✓ $BD = \frac{CD}{\sqrt{3}}$ ✓ $DC = \sqrt{3}r$

	$EC^2 = EA^2 + AC^2$ $= 3r^2 + 9r^2$ $EC = 2\sqrt{3}r$ $\therefore$ ED = EC – DC $= \sqrt{3}r$ $\therefore$ ED = EA = AD $\therefore$ ΔADE is equilateral/*is gelyksydig*	✓ $EC = 2\sqrt{3}r$ ✓ ED = EA = AD (4) **[20]**

TOTAL/*TOTAAL*: 150