MATHEMATICS

Grade 8 - Term 1

CAPS

Learner Book

Revised edition

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Contents

Term 1

Term 2

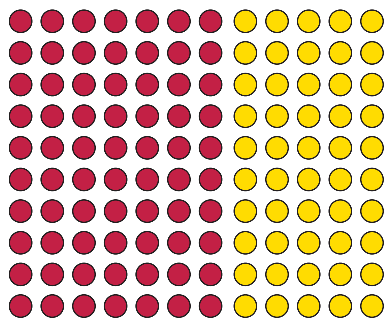
Term 3

Term 4

CHAPTER 1
Whole numbers

1.1 Properties of whole numbers

THE COMMUTATIVE PROPERTY OF ADDITION AND MULTIPLICATION



1. Which of the following calculations would you choose to calculate the number of yellow beads in this pattern? Do not do any calculations now, just make a choice.
   (a) 7 + 7 + 7 + 7 + 7
   (b) 10 + 10 + 10 + 10 + 10 + 10 + 10
   (c) 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5
   (d) 5 + 5 + 5 + 5 + 5 + 5 + 5
   (e) 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7
   (f) 10 + 10 + 10 + 10 + 10

2. (a) How many red beads are there in the pattern? How many yellow beads are there?
   (b) How many beads are there in the pattern in total?

3. (a) Which expression describes what you did to calculate the total number of beads: 70 + 50 *or* 50 + 70?
   (b) Does it make a difference?
   (c) Which expression describes what you did to calculate the number of red beads: 7 × 10 *or* 10 × 7?
   (d) Does it make a difference?

We say: **addition and multiplication are commutative**. The numbers can be swopped around as their order does not change the answer. This does **not** work for subtraction and division, however.

4. Calculate each of the following:

(a) 5 × 8	(b) 10 × 8	(c) 12 × 8	(d) 8 × 12
(e) 6 × 8	(f) 3 × 7	(g) 6 × 7	(h) 7 × 6

THE ASSOCIATIVE PROPERTY OF ADDITION AND MULTIPLICATION

Lebogang and Nathi both have to calculate 25 × 24.

Lebogang calculates 25 × 4 and then multiplies by 6.
Nathi calculates 25 × 6 and then multiplies by 4.

1. Will they get the same answer or not?

> If three or more numbers have to be multiplied, it does not matter which two of the numbers are multiplied first.
>
> This is called the **associative property of multiplication**. We also say **multiplication is associative**.

2. Do the following calculations. **Do not use a calculator now**.
   (a) 4 + 7 + 5 + 6 (b) 7 + 6 + 5 + 4
   (c) 6 + 5 + 7 + 4 (d) 7 + 5 + 4 + 6

3. (a) Is addition associative?
   (b) Illustrate your answer with an example.

4. Find the value of each expression by working in the easiest possible way:
   (a) 2 × 17 × 5 (b) 4 × 7 × 5
   (c) 75 + 37 + 25 (d) 60 + 87 + 40 + 13

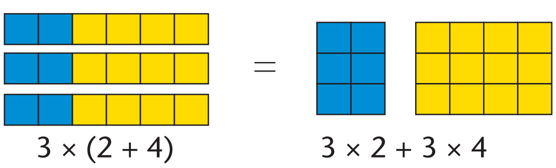
5. What must you add to each of the following numbers to get 100?
   82 44 56 78 24 89 77

6. What must you multiply each of these numbers by to get 1 000?
   250 125 25 500 200 50

7. Calculate each of the following. Note that you can make the work very easy by deciding how to group the operations.
   (a) 82 + 54 + 18 + 46 + 237 (b) 24 + 89 + 44 + 76 + 56 + 11
   (c) 25 × (86 × 4) (d) 32 × 125

MORE CONVENTIONS AND THE DISTRIBUTIVE PROPERTY

The distributive property is a useful property because it allows us to do this:



Both answers are 18. We used brackets in the first example to show that the addition operation must be done first. Otherwise, we would have done the multiplication first. For example, the expression 3 × 2 + 4 means "multiply 3 by 2; then add 4". It does **not** mean "add 2 and 4; then multiply by 3".

The expression $4 + 3 \times 2$ also means "multiply 3 by 2; then add 4".

> If you wish to specify that addition or subtraction should be **done first**, that part of the expression should be enclosed **in brackets**.

The distributive property can be used to break up a difficult multiplication into smaller parts. For example, it can be used to make it easier to calculate 6×204:

6×204 can be rewritten as $6 \times (200 + 4)$ (Remember the brackets!)
$= 6 \times 200 + 6 \times 4$
$= 1\ 200 + 24$
$= 1\ 224$

Multiplication can also be distributed over subtraction, for example to calculate 7×96:

$7 \times 96 = 7 \times (100 - 4)$
$= 7 \times 100 - 7 \times 4$
$= 700 - 28$
$= 672$

1. Here are some calculations with answers. Rewrite them with brackets to make all the answers correct.
   (a) $8 + 6 \times 5 = 70$
   (b) $8 + 6 \times 5 = 38$
   (c) $5 + 8 \times 6 - 2 = 52$
   (d) $5 + 8 \times 6 - 2 = 76$
   (e) $5 + 8 \times 6 - 2 = 51$
   (f) $5 + 8 \times 6 - 2 = 37$

2. Calculate the following:
   (a) $100 \times (10 + 7)$
   (b) $100 \times 10 + 100 \times 7$
   (c) $100 \times (10 - 7)$
   (d) $100 \times 10 - 100 \times 7$

3. Copy and complete the table:

×	8	5	4	9	7	3	6	2	10	11	12
7											
3				27				6			
9											
5											
8											
6											
4					28						
2											
10	80									110	
12											
11											

4. Use the various mathematical conventions for numerical expressions to make the following calculations easier. Show how you work them out.
   (a) 18 × 50 (b) 125 × 28 (c) 39 × 220
   (d) 443 + 2 100 + 557 (e) 318 + 650 + 322 (f) 522 + 3 003 + 78

Two more properties of numbers are:

- **The additive property of 0:** when we add zero to any number, the answer is that number.
- **The multiplicative property of 1:** when we multiply any number by 1, the answer is that number.

1.2 Calculations with whole numbers

ESTIMATING, APPROXIMATING AND ROUNDING

1. Copy and complete the following statements by giving answers to these questions, without doing any calculations with the given numbers.
   (a) Is 8 × 117 more than 2 000 or less than 2 000? .... than 2 000
   (b) Is 27 × 88 more than 3 000 or less than 3 000? .... than 3 000
   (c) Is 18 × 117 more than 3 000 or less than 3 000? .... than 3 000
   (d) Is 47 × 79 more than 3 000 or less than 3 000? .... than 3 000

What you have done when you tried to give answers to questions 1(a) to (d), is called **estimation**. To estimate is to try to get close to an answer without actually doing the calculations with the given numbers.

An estimate may also be called an **approximation**.

2. Look at question 1 again.
   (a) The numbers 1 000, 2 000, 3 000, 4 000, 5 000, 6 000, 7 000, 8 000, 9 000 and 10 000 are all multiples of a thousand. In each case, write down the multiple of 1 000 that you think is closest to the answer. The numbers you write down are called **estimates**.
   (b) In some cases you may achieve a **better estimate** by adding 500 to your estimate, or subtracting 500 from it. If so, you may add or subtract 500.
   (c) If you wish, you may write what you believe is an even better estimate by adding or subtracting some hundreds.

3. (a) Use a calculator to find the exact answers for the calculations in question 1. Calculate the **error** in your last approximation of each of the answers in question 1.
   (b) What was your smallest error?

The difference between an estimate and the actual answer is called the **error**.

4. Think again about what you did in question 2. In 2(a) you tried to approximate the answers to the nearest **1 000**. In 2(c) you tried to approximate the answers to the nearest **100**. Describe what you tried to achieve in question 2(b).

5. Estimate the answers for each of the following products and sums. Try to approximate the answers for the **products** to the nearest thousand, and for the **sums** to the nearest hundred.

(a) 84 × 178	(b) 677 + 638
(c) 124 × 93	(d) 885 + 473
(e) 79 × 84	(f) 921 + 367
(g) 56 × 348	(h) 764 + 829

6. Use a calculator to find the exact answers for the calculations in question 5. Calculate the error in each of your approximations. Use the second line in each question to do this.

Calculating with "easy" numbers that are close to given numbers is a good way to obtain approximate answers, for example:

- To approximate 764 + 829, you may calculate 800 + 800 to get the approximate answer 1 600; with an error of 7.
- To approximate 84 × 178, you may calculate 80 × 200 to get the approximate answer 16 000; with an error of 1 048.

7. Calculate with "easy" numbers close to the given numbers to produce approximate answers for each product below. **Do not use a calculator.** When you have made your approximations, use a calculator to work out the precise answers.

(a) 78 × 46	(b) 67 × 88
(c) 34 × 276	(d) 78 × 178

ROUNDING OFF AND COMPENSATING

> The word **compensate** means to do things that will remove damage.

1. (a) Approximate the answer for 386 + 3 435, by rounding both numbers off to the nearest hundred, and then adding the rounded numbers.
   (b) Because you rounded 386 up to 400, you introduced an error of 14 in your approximate answer. What error did you introduce by rounding 3 435 down to 3 400?
   (c) What combined (total) error did you introduce by rounding both numbers off before calculating?
   (d) Use your knowledge of the total error to correct your approximate answer, so that you have the correct answer for 386 + 3 435.

In question 1, you used **rounding off** and **compensating** to find the correct answer for 386 + 3 435. By rounding the numbers off you introduced errors. You then compensated for the errors by making adjustments to your answer.

2. Round off and compensate to calculate each of the following accurately:
   (a) 473 + 638 (b) 677 + 921

Subtraction can also be done in this way. For example, to work out R5 362 – R2 687, you may round R2 687 up to R3 000. You may do this in the following ways:

- Rounding R2 687 up to R3 000 can be done in two steps: 2 687 + 13 = 2 700, and 2 700 + 300 = 3 000. In total, 313 is added.
- You can now add 313 to 5 362 too: R5 362 + 313 = 5 675.
- Instead of calculating R5 362 – R2 687, which is a bit difficult, you may calculate R5 675 – R3 000. This is easy: R5 675 – R3 000 = R2 675.

This means that R5 362 – R2 687 = R2 675, because
R5 362 – R2 687 = (R5 362 + R313) – (R2 687 + R313).

ADDING NUMBERS IN PARTS WRITTEN IN COLUMNS

Numbers can be added by thinking of their **parts** as we say the numbers.
For example, we say 4 994 as *four thousand nine hundred and ninety-four*.
This can be written in expanded notation as 4 000 + 900 + 90 + 4.

Similarly, we can think of 31 837 as 30 000 + 1 000 + 800 + 30 + 7.

31 837 + 4 994 can be calculated by working with the various kinds of parts separately. To make this easy, you can write the numbers below each other so that the units are below the units, the tens below the tens and so on, as shown on the right.

31 837
4 994

We write only this:	In your mind you can see this:				
31 837	30 000	1 000	800	30	7
4 994		4 000	900	90	4

The numbers in each column can be added to get a new set of numbers.

31 837	30 000	1 000	800	30	7
4 994		4 000	900	90	4
11					11
120				120	
1 700			1 700		
5 000		5 000			
30 000	30 000				
36 831	It is easy to add the new set of numbers to get the answer.				

The work may start with the 10 000s or any other parts. Starting with the units, as shown on page 6, makes it possible to do more of the work mentally, and write less, as shown below.

31 837 4 994 36 831	To achieve this, only the units digit 1 of the 11 is written in the first step. The 10 of the 11 is remembered and added to the 30 and 90 of the tens column, to get 130.

We say the 10 is **carried** from the units column to the tens column. The same is done when the tens parts are added to get 130: only the digit "3" is written (in the tens column, so it means 30), and the 100 is carried to the next step.

1. Calculate each of the following without using a calculator:
   (a) 4 638 + 2 667   (b) 748 + 7 246

2. Impilo Enterprises plans a new computerised training facility in their existing building. The training manager has to keep the total expenditure budget under R1 million. This is what she has written so far:

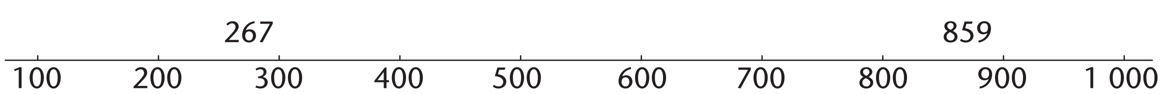
Architects and builders	R 102 700
Painting and carpeting	R 42 600
Security doors and blinds	R 52 000
Data projector	R 4 800
25 new secretary chairs	R 50 400
24 desks for work stations	R123 000
1 desk for presenter	R 28 000
25 new computers	R300 000
12 colour laser printers	R 38 980

   Work out the total cost of all the items for which the training manager has budgeted.

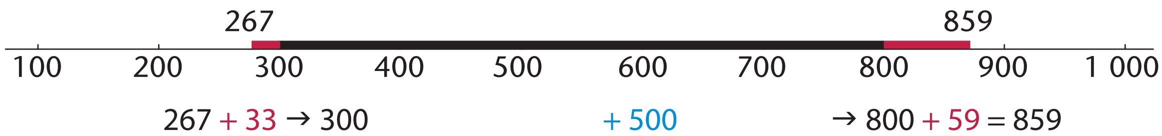
3. Calculate each of the following, without using a calculator:
   (a) 7 828 + 6 284   (b) 7 826 + 888 + 367
   (c) 657 + 32 890 + 6 542   (d) 6 666 + 3 333 + 1

METHODS OF SUBTRACTION

There are many ways to find the difference between two numbers. For example, to find the difference between 267 and 859, you may think of the numbers as they would appear on a number line, for example:



We may think of the distance between 267 and 859 as three steps: from 267 to 300, from 300 to 800, and from 800 to 859. How big are each of these three steps?



This number line shows that 859 – 267 is 33 + 500 + 59.

1. Calculate 33 + 500 + 59 to find the answer for 859 – 267.
2. Calculate each of the following. You may think of working out the distance between the two numbers as shown above, or use any other method you prefer.
   Do not use a calculator now.
   (a) 823 – 456
   (b) 1 714 – 829
   (c) 3 045 – 2 572
   (d) 5 131 – 367

Like addition, subtraction can also be done by working with the different parts in which we say numbers. For example, 8 764 – 2 352 can be calculated as follows:

8 thousand – 2 thousand = 6 thousand
7 hundred – 3 hundred = 4 hundred
6 tens – 5 tens = 1 ten
4 units – 2 units = 2 units
So, 8 764 – 2 352 = 6 412

Subtraction by parts is more difficult in some cases, for example 6 213 – 2 758:

6 000 – 2 000 = 4 000. This step is easy, but the following steps cause problems:

200 – 700 = ?
10 – 50 = ?
3 – 8 = ?

Fortunately, the parts and sequence of work may be rearranged to overcome these problems, as shown below:

One way to overcome these problems is to work with negative numbers:
200 – 700 = (–500)
10 – 50 = (–40)
3 – 8 = (–5)
4 000 – 500 → 3 500 – 45 =

Instead of		we may do		
3 – 8 =	?	13 – 8 =	5	“borrow” 10 from below
10 – 50 =	?	100 – 50 =	50	“borrow” 100 from below
200 – 700 =	?	1 100 – 700 =	400	“borrow” 1 000 from below
6 000 – 2 000 =	?	5 000 – 2 000 =	3 000	

This reasoning can also be set out in columns:

Instead of				we may do				but write only this			
6 000	200	10	3	5 000	1 100	100	13	6	2	1	3
2 000	700	50	8	2 000	700	50	8	2	7	5	8
				3 000	400	50	5	3	4	5	5

3. (a) Complete the above calculations and find the answer for 6 213 – 2 758.
   (b) Use the borrowing technique to calculate 823 – 376 and 6 431 – 4 968.
4. Check your answers in question 3(b) by doing addition.

With some practice, you can learn to subtract using borrowing without writing all the steps. When calculating 6 213 – 2 758, use the column method, as shown on the right.

6 213
2 758
5
50
400
3 000
3 455

If you work more mentally, you will write even less, as shown below:

6 213
2 758
3 455

Do not use a calculator when you do question 5, because the purpose of this work is for you to understand the methods of subtraction. What you learn here will help you to understand **algebra** better at a later stage.

5. Calculate each of the following:
   (a) 7 342 – 3 877 (b) 8 653 – 1 856 (c) 5 671 – 4 528

You may use a calculator to do questions 6 and 7.

6. Estimate the difference between the two car prices in each case, to the nearest R1 000 or closer. Then calculate the difference.
   (a) R102 365 and R98 128 (b) R63 378 and R96 889
7. First estimate the answers to the nearest 100 000, 10 000 or 1 000. Then calculate.
   (a) 238 769 –141 453 (b) 856 333 – 739 878 (c) 65 244 – 39 427

A METHOD OF MULTIPLICATION

You can calculate 7 × 4 598 in parts, as shown here:

$$7 \times 4\,000 = 28\,000$$
$$7 \times 500 = 3\,500$$
$$7 \times 90 = 630$$
$$7 \times 8 = 56$$

The four partial products can now be added to get the answer, which is 32 186. It is convenient to write the work in vertical columns for units, tens, hundreds and so on, as shown on the right.

		4	5	9	8
×					7
				5	6
			6	3	0
		3	5	0	0
	2	8	0	0	0
	3	2	1	8	6

When working from right to left in the columns, you can "carry" parts of the partial answers to the next column. Instead of writing 56, only the 6 of the product 7 × 8 is written down. The 50 is kept in mind, and added to the 630 obtained when 7 × 90 is calculated in the next step.

	4	5	9	8
				7
3	2	1	8	6

1. Calculate each of the following. Do not use a calculator now.
   (a) 27 × 649 (b) 75 × 1 756 (c) 348 × 93
2. Use your calculator to check your answers for question 1. Redo the questions for which you had the wrong answers.
3. Calculate each of the following. Do not use a calculator now.
   (a) 67 × 276 (b) 84 × 178
4. Use a calculator to check your answers for question 3. Redo the questions for which you had the wrong answers.

LONG DIVISION

1. The municipal head gardener wants to buy young trees to plant along the main street of the town. The young trees cost R27 each, and an amount of R9 400 has been budgeted for trees. He needs 324 trees. Do you think he has enough money?
2. (a) How much will 300 trees cost?
   (b) How much money will be left if 300 trees are bought?
   (c) How much money will be left if 20 more trees are bought?

The municipal gardener wants to work out exactly how many trees, at R27 each, he can buy with the budgeted amount of R9 400. His thinking and writing are described below.

Step 1:

What he writes:	What he thinks:
27 ⟌ 9 400	*I want to find out how many chunks of 27 there are in 9 400.*

Step 2:

What he writes:	What he thinks:
300	*I think there are at least 300 chunks of 27 in 9 400.*
27 ⟌ 9 400	
8 100	*300 × 27 = 8 100. I need to know how much is left over.*
1 300	*I want to find out how many chunks of 27 there are in 1 300.*

Step 3: (He has to rub out the one "0" of the 300 on top, to make space.)

What he writes:	What he thinks:
340	*I think there are at least 40 chunks of 27 in 1 300.*
27 ⟌ 9 400	
8 100	
1 300	
1 080	*40 × 27 = 1 080. I need to know how much is left over.*
220	*I want to find out how many chunks of 27 there are in 220.* *Perhaps I can buy some extra trees.*

Step 4: (He rubs out another "0".)

What he writes:	What he thinks:
348	*I think there are at least 8 chunks of 27 in 220.*
27) 9 400	
8 100	
1 300	
1 080	
220	
216	*8 × 27 = 216*
4	*So, I can buy 348 young trees and will have R4 left.*

Do not use a calculator to do questions 3 and 4. The purpose of this work is for you to develop a good understanding of how division can be done. Check all your answers by doing multiplication.

3. (a) Graham bought 64 goats, all at the same price. He paid R5 440 in total. What was the price for each goat? You can start by working out how much he would have paid if he paid R10 per goat. You can start with a bigger step if you wish.
   (b) Mary has R2 850 and she wants to buy candles for her sister's wedding reception. The candles cost R48 each. How many candles can she buy?

4. Calculate each of the following. Do not use a calculator.
   (a) 7 234 ÷ 48 (b) 3 267 ÷ 24
   (c) 9 500 ÷ 364 (d) 8 347 ÷ 24

1.3 Multiples, factors and prime factors

MULTIPLES AND FACTORS

1. The numbers 6; 12; 18; 24; ... are **multiples** of 6.
   The numbers 7; 14; 21; 28; ... are **multiples** of 7.

If n is a natural number, $6n$ represents the multiples of 6.

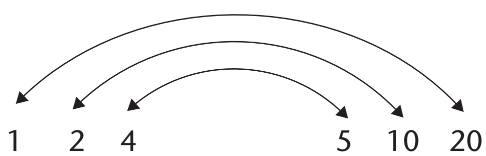
   (a) What is the hundredth number in each sequence above?
   (b) Is 198 a number in the first sequence?
   (c) Is 175 a number in the second sequence?

Of which numbers is 20 a multiple?

20 = 1 × 20 = 2 × 10 = 4 × 5 = 5 × 4 = 10 × 2 = 20 × 1

Factors come in pairs. The following pairs are factors of 20:

20 is a multiple of 1; 2; 4; 5; 10 and 20, and all of these numbers are factors of 20.



2. A rectangle has an area of 30 cm. What are the possible lengths of the sides of the rectangle in centimetres if the lengths of the sides are natural numbers?
3. Are 4, 8, 12 and 16 factors of 48? Simon says that all multiples of 4 smaller than 48 are factors of 48. Is he right?
4. We have defined factors in terms of the product of *two* numbers. What happens if we have a product of *three* or more numbers, for example: 210 = 2 × 3 × 5 × 7?
   (a) Explain why 2, 3, 5 and 7 are factors of 210.
   (b) Are 2 × 3, 3 × 5, 5 × 7, 2 × 5 and 2 × 7 factors of 210?
   (c) Are 2 × 3 × 5, 3 × 5 × 7 and 2 × 5 × 7 factors of 210?
5. Is 20 a factor of 60? What factors of 20 are also factors of 60?

PRIME NUMBERS AND COMPOSITE NUMBERS

1. Express each of the following numbers as a product of as many factors as possible, including repeated factors. Do not use 1 as a factor.
   (a) 66 (b) 67
   (c) 68 (d) 69
   (e) 70 (f) 71
   (g) 72 (h) 73

The number 36 can be formed as 2 × 2 × 3 × 3. Because 2 and 3 are used twice, they are called **repeated factors** of 36.

2. Which of the numbers in question 1 cannot be expressed as a product of two whole numbers, except as the product *1 × the number itself*?

A number that cannot be expressed as a product of two whole numbers, except as the product *1 × the number itself*, is called a **prime number**.

3. Which of the numbers in question 1 are prime numbers?

Composite numbers are natural numbers with more than two different factors. The sequence of composite numbers is 4; 6; 8; 9; 10; 12; ...

4. Are the statements below true or false? If you answer "false", explain why.
   (a) All prime numbers are odd numbers.
   (b) All composite numbers are even numbers.
   (c) 1 is a prime number.
   (d) If a natural number is not prime, then it is composite.
   (e) 2 is a composite number.
   (f) 785 is a prime number.
   (g) A prime number can only end in 1, 3, 7 or 9.
   (h) Every composite number is divisible by at least one prime number.

5. We can find out if a given number is prime by systematically checking whether the primes 2; 3; 5; 7; 11; 13; ... are factors of the given number or not.

   To find possible factors of 131, we need to consider only the primes 2; 3; 5; 7 and 11. Why not 13; 17; 19; ...?

6. Determine whether the following numbers are prime or composite. If the number is composite, write down at least two factors of the number (besides 1 and the number itself).

   (a) 221 (b) 713

PRIME FACTORISATION

To find all the factors of a number you can write the number as the product of prime factors; first by writing it as the product of two convenient (composite) factors and then by splitting these factors into smaller factors until all factors are prime. Then you take all the possible combinations of the products of the prime factors.

> Every composite number can be expressed as the product of prime factors and this can happen in only one way.

Example: Find the factors of 84.

Write 84 as the product of prime factors by starting with different known factors:

$$84 = 4 \times 21 = 2 \times 2 \times 3 \times 7 \quad \text{or} \quad 84 = 7 \times 12 = 7 \times 3 \times 4 = 7 \times 3 \times 2 \times 2 \quad \text{or} \quad 84 = 2 \times 42 = 2 \times 6 \times 7 = 2 \times 2 \times 3 \times 7$$

A more systematic way of finding the prime factors of a number would be to start with the prime numbers and try the consecutive prime numbers 2; 3; 5; 7; ... as possible factors. The work may be set out as shown below.

2	1 430
5	715
11	143
13	13
	1

1 430 = 2 × 5 × 11 × 13

3	2 457
3	819
3	273
7	91
13	13
	1

2 457 = 3 × 3 × 3 × 7 × 13

We can use exponents to write the products of prime factors more compactly as products of powers of prime factors.

$$2\,457 = 3 \times 3 \times 3 \times 7 \times 13 = 3^3 \times 7 \times 13$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$

$$1\,500 = 2 \times 2 \times 3 \times 5 \times 5 \times 5 = 2^2 \times 3 \times 5^3$$

1. Express the following numbers as the product of powers of primes:

   (a) 792 (b) 444

2. Find the prime factors of the following numbers:

   28 32 124 36 42 345 182

COMMON MULTIPLES AND FACTORS

1. Is 4 × 5 a multiple of 4? Is 4 × 5 a multiple of 5?
2. Comment on the following statement:
   The product of numbers is a multiple of each of the numbers in the product.

We use **common multiples** when fractions with different denominators are added. To add $\frac{2}{3} + \frac{3}{4}$, the common denominator is 3 × 4, so the sum becomes $\frac{8}{12} + \frac{9}{12}$. In the same way, we could use 6 × 8 = 48 as a common denominator to add $\frac{1}{6} + \frac{3}{8}$, but 24 is the **lowest common multiple** (**LCM**) of 6 and 8.

Prime factorisation makes it easy to find the **LCM** or highest common factor.

When we simplify a fraction, we divide the same number into the numerator and the denominator. For the simplest fraction, use the **highest common factor** (**HCF**) to divide into both the numerator and denominator.

The HCF is divided into the numerator and the denominator to write the fraction in its **simplest form**.

$$\text{So } \frac{36}{144} = \frac{2 \times 2 \times 3 \times 3}{2 \times 2 \times 2 \times 2 \times 3 \times 3} = \frac{1}{4}$$

Use prime factorisation to determine the LCM and HCF of 32, 48 and 84 in a systematic way:

$$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$$
$$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$$
$$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$$

The LCM is a **multiple** and therefore, all of the factors of all the numbers must divide into it.

All of the factors that are present in the three numbers must also be factors of the LCM, even if it is a factor of only one of the numbers. But because it has to be the lowest common multiple, there are no unnecessary factors in the LCM.

The highest power of each factor is in the LCM, because all of the other factors can divide into it. In 32, 48 and 84, the highest power of 2 is 2^5, the highest power of 3 is 3 and the highest power of 7 is 7.

LCM = $2^5 \times 3 \times 7 = 672$

The HCF is a common factor. Therefore, for a factor to be in the HCF, it must be a factor of *all* of the numbers. The only number that appears as a factor of all three numbers is 2. The lowest power of 2 is 2^2; therefore, the HCF is 2^2.

3. Determine the LCM and the HCF of the numbers in each case:
   (a) 24; 28; 42
   (b) 17; 21; 35
   (c) 75; 120; 200
   (d) 18; 30; 45

INVESTIGATE PRIME NUMBERS

You may use a calculator for this investigation.

1. Find all the prime numbers between 110 and 130.
2. Find all the prime numbers between 210 and 230.
3. Find the biggest prime number smaller than 1 000.

1.4 Solving problems

RATE AND RATIO

You may use a calculator for the work in this section.

1. Tree plantations in the Western Cape are to be cut down in favour of natural vegetation. There are roughly 3 000 000 trees on plantations in the area and it is possible to cut them down at a **rate** of 15 000 trees per day with the labour available. How many working days will it take before all the trees will be cut down?

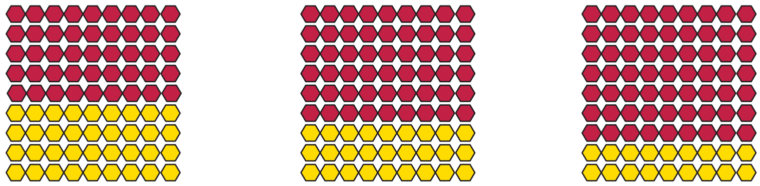
Instead of saying "... per day", people often say "at a **rate** of ... per day". Speed is a way in which to describe the rate of movement.

The word **per** is often used to describe a rate and can mean *for every*, *for*, *in each*, *in*, *out of*, or *every*.

2. A car travels a distance of 180 km in two hours on a straight road. How many kilometres can it travel in three hours at the same speed?

3. Thobeka wants to order a book that costs \$56,67. The rand-dollar exchange rate is R13,79 to a dollar. What is the price of the book in rands?

4. In pattern A below, there are five red beads for every four yellow beads.



Pattern A *Pattern B* *Pattern C*

Describe patterns B and C in the same way.

5. Copy and complete the following table to show how many screws are produced by two machines in different periods of time:

Number of hours	1	2	3	5	8
Number of screws at machine A	1 800				
Number of screws at machine B	2 700				

(a) How much faster is machine B than machine A?
(b) How many screws will machine B produce in the same time that it takes machine A to make 100 screws?

The patterns in question 4 can be described like this: In pattern A, the **ratio** of yellow beads to red beads is 4 to 5. This is written as 4 : 5. In pattern B, the ratio between yellow beads and red beads is 3 : 6. In pattern C the ratio is 2 : 7.

In question 5, machine A produces two screws for every three screws that machine B produces. This can be described by saying that the ratio between the production speeds of machines A and B is 2 : 3.

6. Nathi, Paul and Tim worked in Mr Setati's garden. Nathi worked for five hours, Paul for four hours and Tim for three hours. Mr Setati gave the boys R600 for their work. How should they divide the R600 among the three of them?

We use **ratios** to show how many times more, or less, one quantity is than another.

A **ratio** is a comparison of two (or more) quantities. The number of hours that Nathi, Paul and Tim worked are in the ratio 5 : 4 : 3. To be fair, the money should also be shared in that ratio. Therefore, Nathi should receive five parts, Paul four parts and Tim three parts of the money. There were 12 parts, which means Nathi should receive $\frac{5}{12}$ of the total amount, Paul should get $\frac{4}{12}$ and Tim should get $\frac{3}{12}$.

7. Ntabi uses three packets of jelly to make a pudding for eight people. How many packets of jelly does she need to make a pudding for 16 people? And for 12 people?

8. Which rectangle is more like a square: a 3 × 5 rectangle or a 6 × 8 rectangle? Explain.

To increase 40 in the ratio 2 : 3 means that the 40 represents two parts and must be increased so that the new number represents three parts. If 40 represents two parts, 20 represents one part. The increased number will therefore be 20 × 3 = 60.

Remember that if you multiply by 1, the number does not change. If you multiply by a number greater than 1, the number increases. If you multiply by a number smaller than 1, the number decreases.

9. (a) Increase 56 in the ratio 2 : 3.
   (b) Decrease 72 in the ratio 4 : 3.

10. (a) Divide 840 in the ratio 3 : 4.
    (b) Divide 360 in the ratio 1 : 2 : 3.

11. Look at the following data about the performance of different athletes during a walking event. Investigate the data to find out who walks the fastest and who walks the slowest. Arrange the athletes from the fastest walker to the slowest walker.
    (a) First make estimates to do the investigation.
    (b) Then use your calculator to do the investigation.

Athlete	A	B	C	D	E	F
Distance walked in metres	2 480	4 283	3 729	6 209	3 112	5 638
Time taken in minutes	17	43	28	53	24	45

PROFIT, LOSS, DISCOUNT AND INTEREST

1. (a) How much is one eighth of R800?
   (b) How much is one hundredth of R800?
   (c) How much is seven hundredths of R800?

Rashid is a furniture dealer. He buys a couch for R2 420. He displays the couch in his showroom with the price marked as R3 200. Rashid offers a discount of R320 to customers who pay cash.

The amount for which a dealer buys an article from a producer or manufacturer is called the **cost price**. The price marked on the article is called the **marked price** and the price of the article after discount is the **selling price**.

2. (a) What is the cost price of the couch in Rashid's furniture shop?
   (b) What is the marked price?
   (c) What is the selling price for a customer who pays cash?
   (d) How much is ten hundredths of R3 200?

The **discount** on an article is always less than the marked price of the article. In fact, it is only a fraction of the marked price. The discount of R320 that Rashid offers on the couch is ten hundredths of the marked price.

Another word for hundredths is **percentage**, and the symbol for percentage is %. We can therefore say that Rashid offers a discount of 10%.

A percentage is a number of hundredths.

18% is 18 hundredths and 25% is 25 hundredths.

% is a symbol for hundredths. 8% means eight hundredths and 15% means 15 hundredths.

The symbol % is just a variation of the $\frac{\quad}{100}$ that is used in the common fraction notation for hundredths.

For example: 8% is $\frac{8}{100}$.

A discount of 6% on an article can be calculated in two steps:

Step 1: Calculate one hundredth of the marked price (divide by 100).

Step 2: Calculate six hundredths of the marked price (multiply by six).

3. Calculate a discount of 6% on each of the following marked prices of articles:
   (a) R3 600 (b) R9 360

4. (a) How much is one hundredth of R700?
   (b) A customer pays cash for a coat marked at R700.
   He is given a R63 discount. How many hundredths of R700 is this?
   (c) What is the percentage discount?

5. A client buys a blouse marked at R300 and she is given R36 discount for paying cash. Work as in question 4 to determine what percentage discount she was given.

You may use a calculator to do questions 6, 7 and 8.

6. A dealer buys an article for R7 500 and makes the price 30% higher. The article is sold at a 20% discount.
   (a) What is the selling price of the article?
   (b) What is the dealer's percentage profit?

When a person borrows money from a bank or some other institution, he or she normally has to pay for the use of the money. This is called **interest**.

7. Sam borrows R7 000 from a bank at 14% interest for one year. How much does he have to pay back to the bank at the end of the period?

8. Jabu invests R5 600 for one year at 8% interest.
   (a) What will the value of his investment be at the end of that year?
   (b) At the end of the year, Jabu does not withdraw the investment or the interest earned; instead, he reinvests it for another year. How much will it be worth at the end of the second year?
   (c) What will the value of Jabu's investment be after five years?

Chapter 2
Integers

2.1 What is beyond 0?

WHY PEOPLE DECIDED TO HAVE NEGATIVE NUMBERS

On the right, you can see how Jimmy prefers to work when doing calculations, such as 542 + 253.

500 + 200 = 700
40 + 50 = 90
2 + 3 = 5
700 + 90 + 5 = 795

He tries to calculate 542 – 253 in a similar way:

500 – 200 = 300
40 – 50 = ?

Jimmy clearly has a problem. He reasons as follows:

I can subtract 40 from 40; that gives 0. But then there is still 10 that I have to subtract.

He decides to deal with the 10 that he still has to subtract later, and continues:

500 – 200 = 300
40 – 50 = 0, *but there is still 10 that I have to subtract.*
2 – 3 = 0, *but there is still 1 that I have to subtract.*

1. (a) What must Jimmy still subtract, and what will his final answer be?
   (b) When Jimmy did another subtraction problem, he ended up with this writing at one stage:

   600 and (–)50 and (–)7

   What do you think Jimmy's final answer for this subtraction problem is?

About 500 years ago, some **mathematicians** proposed that a "negative number" may be used to describe the result in a situation, such as in Jimmy's subtraction problem above, where a number is subtracted from a number smaller than itself.

For example, we may say 10 – 20 = (–10).

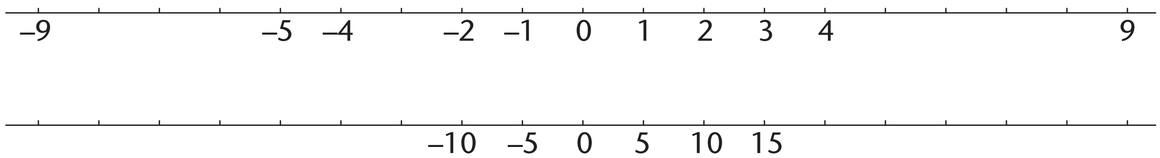
This proposal was soon accepted by other mathematicians, and it is now used all over the world.

Mathematicians are people who do mathematics for a living. Mathematics is their profession, like healthcare is the profession of nurses and medical doctors.

2. Calculate each of the following:

(a) 16 – 20 (b) 16 – 30
(c) 16 – 40 (d) 16 – 60
(e) 16 – 200 (f) 5 – 1 000

3. Some numbers are shown on the number lines below. Copy the number lines and numbers shown, and fill in the missing numbers.



The numbers 1; 2; 3; 4 etc. are called the **natural numbers**. The natural numbers, 0 and the negative whole numbers together, are called the **integers**.

The following statement is true if the number is 5:

15 – (a certain number) = 10

A few centuries ago, some mathematicians decided they wanted to have numbers that will also make sentences like the following true:

15 + (a certain number) = 10

But to go from 15 to 10 you have to subtract 5.

The number we need to make the sentence *15 + (a certain number) = 10* true must have the following strange property:

If you **add** this number, it should have the **same effect** as to **subtract 5**.

Now the mathematicians of a few centuries ago really wanted to have numbers for which such strange sentences would be true. So they thought:

> *Let us decide, and agree amongst ourselves, that the number we call* ***negative 5*** *will have the property that if you add it to another number, the effect will be the same as when you subtract the natural number 5.*

This means that the mathematicians agreed that 15 + (–5) is equal to 15 – 5.

Stated differently, instead of adding *negative 5* to a number, you may subtract 5.

> Adding a negative number has the same effect as subtracting a natural number.
> For example: 20 + (–15) = 20 – 15 = 5

4. Calculate each of the following:

(a) 500 + (–300) (b) 100 + (–20) + (–40)
(c) 500 + (–200) + (–100) (d) 100 + (–60)

5. Make a suggestion of what the answer for (–20) + (–40) should be. Give reasons for your suggestion.

6. Copy and continue the lists of numbers to complete the following table:

(a)	(b)	(c)	(d)	(e)	(f)	(g)
10	100	3	–3	–20	150	0
9	90	6	–6	–18	125	–5
8	80	9	–9	–16	100	–10
7	70	12	–12	–14	75	–15
6	60	15	–15		50	–20
5	50					–25
4	40					
3	30					
2	20					
1	10					
0	0					
–1	–10					

The following statement is true if the number is 5:

15 + (a certain number) = 20

What properties should a number have so that it makes the following statement true?

15 – (a certain number) = 20

To go from 15 to 20 you have to add 5. The number we need to make the sentence *15 – (a certain number)* = 20 true, must have the following property:

If you **subtract** this number, it should have the same effect as to **add 5**.

Let us agree that 15 – (–5) is equal to 15 + 5.

Stated differently, instead of subtracting *negative* 5 from a number, you may add 5.

Subtracting a negative number has the same effect as adding a natural number.

For example: 20 – (–15) = 20 + 15 = 35

7. Calculate each of the following:

(a) 30 – (–10) (b) 30 + 10 (c) 30 + (–10) (d) 30 – 10

(e) 30 – (–30) (f) 30 + 30 (g) 30 + (–30) (h) 30 – 30

You probably agree that:

5 + (–5) = 0   10 + (–10) = 0   and   20 + (–20) = 0

We may say that for each "positive" number there is a **corresponding** or **opposite** negative number. Two positive and negative numbers that correspond, for example 3 and (–3), are called **additive inverses**. They wipe each other out when you add them.

When you add any number to its additive inverse, the answer is 0 (the additive property of 0). For example, 120 + (–120) = 0.

What may each of the following be equal to?

(–8) + 5

(–5) + (–8)

8. Write the additive inverse of each of the following numbers:
   (a) 24 (b) –24
   (c) –103 (d) 2 348

The idea of additive inverses may be used to explain why 8 + (–5) is equal to 3:

$$8 + (-5) = 3 + \boxed{5 + (-5)} = 3 + 0 = 3$$

9. Use the idea of additive inverses to explain why each of these statements is true:
   (a) 43 + (–30) = 13 (b) 150 + (–80) = 70

STATEMENTS THAT ARE TRUE FOR MANY DIFFERENT NUMBERS

For how many different pairs of numbers can the following statement be true, if only natural (positive) numbers are allowed?

a number + another number = 10

For how many different pairs of numbers can the statement be true if negative numbers are also allowed?

2.2 Adding and subtracting with integers

ADDING CAN MAKE LESS AND SUBTRACTION CAN MAKE MORE

1. Calculate each of the following:
   (a) 10 + 4 + (–4) (b) 10 + (–4) + 4
   (c) 3 + 8 + (–8) (d) 3 + (–8) + 8

Natural numbers can be arranged in any order to add and subtract them. This is also the case for integers.

The numbers 1; 2; 3; 4; ... that we use to count, are called **natural numbers**.

2. Calculate each of the following:
   (a) 18 + 12 (b) 12 + 18 (c) 2 + 4 + 6
   (d) 6 + 4 + 2 (e) 2 + 6 + 4 (f) 4 + 2 + 6
   (g) 4 + 6 + 2 (h) 6 + 2 + 4 (i) 6 + (–2) + 4
   (j) 4 + 6 + (–2) (k) 4 + (–2) + 6 (l) (–2) + 4 + 6
   (m) 6 + 4 + (–2) (n) (–2) + 6 + 4 (o) (–6) + 4 + 2

3. Calculate each of the following:
   (a) (–5) + 10 (b) 10 + (–5) (c) (–8) + 20
   (d) 20 – 8 (e) 30 + (–10) (f) 30 + (–20)
   (g) 30 + (–30) (h) 10 + (–5) + (–3)
   (i) (–5) + 7 + (–3) + 5 (j) (–5) + 2 + (–7) + 4

4. In each case, find the number that makes the statement true. Give your answer by writing a closed number sentence.
   (a) 20 + (an unknown number) = 50
   (b) 50 + (an unknown number) = 20
   (c) 20 + (an unknown number) = 10
   (d) (an unknown number) + (–25) = 50
   (e) (an unknown number) + (–25) = –50

Statements like these are also called number sentences.
An incomplete number sentence, where some numbers are not known at first, is sometimes called an **open number sentence**:
8 – (a number) = 10
A **closed number sentence** is where all the numbers are known:
8 + 2 = 10

5. Use the idea of additive inverses to explain why each of the following statements is true:
   (a) 43 + (–50) = –7 (b) 60 + (–85) = –25

6. Copy and complete the following table as far as you can:

(a)	(b)	(c)
5 – 8 =	5 + 8 =	8 – 3 =
5 – 7 =	5 + 7 =	7 – 3 =
5 – 6 =	5 + 6 =	6 – 3 =
5 – 5 =	5 + 5 =	5 – 3 =
5 – 4 =	5 + 4 =	4 – 3 =
5 – 3 =	5 + 3 =	3 – 3 =
5 – 2 =	5 + 2 =	2 – 3 =
5 – 1 =	5 + 1 =	1 – 3 =
5 – 0 =	5 + 0 =	0 – 3 =
5 – (–1) =	5 + (–1) =	(–1) – 3 =
5 – (–2) =	5 + (–2) =	(–2) – 3 =
5 – (–3) =	5 + (–3) =	(–3) – 3 =
5 – (–4) =	5 + (–4) =	(–4) – 3 =
5 – (–5) =	5 + (–5) =	(–5) – 3 =
5 – (–6) =	5 + (–6) =	(–6) – 3 =

7. Calculate each of the following:
   (a) 80 + (–60) (b) 500 + (–200) + (–200)

8. (a) Is 100 + (–20) + (–20) = 60, or does it equal to something else?
   (b) What do you think (–20) + (–20) is equal to?

9. Calculate each of the following:
   (a) 20 – 20 (b) 50 – 20 (c) (–20) – (–20) (d) (–50) – (–20)

10. Calculate each of the following:
   (a) 20 – (–10) (b) 100 – (–100) (c) 20 + (–10) (d) 100 + (–100)
   (e) (–20) – (–10) (f) (–100) – (–100) (g) (–20) + (–10) (h) (–100) + (–100)

11. Copy and complete the following table as far as you can:

(a)	(b)	(c)
5 – (–8) =	(–5) + 8 =	8 – (–3) =
5 – (–7) =	(–5) + 7 =	7 – (–3) =
5 – (–6) =	(–5) + 6 =	6 – (–3) =
5 – (–5) =	(–5) + 5 =	5 – (–3) =
5 – (–4) =	(–5) + 4 =	4 – (–3) =
5 – (–3) =	(–5) + 3 =	3 – (–3) =
5 – (–2) =	(–5) + 2 =	2 – (–3) =
5 – (–1) =	(–5) + 1 =	1 – (–3) =
5 – 0 =	(–5) + 0 =	0 – (–3) =
5 – 1 =	(–5) + (–1) =	(–1) – (–3) =
5 – 2 =	(–5) + (–2) =	(–2) – (–3) =
5 – 3 =	(–5) + (–3) =	(–3) – (–3) =
5 – 4 =	(–5) + (–4) =	(–4) – (–3) =
5 – 5 =	(–5) + (–5) =	(–5) – (–3) =

12. In each case, state whether the statement is true or false and give a numerical example to demonstrate your answer:
   (a) Subtracting a positive number from a negative number has the same effect as adding the additive inverse of the positive number.
   (b) Adding a negative number to a positive number has the same effect as adding the additive inverse of the negative number.
   (c) Subtracting a negative number from a positive number has the same effect as subtracting the additive inverse of the negative number.
   (d) Adding a negative number to a positive number has the same effect as subtracting the additive inverse of the negative number.

Maths_Gr8_LB_book.indb 24 2017/11/20 10:38:36 AM

(e) Adding a positive number to a negative number has the same effect as adding the additive inverse of the positive number.
(f) Adding a positive number to a negative number has the same effect as subtracting the additive inverse of the positive number.
(g) Subtracting a positive number from a negative number has the same effect as subtracting the additive inverse of the positive number.
(h) Subtracting a negative number from a positive number has the same effect as adding the additive inverse of the negative number.

COMPARING INTEGERS AND SOLVING PROBLEMS

1. Rewrite and include <, > or = to make the relationship between the numbers true:
   (a) –103 ☐ –99
   (b) –699 ☐ –701
   (c) 30 ☐ –30
   (d) 10 – 7 ☐ –(10 – 7)
   (e) –121 ☐ –200
   (f) 12 – 5 ☐ –(12 + 5)
   (g) –199 ☐ –110

2. At 5 a.m. in Bloemfontein the temperature was –5 °C. At 1 p.m., it was 19 °C. By how many degrees did the temperature rise?

3. A diver swims 150 m below the surface of the sea. She moves 75 m towards the surface. How far below the surface is she now?

4. One trench in the ocean is 800 m deep and another is 2 200 m deep. What is the difference in their depths?

5. An island has a mountain which is 1 200 m high. The surrounding ocean has a depth of 860 m. What is the difference in height?

6. On a winter's day in Upington the temperature rose by 19 °C. If the minimum temperature was –4 °C, what was the maximum temperature?

2.3 Multiplying and dividing with integers

MULTIPLICATION WITH INTEGERS

1. Calculate each of the following:
   (a) –5 + –5 + –5 + –5 + –5 + –5 + –5 + –5 + –5 + –5
   (b) –10 + –10 + –10 + –10 + –10
   (c) –6 + –6 + –6 + –6 + –6 + –6 + –6 + –6
   (d) –8 + –8 + –8 + –8 + –8 + –8
   (e) –20 + –20 + –20 + –20 + –20 + –20 + –20

2. In each case, show whether you agree (✓) or disagree (✗) with the given statement:
   (a) $10 \times (-5) = 50$ (b) $8 \times (-6) = (-8) \times 6$
   (c) $(-5) \times 10 = 5 \times (-10)$ (d) $6 \times (-8) = -48$
   (e) $(-5) \times 10 = 10 \times (-5)$ (f) $8 \times (-6) = 48$
   (g) $4 \times 12 = -48$ (h) $(-4) \times 12 = -48$

Multiplication of integers is commutative:
$(-20) \times 5 = 5 \times (-20)$

3. Is addition of integers commutative? Demonstrate your answer with three different examples.
4. Calculate each of the following:
   (a) $20 \times (-10)$ (b) $(-5) \times 4$ (c) $(-20) \times 10$
   (d) $4 \times (-25)$ (e) $29 \times (-20)$ (f) $(-29) \times (-2)$
5. Calculate each of the following:
   (a) $10 \times 50 + 10 \times (-30)$ (b) $50 + (-30)$ (c) $10 \times \mathbf{(}50 + (-30)\mathbf{)}$
   (d) $(-50) + (-30)$ (e) $10 \times (-50) + 10 \times (-30)$ (f) $10 \times \mathbf{(}(-50) + (-30)\mathbf{)}$

The product of two positive numbers is a positive number, for example: $5 \times 6 = 30$.

The product of a positive number and a negative number is a negative number, for example: $5 \times (-6) = -30$.

The product of a negative number and a positive number is a negative number, for example: $(-5) \times 6 = -30$.

6. (a) Four numerical expressions are given below. Which expressions do you expect to have the same answers? Do not do the calculations.

   $14 \times (23 + 58)$ $23 \times (14 + 58)$ $14 \times 23 + 14 \times 58$ $14 \times 23 + 58$

   (b) What property of operations is demonstrated by the fact that two of the above expressions have the same value?
7. Consider your answers for question 6.
   (a) Does multiplication distribute over addition in the case of integers?
   (b) Illustrate your answer with two examples.
8. Three numerical expressions are given below. Which expressions do you expect to have the same answers? Do not do the calculations.

   $10 \times \mathbf{(}(-50) - (-30)\mathbf{)}$ $10 \times (-50) - (-30)$ $10 \times (-50) - 10 \times (-30)$

9. Do the three sets of calculations given in question 8.

Maths_Gr8_LB_book.indb 26 2017/11/20 10:38:36 AM

Your work in questions 5, 8 and 9 demonstrates that multiplication with a positive number distributes over addition and subtraction of integers. For example:

10 × **(**5 + (–3)**)** = 10 × 2 = **20** and 10 × 5 + 10 × (–3) = 50 + (–30) = **20**

10 × **(**5 – (–3)**)** = 10 × 8 = **80** and 10 × 5 – 10 × (–3) = 50 – (–30) = **80**

10. Calculate: (–10) × **(**5 + (–3)**)**

Now consider the question of whether or not multiplication with a negative number distributes over addition and subtraction of integers. For example, would (–10) × 5 + (–10) × (–3) also have the answer –20, as does (–10) × **(**5 + (–3)**)**?

11. What must (–10) × (–3) be equal to, if we want (–10) × 5 + (–10) × (–3) to be equal to –20?

In order to ensure that multiplication distributes over addition and subtraction in the system of integers, we have to agree that

(a negative number) × (a negative number) is a positive number.

For example: (–10) × (–3) = 30.

12. Calculate each of the following:

(a) (–10) × (–5)
(b) (–10) × 5
(c) 10 × 5
(d) 10 × (–5)
(e) (–20) × (–10) + (–20) × (–6)
(f) (–20) × **(**(–10) + (–6)**)**
(g) (–20) × (–10) – (–20) × (–6)
(h) (–20) × **(**(–10) – (–6)**)**

Here is a summary of the **properties of integers** that make it possible to do calculations with integers:

- When a number is added to its additive inverse, the result is 0, for example (+12) + (–12) = 0.
- Adding an integer has the same effect as subtracting its additive inverse. For example, 3 + (–10) can be calculated by doing 3 – 10, and the answer is –7.
- Subtracting an integer has the same effect as adding its additive inverse. For example, 3 – (–10) can be calculated by doing 3 + 10, and the answer is 13.
- The product of a positive and a negative integer is negative, for example (–15) × 6 = –90.
- The product of a negative and a negative integer is positive, for example (–15) × (–6) = 90.

DIVISION WITH INTEGERS

1. (a) Calculate 25 × 8.
   (b) How much is 200 ÷ 25?
   (c) How much is 200 ÷ 8?

Division is the inverse of multiplication. Therefore, if two numbers and the value of their product are known, the answers to two division problems are also known.

2. Calculate each of the following:
   (a) 25 × (–8)
   (b) (–125) × 8

3. Use the work you have done for question 2 to write the answers for the following division questions:
   (a) (–1 000) ÷ (–125)
   (b) (–1 000) ÷ 8
   (c) (–200) ÷ 25
   (d) (–200) ÷ 8

4. Can you also work out the answers for the following division questions by using the work you have done for question 2?
   (a) 1 000 ÷ (–125)
   (b) (–1 000) ÷ (–8)
   (c) (–100) ÷ (–25)
   (d) 100 ÷ (–25)

When two numbers are multiplied, for example 30 × 4 = 120, the word "product" can be used in various ways to describe the situation:

- An expression that specifies multiplication only, such as 30 × 4, is called a **product** or a product expression.
- The answer obtained is also called the product of the two numbers. For example, 120 is called the **product of 30 and 4**.

An expression that specifies division only, such as 30 ÷ 5, is called a **quotient** or a **quotient expression**. The answer obtained is also called the quotient of the two numbers. For example, 6 is called the **quotient of 30 and 5**.

5. In each case, state whether you agree or disagree with the statement, and give an example to illustrate your answer:
   (a) The quotient of a positive and a negative integer is negative.
   (b) The quotient of a positive and a positive integer is negative.
   (c) The quotient of a negative and a negative integer is negative.
   (d) The quotient of a negative and a negative integer is positive.

6. Do the necessary calculations to enable you to provide the values of the quotients:
   (a) (–500) ÷ (–20)
   (b) (–144) ÷ 6
   (c) 1 440 ÷ (–60)
   (d) (–1 440) ÷ (–6)
   (e) –14 400 ÷ 600
   (f) 500 ÷ (–20)

THE ASSOCIATIVE PROPERTIES OF OPERATIONS WITH INTEGERS

Multiplication of whole numbers is **associative**. This means that in a product with several factors, the factors can be placed in any sequence, and the calculations can be performed in any sequence. For example, the following sequences of calculations will all produce the same answer:

A. 2 × 3, the answer of 2 × 3 multiplied by 5, the new answer multiplied by 10
B. 2 × 5, the answer of 2 × 5 multiplied by 10, the new answer multiplied by 3
C. 10 × 5, the answer of 10 × 5 multiplied by 3, the new answer multiplied by 2
D. 3 × 5, the answer of 3 × 5 multiplied by 2, the new answer multiplied by 10

1. Do the four sets of calculations given in A to D to check whether or not they really produce the same answers.
2. (a) If the numbers 3 and 10 in the calculation sequences A, B, C and D are replaced with –3 and –10, do you think the four answers will still be the same?
   (b) Investigate to check your expectation.

Multiplication with integers is associative.

The calculation sequence A can be represented in symbols in only two ways:

- 2 × 3 × 5 × 10: The convention to work from left to right unless otherwise indicated with brackets, ensures that this representation corresponds to A.
- 5 × (2 × 3) × 10, where brackets are used to indicate that 2 × 3 should be calculated first. When brackets are used, there are different possibilities to describe the same sequence.

3. Express the calculation sequences B, C and D given above symbolically, without using brackets.
4. Investigate, in the same way that you did for multiplication in question 2, whether or not addition with integers is associative. Use sequences of four integers.
5. (a) Calculate: 80 – 30 + 40 – 20
   (b) Calculate: 80 + (–30) + 40 + (–20)
   (c) Calculate: 30 – 80 + 20 – 40
   (d) Calculate: (–30) + 80 + (– 20) + 40
   (e) Calculate: 20 + 30 – 40 – 80

MIXED CALCULATIONS WITH INTEGERS

1. Calculate each of the following:
   (a) –3 × 4 + (–7) × 9
   (b) –20(–4 – 7)
   (c) 20 × (–5) – 30 × 7
   (d) –9(20 – 15)
   (e) –8 × (–6) – 8 × 3
   (f) (–26 – 13) ÷ (–3)
   (g) –15 × (–2) + (–15) ÷ (–3)
   (h) –15(2 – 3)
   (i) (–5 + –3) × 7
   (j) –5 × (–3 + 7) + 20 ÷ (–4)
2. Calculate each of the following:
   (a) 20 × (–15 + 6) – 5 × (–2 – 8) – 3 × (–3 – 8)
   (b) 40 × (7 + 12 – 9) + 25 ÷ (–5) – 5 ÷ 5

(c) $-50(20-25)+30(-10+7)-20(-16+12)$
(d) $-5\times(-3+12-9)$
(e) $-4\times(30-50)+7\times(40-70)-10\times(60-100)$
(f) $-3\times(-14+6)\times(-13+7)\times(-20+5)$
(g) $20\times(-5)+10\times(-3)+(-5)\times(-6)-(3\times5)$
(h) $-5(-20-5)+10(-7-3)-20(-15-5)+30(-40-35)$
(i) $(-50+15-75)\div(-11)+(6-30+12)\div(-6)$

2.4 Squares, cubes and roots with integers

SQUARES AND CUBES OF INTEGERS

1. Calculate each of the following:
   (a) 20×20
   (b) $20\times(-20)$
2. Write the answers for each of the following:
   (a) $(-20)\times20$
   (b) $(-20)\times(-20)$
3. Copy and complete the following table:

x	1	–1	2	–2	5	–5	10	–10
x^2 which is $x\times x$								
x^3								

4. In each case, state for which values of x, in the table in question 3, the given statement is true.
   (a) x^3 is a negative number
   (b) x^2 is a negative number
   (c) $x^2>x^3$
   (d) $x^2<x^3$
5. Copy and complete the following table:

x	3	–3	4	–4	6	–6	7	–7
x^2								
x^3								

6. Ben thinks of a number. He adds 5 to it and his answer is 12.
   (a) What number did he think of?
   (b) Is there another number that would also give 12 when 5 is added to it?
7. Lebo also thinks of a number. She multiplies the number by itself and gets 25.
   (a) What number did she think of?
   (b) Is there more than one number that will give 25 when multiplied by itself?

8. Mary thinks of a number and calculates (the number) × (the number) × (the number). Her answer is 27. What number did Mary think of?

10^2 is 100 and $(-10)^2$ is also 100.
Both 10 and (–10) are called **square roots** of 100.
10 may be called the **positive square root** of 100, and (–10) may be called the **negative square root** of 100.

9. Write the positive square root and the negative square root of each number:
(a) 64 (b) 9

10. Copy and complete the following table:

Number	1	4	9	16	25	36	49	64
Positive square root			3					8
Negative square root			–3					–8

11. Copy and complete the following tables:

(a)

x	1	2	3	4	5	6	7	8
x^3								

(b)

x	–1	–2	–3	–4	–5	–6	–7	–8
x^3								

3^3 is 27 and $(-5)^3$ is –125.
3 is called the **cube root** of 27, because $3^3 = 27$.
–5 is called the cube root of –125 because $(-5)^3 = -125$.

12. Copy and complete the following table:

Number	–1	8	–27	–64	–125	–216	1 000
Cube root			–3				10

The symbol $\sqrt{\ }$ is used to indicate "root".

$\sqrt[3]{-125}$ represents the **cube root** of –125. That means $\sqrt[3]{-125} = -5$.

$\sqrt[2]{36}$ represents the **positive square root** of 36, and $-\sqrt[2]{36}$ represents the **negative square root**. The "2" that indicates "square" is normally omitted, so $\sqrt{36} = 6$ and $-\sqrt{36} = -6$.

13. Copy and complete the following table:

$\sqrt[3]{-8}$	$\sqrt{121}$	$\sqrt[3]{-64}$	$-\sqrt{64}$	$\sqrt{64}$	$\sqrt[3]{-1}$	$-\sqrt{1}$	$\sqrt[3]{-216}$

WORKSHEET

1. Use the numbers –8, –5 and –3 to demonstrate each of the following:
   (a) Multiplication with integers distributes over addition.
   (b) Multiplication with integers distributes over subtraction.
   (c) Multiplication with integers is associative.
   (d) Addition with integers is associative.

2. Calculate each of the following without using a calculator:
   (a) $5 \times (-2)^3$
   (b) $3 \times (-5)^2$
   (c) $2 \times (-5)^3$
   (d) $10 \times (-3)^2$

3. Use a calculator to work out each of the following:
   (a) $24 \times (-53) + (-27) \times (-34) - (-55) \times 76$
   (b) $64 \times (27 - 85) - 29 \times (-47 + 12)$

4. Use a calculator to work out each of the following:
   (a) $-24 \times 53 + 27 \times 34 + 55 \times 76$
   (b) $64 \times (-58) + 29 \times (47 - 12)$

If you do not get the same answers in questions 3 and 4, you have made mistakes.

CHAPTER 3
Exponents

3.1 Revision

EXPONENTIAL NOTATION

1. Calculate each of the following:
   (a) $2 \times 2 \times 2$ (b) $2 \times 2 \times 2 \times 2 \times 2 \times 2$
   (c) $3 \times 3 \times 3$ (d) $3 \times 3 \times 3 \times 3 \times 3 \times 3$

Instead of writing $3 \times 3 \times 3 \times 3 \times 3 \times 3$, we can write 3^6.
We read this as "3 to the power of 6". The number 3 is the **base** and 6 is the **exponent**.
When we write $3 \times 3 \times 3 \times 3 \times 3 \times 3$ as 3^6, we are using **exponential notation**.

2. Write each of the following in exponential form:
   (a) $2 \times 2 \times 2$ (b) $2 \times 2 \times 2 \times 2 \times 2 \times 2$
   (c) $3 \times 3 \times 3$ (d) $3 \times 3 \times 3 \times 3 \times 3 \times 3$

3. Calculate each of the following:
   (a) 5^2 (b) 2^5 (c) 10^2 (d) 15^2
   (e) 3^4 (f) 4^3 (g) 2^3 (h) 3^2

SQUARES

To square a number is to multiply it by itself.
The square of 8 is 64 because 8×8 equals 64.
We write 8×8 as 8^2 in exponential form.
We read 8^2 as **eight squared**.

1. Copy and complete the following table:

	Number	Square the number	Exponential form	Square
(a)	1			
(b)	2			
(c)	3			

	Number	Square the number	Exponential form	Square
(d)	4			
(e)	5			
(f)	6			
(g)	7			
(h)	8	8×8	8^2	64
(i)	9			
(j)	10			
(k)	11			
(l)	12			

2. Calculate each of the following:
   (a) $3^2 \times 4^2$ (b) $2^2 \times 3^2$ (c) $2^2 \times 5^2$ (d) $2^2 \times 4^2$

3. Copy and complete the following statements to make them true:
   (a) $3^2 \times 4^2 = \ldots\ldots^2$ (b) $2^2 \times 3^2 = \ldots\ldots^2$
   (c) $2^2 \times 5^2 = \ldots\ldots^2$ (d) $2^2 \times 4^2 = \ldots\ldots^2$

CUBES

To cube a number is to multiply it by itself and then by itself again. The cube of 3 is 27 because $3 \times 3 \times 3$ equals 27.
We write $3 \times 3 \times 3$ as 3^3 in exponential form.
We read 3^3 as **three cubed**.

1. Copy and complete the following table:

	Number	Cube the number	Exponential form	Cube
(a)	1			
(b)	2			
(c)	3	$3 \times 3 \times 3$	3^3	27
(d)	4			
(e)	5			
(f)	6			
(g)	7			

	Number	Cube the number	Exponential form	Cube
(h)	8			
(i)	9			
(j)	10			

2. Calculate each of the following:
   (a) $2^3 \times 3^3$ (b) $2^3 \times 5^3$ (c) $2^3 \times 4^3$ (d) $1^3 \times 9^3$
3. Which of the following statements are true? If a statement is false, rewrite it as a true statement.
   (a) $2^3 \times 3^3 = 6^3$ (b) $2^3 \times 5^3 = 7^3$ (c) $2^3 \times 4^3 = 8^3$ (d) $1^3 \times 9^3 = 10^3$

SQUARE AND CUBE ROOTS

To find the square root of a number we ask the question:
Which number was multiplied by itself to get a square?

The square root of 16 is 4 because $4 \times 4 = 16$.

The question: **Which number was multiplied by itself to get 16?** is written mathematically as $\sqrt{16}$.
The answer to this question is written as $\sqrt{16} = 4$.

1. Copy and complete the following table:

	Number	Square of the number	Square root of the square of the number	Reason
(a)	1			
(b)	2			
(c)	3			
(d)	4	16	4	$4 \times 4 = 16$
(e)	5			
(f)	6			
(g)	7			
(h)	8			
(i)	9			
(j)	10			
(k)	11			
(l)	12			

2. Calculate the following. Justify your answer.

(a) $\sqrt{144}$ (b) $\sqrt{100}$ (c) $\sqrt{81}$ (d) $\sqrt{64}$

To find the cube root of a number we ask the question: Which number was multiplied by itself and again by itself to get a cube?

The cube root of 64 is 4 because $4 \times 4 \times 4 = 64$.

The question: **Which number was multiplied by itself and again by itself (or cubed) to get 64?** is written mathematically as $\sqrt[3]{64}$. The answer to this question is written as $\sqrt[3]{64} = 4$.

3. Copy and complete the following table:

	Number	Cube of the number	Cube root of the cube of the number	Reason
(a)	1			
(b)	2			
(c)	3			
(d)	4	64	4	$4 \times 4 \times 4 = 64$
(e)	5			
(f)	6			
(g)	7			
(h)	8			
(i)	9			
(j)	10			

4. Calculate the following and give reasons for your answers:

(a) $\sqrt[3]{216}$ (b) $\sqrt[3]{8}$ (c) $\sqrt[3]{125}$

(d) $\sqrt[3]{27}$ (e) $\sqrt[3]{64}$ (f) $\sqrt[3]{1\,000}$

3.2 Working with integers

REPRESENTING INTEGERS IN EXPONENTIAL FORM

1. Calculate the following, without using a calculator:

(a) $-2 \times -2 \times -2$ (b) $-2 \times -2 \times -2 \times -2$

(c) -5×-5 (d) $-5 \times -5 \times -5$

(e) $-1 \times -1 \times -1 \times -1$ (f) $-1 \times -1 \times -1$

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2. Calculate each of the following:

(a) -2^2 (b) $(-2)^2$

(c) $(-5)^3$ (d) -5^3

3. Use your calculator to work out the answers to question 2.

(a) Are your answers to question 2(a) and (b) different or the same as those of the calculator?

(b) If your answers are different to those of the calculator, try to explain how the calculator did the calculations differently from you.

The calculator "understands" $\mathbf{-5^2}$ and $\mathbf{(-5)^2}$ as two different numbers.

It understands $\mathbf{-5^2}$ as $\mathbf{-5 \times 5 = -25}$ and $\mathbf{(-5)^2}$ as $\mathbf{-5 \times -5 = 25}$.

4. Write the following in exponential form:

(a) $-2 \times -2 \times -2$ (b) $-2 \times -2 \times -2 \times -2$

(c) -5×-5 (d) $-5 \times -5 \times -5$

(e) $-1 \times -1 \times -1 \times -1$ (f) $-1 \times -1 \times -1$

5. Calculate each of the following:

(a) $(-3)^2$ (b) $(-3)^3$

(c) $(-2)^4$ (d) $(-2)^6$

(e) $(-2)^5$ (f) $(-3)^4$

6. Say whether the sign of the answer is negative or positive. Explain why.

(a) $(-3)^6$ (b) $(-5)^{11}$

(c) $(-4)^{20}$ (d) $(-7)^5$

7. Say whether the following statements are true or false. If a statement is false, rewrite it as a correct statement.

(a) $(-3)^2 = -9$ (b) $-3^2 = 9$

(c) $(-5^2) = -5^2$ (d) $(-1)^3 = -1^3$

(e) $(-6)^3 = -18$ (f) $(-2)^6 = 2^6$

3.3 Laws of exponents

PRODUCT OF POWERS

1. A product of 2s is given below. Describe it using exponential notation, that is, write it as a power of 2.

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

2. Express each of the following as a product of the powers of 2, as indicated by the brackets.
   (a) $(2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)$
   (b) $(2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2)$
   (c) $(2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)$
   (d) $(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$
   (e) $(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2)$

3. Copy and complete the following statements so that they are true. You may want to refer to your answers to question 2(a) to (e) to help you.
   (a) $2^3 \times \ldots\ldots = 2^{12}$
   (b) $2^5 \times \ldots\ldots \times 2^2 = 2^{12}$
   (c) $2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 = \ldots\ldots$
   (d) $2^8 \times \ldots\ldots = 2^{12}$
   (e) $2^3 \times 2^3 \times 2^3 \times \ldots\ldots = 2^{12}$
   (f) $2^6 \times \ldots\ldots = 2^{12}$
   (g) $2^2 \times 2^{10} = \ldots\ldots$

Suppose we are asked to simplify: $3^2 \times 3^4$.

The solution is: $3^2 \times 3^4 = 9 \times 81$

$= 729$

$= 3^6$

The base (3) is a repeated factor. The exponents (2 and 4) tell us the number of times each factor is repeated.

We can explain this solution in the following manner:

$$3^2 \times 3^4 = \underbrace{3 \times 3}_{\text{2 factors}} \times \underbrace{3 \times 3 \times 3 \times 3}_{\text{4 factors}} = \underbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3}_{\text{6 factors}} = 3^6$$

4. Copy and complete the following table:

	Product of powers	Repeated factor	Total number of times the factor is repeated	Simplified form
(a)	$2^7 \times 2^3$			
(b)	$5^2 \times 5^4$			
(c)	$4^1 \times 4^5$			
(d)	$6^3 \times 6^2$			
(e)	$2^8 \times 2^2$			
(f)	$5^3 \times 5^3$			
(g)	$4^2 \times 4^4$			
(h)	$2^1 \times 2^9$			

When you multiply two or more powers that have the same base, the answer has the same base, but its exponent is equal to the sum of the exponents of the numbers you are multiplying.

We can express this symbolically as $a^m \times a^n = a^{m+n}$, where m and n are natural numbers and a is not zero.

5. What is wrong with these statements? Correct each one.
   (a) $2^3 \times 2^4 = 2^{12}$
   (b) $10 \times 10^2 \times 10^3 = 10^{1 \times 2 \times 3} = 10^6$
   (c) $3^2 \times 3^3 = 3^6$
   (d) $5^3 \times 5^2 = 15 \times 10$

6. Express each of the following numbers as a single power of 10.
   Example: 1 000 000 as a power of 10 is 10^6.
   (a) 100
   (b) 1 000
   (c) 10 000
   (d) $10^2 \times 10^3 \times 10^4$
   (e) 100 × 1 000 × 10 000
   (f) 1 000 000 000

7. Write each of the following products in exponential form:
   (a) $x \times x \times x \times x \times x \times x \times x \times x \times x$
   (b) $(x \times x) \times (x \times x \times x) \times (x \times x \times x \times x)$
   (c) $(x \times x \times x \times x) \times (x \times x) \times (x \times x) \times x$
   (d) $(x \times x \times x \times x \times x \times x) \times (x \times x \times x)$
   (e) $(x \times x \times x) \times (y \times y \times y)$
   (f) $(a \times a) \times (b \times b)$

8. Copy and complete the following table:

	Product of powers	Repeated factor	Total number of times the factor is repeated	Simplified form
(a)	$x^7 \times x^3$			
(b)	$x^2 \times x^4$			
(c)	$x^1 \times x^5$			
(d)	$x^3 \times x^2$			
(e)	$x^8 \times x^2$			
(f)	$x^3 \times x^3$			
(g)	$x^1 \times x^9$			

RAISING A POWER TO A POWER

1. Copy and complete the table of powers of 2:

x	1	2	3	4	5	6	7	8	9	10	11
2^x	2	4									
	2^1	2^2	2^3								

x	12	13	14	15	16	17	18
2^x							

2. Copy and complete the table of powers of 3:

x	1	2	3	4	5	6	7	8	9
3^x									
	3^1	3^2	3^3						

x	10	11	12	13	14
3^x					

3. Copy and complete the table. You can read the values from the tables you made in questions 1 and 2.

Product of powers	Repeated factor	Power of power notation	Total number of repetitions	Simplified form	Value
$2^4 \times 2^4 \times 2^4$	2	$(2^4)^3$	12	2^{12}	4 096
$3^2 \times 3^2 \times 3^2 \times 3^2$					
$2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3$					
$3^4 \times 3^4 \times 3^4$					
$2^6 \times 2^6 \times 2^6$					

4. Copy and complete by using your table of powers of 2 to find the answers for the following:
   (a) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ = ... = ...
   (b) $(2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2)$ = ... = ...
   (c) 16^3 = ... = ... = ...

5. Use your table of powers of 2 to find the answers for the following:
   (a) Is $16^3 = 2^{12}$?
   (b) Is $2^4 \times 2^4 \times 2^4 = 2^{12}$?
   (c) Is $2^4 \times 2^3 = 2^{12}$?
   (d) Is $(2^4)^3 = 2^4 \times 2^4 \times 2^4$?
   (e) Is $(2^4)^3 = 2^{12}$?
   (f) Is $(2^4)^3 = 2^{4+3}$?
   (g) Is $(2^4)^3 = 2^{4 \times 3}$?
   (h) Is $(2^2)^5 = 2^{2+5}$?

6. (a) Express 8^5 as a power of 2. It may help to first express 8 as a power of 2.
   (b) Can $(2^3) \times (2^3) \times (2^3) \times (2^3) \times (2^3)$ be expressed as $(2^3)^5$?
   (c) Is $(2^3)^5 = 2^{3+5}$ or is $(2^3)^5 = 2^{3 \times 5}$?

7. (a) Express 4^3 as a power of 2.
   (b) Calculate $2^2 \times 2^2 \times 2^2$ and express your answer as a single power of 2.
   (c) Can $(2^2) \times (2^2) \times (2^2)$ be expressed as $(2^2)^3$?
   (d) Is $(2^2)^3 = 2^{2+3}$ or is $(2^2)^3 = 2^{2 \times 3}$?

8. Simplify the following:
   Example: $(10^2)^2 = 10^2 \times 10^2 = 10^{2+2} = 10^4 = 10\,000$

   (a) $(3^3)^2$ (b) $(4^3)^2$ (c) $(2^4)^2$ (d) $(9^2)^2$
   (e) $(3^3)^3$ (f) $(4^3)^3$ (g) $(5^4)^3$ (h) $(9^2)^3$

$(a^m)^n = a^{m \times n}$, where m and n are natural numbers and a is not equal to zero.

9. Simplify:
   (a) $(5^4)^{10}$ (b) $(10^4)^5$ (c) $(6^4)^4$ (d) $(5^4)^{10}$

10. Write 5^{12} as a power of powers of 5 in two different ways.

To simplify $(x^2)^5$ we can write it out as a product of powers or we can use a shortcut.

$$(x^2)^5 = x^2 \times x^2 \times x^2 \times x^2 \times x^2$$
$$= \underbrace{x \times x}\times\underbrace{x \times x}\times\underbrace{x \times x}\times\underbrace{x \times x}\times\underbrace{x \times x} = x^{10}$$

2×5 factors = 10 factors

11. Copy and complete the following table:

	Expression	Write as a product of the powers and then simplify	Use the rule $(a^m)^n$ to simplify
(a)	$(a^4)^5$	$a^4 \times a^4 \times a^4 \times a^4 \times a^4$ $= a^{4+4+4+4+4} = a^{20}$	$(a^4)^5 = a^{4 \times 5} = a^{20}$
(b)	$(b^{10})^5$		
(c)	$(x^7)^3$		
(d)		$s^6 \times s^6 \times s^6 \times s^6$ $= s^{6+6+6+6}$ $= s^{24}$	
(e)			$y^{3 \times 7} = y^{21}$

POWER OF A PRODUCT

1. Copy and complete the table. You may use your calculator when you are not sure of a value.

	x	1	2	3	4	5
(a)	2^x	$2^1 = 2$				
(b)	3^x		$3^2 = 9$			
(c)	6^x			$6^3 = 216$		

2. Use the table in question 1 to answer the questions below. Are these statements true or false? If a statement is false, rewrite it as a correct statement.
   (a) $6^2 = 2^2 \times 3^2$
   (b) $6^3 = 2^3 \times 3^3$
   (c) $6^5 = 2^5 \times 3^5$
   (d) $6^8 = 2^4 \times 3^4$

3. Copy and complete the following table:

	Expression	The bases of the expression are factors of ...	Equivalent expression
(a)	$2^6 \times 5^6$	10	10^6
(b)	$3^2 \times 4^2$		
(c)	$4^2 \times 2^2$		
(d)			56^5
(e)			30^3
(f)	$3^5 \times x^5$	$3x$	$(3x)^5$
(g)	$7^2 \times z^2$		
(h)	$4^3 \times y^3$		
(i)			$(2m)^6$
(j)			$(2m)^3$
(k)	$2^{10} \times y^{10}$		$(2y)^{10}$

12^2 can be written in terms of its factors as $(2 \times 6)^2$ or as $(3 \times 4)^2$.

We already know that $12^2 = 144$.

What this tells us is that both $(2 \times 6)^2$ and $(3 \times 4)^2$ also equal to 144.

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We write $12^2 = (2 \times 6)^2$ or $12^2 = (3 \times 4)^2$

$= 2^2 \times 6^2$ $= 3^2 \times 4^2$

$= 4 \times 36$ $= 9 \times 16$

$= 144$ $= 144$

A product raised to a power is the product of the factors each raised to the same power.

Using symbols, we write $(a \times b)^m = a^m \times b^m$, where m is a natural number and a and b are not equal to zero.

4. Write each of the following expressions as an expression with one base.

Example: $3^{10} \times 2^{10} = (3 \times 2)^{10} = 6^{10}$

(a) $3^2 \times 5^2$ (b) $5^3 \times 2^3$ (c) $7^4 \times 4^4$

(d) $2^3 \times 6^3$ (e) $4^4 \times 2^4$ (f) $5^2 \times 7^2$

5. Write the following as a product of powers.

Example: $(3x)^3 = 3^3 \times x^3 = 27x^3$

(a) 6^3 (b) 15^2 (c) 21^4 (d) 6^5

(e) 18^2 (f) $(st)^7$ (g) $(ab)^3$ (h) $(2x)^2$

(i) $(3y)^5$ (j) $(3c)^2$ (k) $(gh)^4$ (l) $(4x)^3$

6. Simplify the following expressions.

Example: $3^2 \times m^2 = 9 \times m^2 = 9m^2$

(a) $3^5 \times b^5$ (b) $2^6 \times y^6$ (c) $x^2 \times y^2$

(d) $10^4 \times x^4$ (e) $3^3 \times x^3$ (f) $5^2 \times t^2$

(g) $6^3 \times m^7$ (h) $12^2 \times a^2$ (i) $n^3 \times p^9$

A QUOTIENT OF POWERS

Consider the following table:

x	1	2	3	4	5	6
2^x	2	4	8	16	32	64
3^x	3	9	27	81	243	729
5^x	5	25	125	625	3 125	15 625

Answer questions 1 to 4 by referring to the table when needed.

1. Give the value of each of the following:

(a) 3^4 (b) 2^5 (c) 5^6

2. (a) Calculate $3^6 \div 3^3$. (Read the values of 3^6 and 3^3 from the table and then divide. You may use a calculator where necessary.)

(b) Calculate 3^{6-3}.

(c) Is $3^6 \div 3^3$ equal to 3^3? Explain.

3. (a) Calculate the value of 2^{6-2}.
(b) Calculate the value of $2^6 \div 2^2$.
(c) Calculate the value of $2^{6 \div 2}$.
(d) Read the value of 2^3 from the table.
(e) Read the value of 2^4 from the table.
(f) Which of the statements below is true? Give an explanation for your answer.

A. $2^6 \div 2^2 = 2^{6-2} = 2^4$ B. $2^6 \div 2^2 = 2^{6 \div 2} = 2^3$

To calculate 4^{5-3}, we first do the calculation in the exponent, that is, we subtract 3 from 5. Then we can calculate 4^2 as $4 \times 4 = 16$.

4. Say which of the statements below are true and which are false. Rewrite false statements as correct statements.
(a) $5^6 \div 5^4 = 5^{6 \div 4}$ (b) $3^{4-1} = 3^4 \div 3$
(c) $5^6 \div 5 = 5^{6-1}$ (d) $2^5 \div 2^3 = 2^2$

$a^m \div a^n = a^{m-n}$
Here, m and n are natural numbers, and m is a number greater than n and a is not zero.

5. Simplify the following. Do not use a calculator.
Example: $3^{17} \div 3^{12} = 3^{17-12} = 3^5 = 243$
(a) $2^{12} \div 2^{10}$ (b) $6^{17} \div 6^{14}$ (c) $10^{20} \div 10^{14}$ (d) $5^{11} \div 5^8$

6. Simplify the following:
(a) $x^{12} \div x^{10}$ (b) $y^{17} \div y^{14}$ (c) $t^{20} \div t^{14}$ (d) $n^{11} \div n^8$

THE POWER OF ZERO

1. Simplify the following:
(a) $2^{12} \div 2^{12}$ (b) $6^{17} \div 6^{17}$ (c) $6^{14} \div 6^{14}$ (d) $2^{10} \div 2^{10}$

We define $a^0 = 1$.
Any number raised to the power of zero is always equal to 1.

2. Simplify the following:
(a) 100^0 (b) x^0 (c) $(100x)^0$ (d) $(5x^3)^0$

3.4 Calculations

MIXED OPERATIONS

Simplify the following:

1. $3^3 + \sqrt[3]{-27} \times 2$ 2. $5 \times (2 + 3)^2 + (-1)^0$

3. $3^2 \times 2^3 + 5 \times \sqrt{100}$

4. $\frac{\sqrt[3]{1\,000}}{\sqrt{100}} + (4 - 1)^2$

5. $\sqrt{16} \times \sqrt{16} + \sqrt[3]{216} + 3^2 \times 10$

6. $4^3 \div 2^3 + \sqrt{144}$

3.5 Squares, cubes and roots of rational numbers

SQUARING A FRACTION

Squaring or cubing a fraction or a decimal fraction is no different from squaring or cubing an integer.

1. Copy and complete the following table:

	Fraction	Square the fraction	Value of the square of the fraction
(a)	$\frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
(b)	$\frac{2}{3}$		
(c)	$\frac{3}{4}$		
(d)	$\frac{2}{5}$		
(e)	$\frac{3}{5}$		
(f)	$\frac{2}{6}$		
(g)	$\frac{3}{7}$		
(h)	$\frac{11}{12}$		

2. Calculate each of the following:

(a) $\left(\frac{3}{2}\right)^2$ (b) $\left(\frac{4}{5}\right)^2$ (c) $\left(\frac{7}{8}\right)^2$

3. (a) Use the fact that 0,6 can be written as $\frac{6}{10}$ to calculate $(0{,}6)^2$.

(b) Use the fact that 0,8 can be written as $\frac{8}{10}$ to calculate $(0{,}8)^2$.

FINDING THE SQUARE ROOT OF A FRACTION

1. Copy and complete the following table:

	Fraction	Writing the fraction as a product of factors	Square root
(a)	$\frac{81}{121}$		
(b)	$\frac{64}{81}$		
(c)	$\frac{49}{169}$		
(d)	$\frac{100}{225}$		

2. Determine the following:

(a) $\sqrt{\frac{25}{16}}$ (b) $\sqrt{\frac{81}{144}}$ (c) $\sqrt{\frac{400}{900}}$ (d) $\sqrt{\frac{36}{81}}$

3. (a) Use the fact that 0,01 can be written as $\frac{1}{100}$ to calculate $\sqrt{0{,}01}$.

(b) Use the fact that 0,49 can be written as $\frac{49}{100}$ to calculate $\sqrt{0{,}49}$.

4. Calculate each of the following:

(a) $\sqrt{0{,}09}$ (b) $\sqrt{0{,}64}$ (c) $\sqrt{1{,}44}$

CUBING A FRACTION

One half cubed is equal to one eighth.

We write this as $\left(\frac{1}{2}\right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

1. Calculate each of the following:

(a) $\left(\frac{2}{3}\right)^3$ (b) $\left(\frac{5}{10}\right)^3$ (c) $\left(\frac{5}{6}\right)^3$ (d) $\left(\frac{4}{5}\right)^3$

2. (a) Use the fact that 0,6 can be written as $\frac{6}{10}$ to find $(0{,}6)^3$.

(b) Use the fact that 0,8 can be written as $\frac{8}{10}$ to calculate $(0{,}8)^3$.

(c) Use the fact that 0,7 can be written as $\frac{7}{10}$ to calculate $(0{,}7)^3$.

3.6 Scientific notation

VERY LARGE NUMBERS

1. Express each of the following as a single number. Do not use a calculator.
   Example: $7{,}56 \times 100$ can be written as 756.

   (a) $3{,}45 \times 100$ (b) $3{,}45 \times 10$ (c) $3{,}45 \times 1\,000$
   (d) $2{,}34 \times 10^2$ (e) $2{,}34 \times 10$ (f) $2{,}34 \times 10^3$
   (g) $10^4 \times 10^2$ (h) $10^0 \times 10^6$ (i) $3{,}4 \times 10^5$

We can write 136 000 000 as $1{,}36 \times 10^8$.
$1{,}36 \times 10^8$ is called the **scientific notation** for 136 000 000.
In scientific notation, a number is expressed in two parts: a number between 1 and 10 multiplied by a power of 10. The exponent must always be an integer.

2. Write the following numbers in scientific notation:
   (a) 367 000 000 (b) 21 900 000
   (c) 600 000 000 000 (d) 178

3. Write each of the following numbers in the ordinary way.
   For example: $3{,}4 \times 10^5$ written in the ordinary way is 340 000.

   (a) $1{,}24 \times 10^8$ (b) $9{,}2074 \times 10^4$ (c) $1{,}04 \times 10^6$ (d) $2{,}05 \times 10^3$

4. The universe is 15 000 000 000 years old. Express the age of the universe in scientific notation.

5. The average distance from the earth to the sun is 149 600 000 km. Express this distance in scientific notation.

Because it is easier to multiply powers of ten without a calculator, **scientific notation** makes it possible to do calculations in your head.

6. Explain why the number 24×10^3 is not in scientific notation.

7. Calculate the following. Do not use a calculator.
   Example: $3\,000\,000 \times 90\,000\,000 = 3 \times 10^6 \times 9 \times 10^7 = 3 \times 9 \times 10^{6+7}$
   $= 27 \times 10^{13} = 270\,000\,000\,000\,000$

   (a) $13\,000 \times 150\,000$ (b) $200 \times 6\,000\,000$
   (c) $120\,000 \times 120\,000\,000$ (d) $2{,}5 \times 40\,000\,000$

8. Copy the statements and use > or < to compare these numbers:
   (a) $1{,}3 \times 10^9$ ☐ $2{,}4 \times 10^7$ (b) $6{,}9 \times 10^2$ ☐ $4{,}5 \times 10^3$
   (c) $7{,}3 \times 10^4$ ☐ $7{,}3 \times 10^2$ (d) $3{,}9 \times 10^6$ ☐ $3{,}7 \times 10^7$

WORKSHEET

1. Calculate:

(a) 11^2

(b) $3^2 \times 4^2$

(c) 6^3

(d) $\sqrt{121}$

(e) $(-3)^2$

(f) $\sqrt[3]{125}$

2. Simplify:

(a) $3^4 \times m^6$

(b) $b^2 \times n^6$

(c) $y^{12} \div y^5$

(d) $(10^2)^3$

(e) $(2w^2)^3$

(f) $(3d^5)(2d)^3$

3. Calculate:

(a) $\left(\frac{2}{5}\right)^2$

(b) $\sqrt{\frac{9}{25}}$

(c) $(6^4y^2)^0$

(d) $(0{,}7)^2$

4. Simplify:

(a) $(2^2 + 4)^2 + \frac{6^2}{3^2}$

(b) $\sqrt[3]{-125} - 5 \times 3^2$

5. Write 3×10^9 in the ordinary way.

6. The first birds appeared on earth about 208 000 000 years ago. Write this number in scientific notation.

Chapter 4
Numeric and geometric patterns

4.1 The term-term relationship in a sequence

GOING FROM ONE TERM TO THE NEXT

Write down the next three numbers in each of the sequences below. Also explain in writing, in each case, how you figured out what the numbers should be.

A list of numbers which form a pattern is called a **sequence**. Each number in a sequence is called a **term** of the sequence. The first number is the first term of the sequence.

1. Sequence A: 2; 5; 8; 11; 14; 17; 20; 23; .....
2. Sequence B: 4; 5; 8; 13; 20; 29; 40; .....
3. Sequence C: 1; 2; 4; 8; 16; 32; 64; .....
4. Sequence D: 3; 5; 7; 9; 11; 13; 15; 17; 19; .....
5. Sequence E: 4; 5; 7; 10; 14; 19; 25; 32; 40; .....
6. Sequence F: 2; 6; 18; 54; 162; 486; .....
7. Sequence G: 1; 5; 9; 13; 17; 21; 25; 29; 33; .....
8. Sequence H: 2; 4; 8; 16; 32; 64; .....

Numbers that follow one another are said to be **consecutive**.

ADDING OR SUBTRACTING THE SAME NUMBER

1. Which sequences above are of the same kind as sequence A? Explain your answer.

Amanda explains how she figured out how to continue sequence A:

I looked at the first two numbers in the sequence and saw that I needed 3 to go from 2 to 5. I looked further and saw that I also needed 3 to go from 5 to 8. I tested that and it worked for all the next numbers.

This gave me a rule I could use to extend the sequence: add 3 to each number to find the next number in the pattern.

Tamara says you can also find the pattern by working backwards and subtracting 3 each time:

$14 - 3 = 11; \; 11 - 3 = 8; \; 8 - 3 = 5; \; 5 - 3 = 2$

When the **differences** between consecutive terms of a sequence are the same, we say the difference is **constant**.

2. Provide a rule to describe the relationship between the numbers in the sequence. Use this rule to calculate the missing numbers in each sequence.
   (a) 1; 8; 15; .....; .....; .....; .....; .....; ...
   (b) 10 020; ....; ....; ....; 9 980; 9 970; ....; ....; 9 940; 9 930; ...
   (c) 1,5; 3,0; 4,5; .....; .....; .....; .....; .....; ...
   (d) 2,2; 4,0; 5,8; ......; ......; ......; ......; ......; ...
   (e) $45\frac{3}{4}$; $46\frac{1}{2}$; $47\frac{1}{4}$; 48; ......; ......; ......; ......; ......; ...
   (f) ....; 100,49; 100,38; 100,27; ....; ....; 99,94; 99,83; 99,72; ...
3. Copy and complete the following table:

Input number	1	2	3	4	5		12		n
Input number + 7	8			11		15		30	

MULTIPLYING OR DIVIDING WITH THE SAME NUMBER

Take another look at sequence F on page 49: 2; 6; 18; 54; 162; 486; ...

Piet explains that he figured out how to continue the sequence F:

I looked at the first two terms in the sequence and wrote $2 \times ? = 6$.
When I multiplied the first number by 3, I got the second number: $2 \times 3 = 6$.
I then checked to see if I could find the next number if I multiplied 6 by 3: $6 \times 3 = 18$.
I continued checking in this way: $18 \times 3 = 54$; $54 \times 3 = 162$ and so on.
This gave me a rule I can use to extend the sequence and my rule was: multiply each number by 3 to calculate the next number in the sequence.

Zinhle says you can also find the pattern by working backwards and dividing by 3 each time:

$54 \div 3 = 18; \; 18 \div 3 = 6; \; 6 \div 3 = 2$

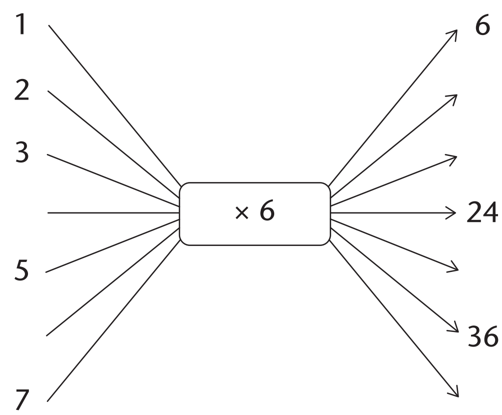
The number that we multiply with to get the next term in the sequence is called a **ratio**. If the number we multiply with remains the same throughout the sequence, we say it is a **constant ratio**.

1. Check whether Piet's reasoning works for sequence H on page 49: 2; 4; 8; 16; 32; 64; ...
2. Describe, in words, the rule for finding the next number in the sequences on the next page. Write down the next five terms of each sequence.

(a) 1; 10; 100; 1 000; . . . . . (b) 16; 8; 4; 2; . . . . . (c) 7; –21; 63; –189; . . . . .

(d) 3; 12, 48; . . . . . (e) 2 187; –729; 243; –81; . . . . .

3. (a) Copy and fill in the missing output and input numbers:



What is the term-to-term rule for the output numbers here, + 6 or × 6?

(b) Copy and complete the following table:

Input numbers	1	2	3	4	5		12	x
Output numbers	6			24		36		

NEITHER ADDING NOR MULTIPLYING BY THE SAME NUMBER

1. Consider sequences A to H again and answer the questions that follow:
   Sequence A: 2; 5; 8; 11; 14; 17; 20; 23; ...
   Sequence B: 4; 5; 8; 13; 20; 29; 40; ...
   Sequence C: 1; 2; 4; 8; 16; 32; 64; ...
   Sequence D: 3; 5; 7; 9; 11; 13; 15; 17; 19; ...
   Sequence E: 4; 5; 7; 10; 14; 19; 25; 32; 40; ...
   Sequence F: 2; 6; 18; 54; 162; 486; ...
   Sequence G: 1; 5; 9; 13; 17; 21; 25; 29; 33; ...
   Sequence H: 2; 4; 8; 16; 32; 64; ....

   (a) Which other sequence(s) is/are of the same kind as sequence B? Explain.
   (b) In what way are sequences B and E different from the other sequences?

There are sequences where there is neither a constant difference nor a constant ratio between consecutive terms and yet a pattern still exists, as in the case of sequences B and E.

2. Consider the sequence: 10; 17; 26; 37; 50; ...
   (a) Write down the next five numbers in the sequence.
   (b) Eric observed that he can calculate the next term in the sequence as follows: 10 + **7** = 17; 17 + **9** = 26; 26 + **11** = 37. Use Eric's method to check whether your numbers in question 2(a) above are correct.

3. Which of the statements below can Eric use to describe the relationship between the numbers in the sequence in question 2? Test the rule for the first three terms of the sequence and then simply write "yes" or "no" next to each statement.
   (a) Increase the difference between consecutive terms by two each time.
   (b) Increase the difference between consecutive terms by one each time.
   (c) Add two more than you added to get the previous term.
4. Provide a rule to describe the relationship between the numbers in the sequences below. Use your rule to provide the next five numbers in each sequence.
   (a) 1; 4; 9; 16; 25; .....
   (b) 2; 13; 26; 41; 58; .....
   (c) 4; 14; 29; 49; 74; .....
   (d) 5; 6; 8; 11; 15; 20; .....

4.2 The position-term relationship in a sequence

USING POSITION TO MAKE PREDICTIONS

1. Take another look at sequences A to H. Which sequence(s) are of the same kind as sequence A? Explain.

Sequence A: 2; 5; 8; 11; 14; 17; 20; 23;...
Sequence B: 4; 5; 8; 13; 20; 29; 40; ...
Sequence C: 1; 2; 4; 8; 16; 32; 64; ...
Sequence D: 3; 5; 7; 9; 11; 13; 15; 17; 19; ...
Sequence E: 4; 5; 7; 10; 14; 19; 25; 32; 40; ...
Sequence F: 2; 6; 18; 54; 162; 486; ...
Sequence G: 1; 5; 9; 13; 17; 21; 25; 29; 33; ...
Sequence H: 2; 4; 8; 16; 32; 64; ...

Sizwe has been thinking about Amanda and Tamara's explanations of how they worked out the rule for sequence A and has drawn up a table. He agrees with them but says that there is another rule that will also work. He explains:

My table shows the terms in the sequence and the difference between consecutive terms:

	1st term	2nd term	3rd term	4th term						
A:	5	8	11	14						
differences	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	

Sizwe reasons that the following rule will also work:

Multiply the position of the number by 3 and add 2 to the answer.
I can write this rule as a number sentence: Position of the number $\times 3 + 2$
I use my number sentence to check: $\mathbf{1} \times 3 + 2 = 5$; $\mathbf{2} \times 3 + 2 = 8$; $\mathbf{3} \times 3 + 2 = 11$

2. (a) What do the numbers in bold in Sizwe's number sentence stand for?
   (b) What does the number 3 in Sizwe's number sentence stand for?
3. Consider the sequence 5; 8; 11; 14; ...
   Apply Sizwe's rule to the sequence and determine:
   (a) term number 7 of the sequence
   (b) term number 10 of the sequence
   (c) the hundredth term of the sequence
4. Consider the sequence: 3; 5; 7; 9; 11; 13; 15; 17; 19; ...
   (a) Use Sizwe's explanation to find a rule for this sequence.
   (b) Determine the 28th term of the sequence.

MORE PREDICTIONS

Copy and complete the following tables by calculating the missing terms:

1.

Position in sequence	1	2	3	4	10	54
Term	4	7	10	13		

2.

Position in sequence	1	2	3	4	8	16
Term	4	9	14	19		

3.

Position in sequence	1	2	3	4	7	30
Term	3	15	27			

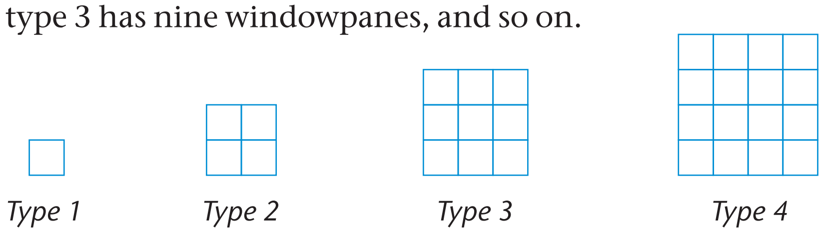
4. Copy the following table. Use the rule **position in the sequence** × **(position in the sequence + 1)** to complete it.

Position in sequence	1	2	3	4	5	6
Term	2					

4.3 Investigating and extending geometric patterns

SQUARE NUMBERS

A factory makes window frames. Type 1 has one windowpane, type 2 has four windowpanes, type 3 has nine windowpanes, and so on.

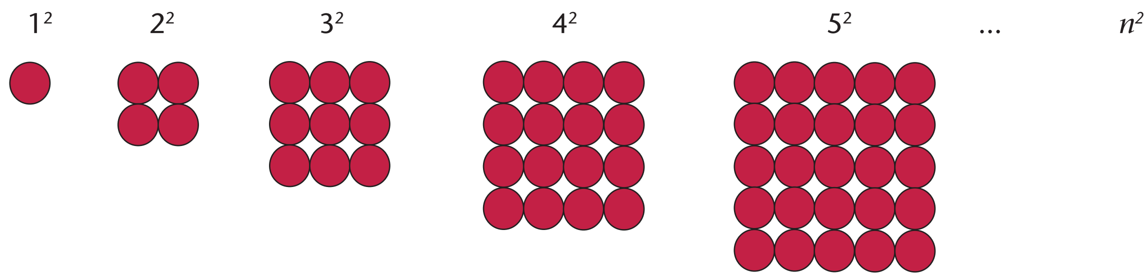


1. How many windowpanes will there be in type 5?
2. How many windowpanes will there be in type 6?
3. How many windowpanes will there be in type 7?
4. How many windowpanes will there be in type 12? Explain.
5. Copy and complete the table. Show your calculations.

Frame type	1	2	3	4	15	20
Number of windowpanes	1	4	9	16		

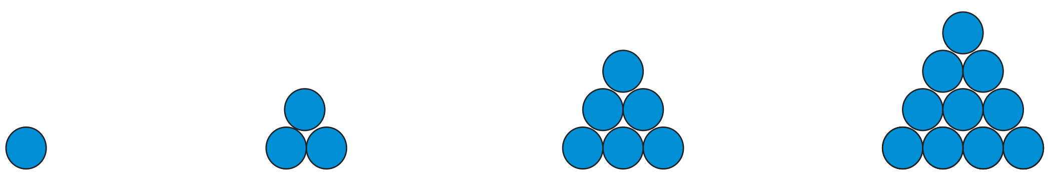
In algebra we think of a square as a number that is obtained by multiplying a number by itself. So, 1 is also a square because $1 \times 1 = 1$.

The symbol n is used below to represent the *position number* in the expression that gives the rule (n^2) when generalising.



TRIANGULAR NUMBERS

Therese uses circles to form a pattern of triangular shapes:



Picture 1 *Picture 2* *Picture 3* *Picture 4*

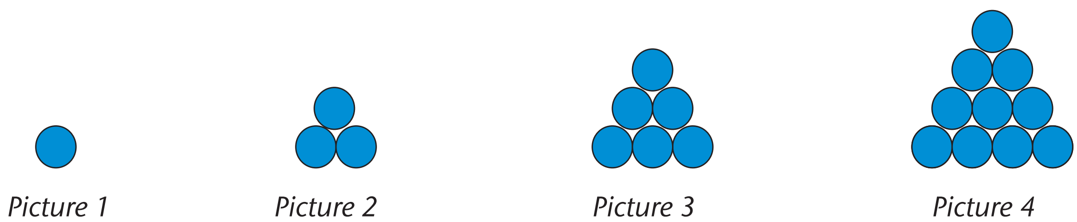
1. If the pattern is continued, how many circles must Therese have:
   (a) in the bottom row of picture 5?
   (b) in the second row from the bottom of picture 5?
   (c) in the third row from the bottom of picture 5?
   (d) in the second row from the top of picture 5?
   (e) in the top row of picture 5?
   (f) in total in picture 5? Show your calculations.
2. How many circles does Therese need to form triangle picture 7? Show the calculation.
3. How many circles does Therese need to form triangle picture 8?

4. Copy and complete the following table. Show all your work.

Picture number	1	2	3	4	5	6	12	15
Number of circles	1	3	6	10				

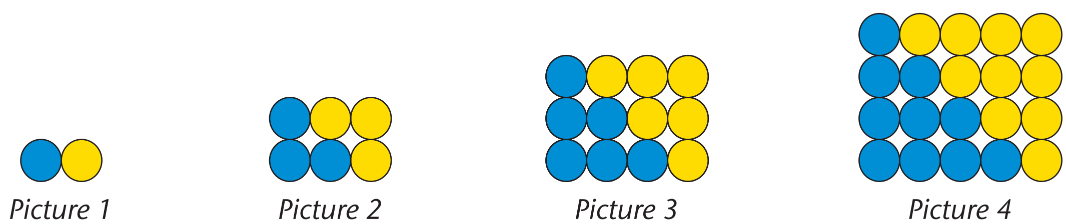
More than 2 500 years ago, Greek mathematicians already knew that the numbers 3, 6, 10, 15 and so on could form a triangular pattern. They represented these numbers with dots which they arranged in such a way that they formed equilateral triangles, hence the name **triangular numbers**. Algebraically, we think of them as sums of consecutive natural numbers starting with 1.

Let us revisit the activity on triangular numbers that we did on the previous page.



So far, we have determined the number of circles in the pattern by adding consecutive natural numbers. If we were asked to determine the number of circles in picture 200, for example, it would take us a very long time to do so. We need to find a quicker method of finding any triangular number in the sequence.

Consider the arrangement below.



We have added the yellow circles to the original blue circles and then rearranged the circles in such a way that they are in a rectangular form.

5. Picture 2 is three circles long and two circles wide. Copy and complete the following sentences:
   (a) Picture 3 is ...... circles long and ...... circles wide.
   (b) Picture 1 is ...... circles long and ...... circle wide.
   (c) Picture 4 is ...... circles long and ...... circles wide.
   (d) Picture 5 is ...... circles long and ...... circles wide.

6. How many circles will there be in a picture that is:
   (a) ten circles long and nine circles wide?
   (b) seven circles long and six circles wide?
   (c) six circles long and five circles wide?
   (d) 20 circles long and 19 circles wide?

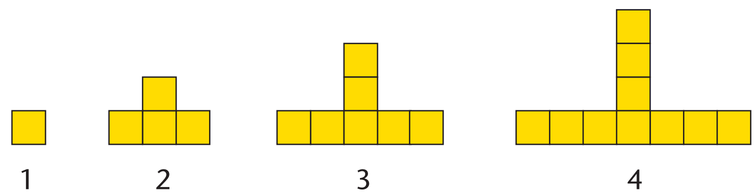
Suppose we want to use a quicker method to determine the number of circles in picture 15. We know that picture 15 is 16 circles long and 15 circles wide. This gives a total of $15 \times 16 = 240$ circles. But we must compensate for the fact that the yellow circles were originally not there by halving the total number of circles. In other words, the original figure has $240 \div 2 = 120$ circles.

7. Use the above reasoning to calculate the number of circles in:
   (a) picture 20 (b) picture 35

4.4 Describing patterns in different ways

T-SHAPED NUMBERS

The pattern below is made from squares.



1. (a) How many squares will there be in pattern 5?
   (b) How many squares will there be in pattern 15?
   (c) Copy and complete the following table:

Pattern number	1	2	3	4	5	6	20
Number of squares	1	4	7	10			

You can use the following three plans (or methods) to calculate the number of squares for pattern 20. Study each one carefully.

Plan A:

To get from one square to four squares, you have to add three squares. To get from four squares to seven squares, you have to add three squares. To get from seven squares to ten squares, you have to add three squares. Continue to add three squares for each pattern until pattern 20.

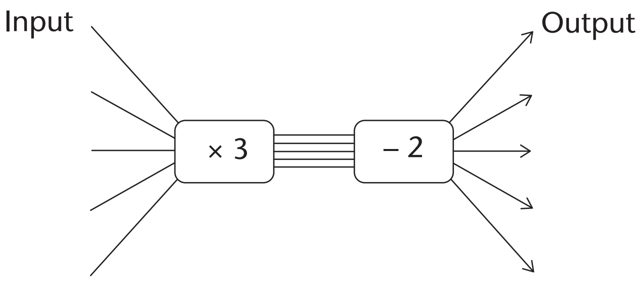
Plan B:

Multiply the pattern number by three and subtract two. Pattern 20 will therefore have $20 \times 3 - 2$ squares.

Plan C:

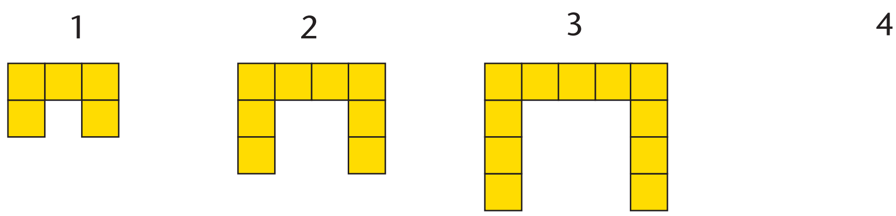
The number of squares in pattern 5 is 13. Pattern 20 will therefore have 13 × 4 = 52 squares because 20 = 5 × 4.

2. (a) Which method or plan (A, B or C) will give the right answer? Explain why.
   (b) Which of the above plans did you use? Explain why?
   (c) Can this flow diagram be used to calculate the number of squares?

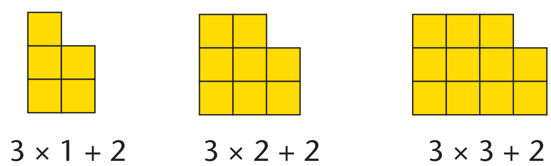


... AND SOME OTHER SHAPES

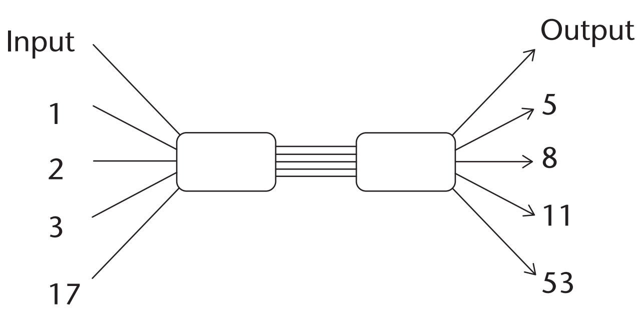
1. Three figures are given below. Draw the next figure in the tile pattern.



2. (a) If the pattern is continued, how many tiles will there be in the 17th figure? Answer this question by analysing what happens.
   (b) Thato decides that it easier for him to see the pattern when the tiles are rearranged as shown on the right:



   Use Thato's method to determine the number of tiles in the 23rd figure.
   (c) Copy and complete the following flow diagram by writing the appropriate operators so that it can be used to calculate the number of tiles in any figure of the pattern.



   (d) How many tiles will there be in the 50th figure if the pattern is continued?

WORKSHEET

1. Write down the next four terms in each sequence. Also explain, in each case, how you figured out what the terms are.

   (a) 2; 4; 8; 14; 22; 32; 44; .....

   (b) 2; 6; 18; 54; 162; .....

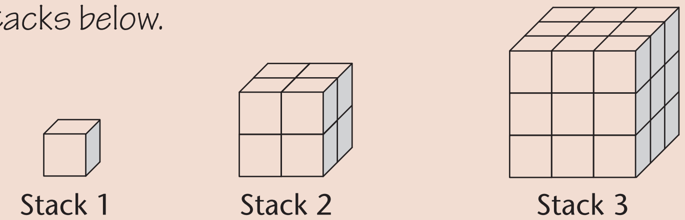
   (c) 1; 7; 13; 19; 25; .....

2. (a) Copy and complete the following table by calculating the missing terms:

Position in sequence	1	2	3	4	5	7	10
Term	3	10	17				

   (b) Write the rule to calculate the term from the position in the sequence in words.

3. Consider the stacks below.



   (a) How many cubes will there be in stack 5?

   (b) Copy and complete the following table:

Stack number	1	2	3	4	5	6	10
Number of cubes	1	8	27				

   (c) Write down the rule to calculate the number of cubes for any stack number.

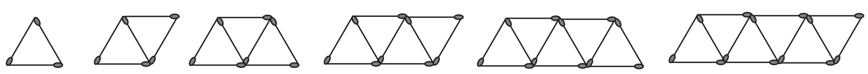
Chapter 5
Functions and relationships

5.1 Constant and variable quantities

LOOKING FOR CONNECTIONS BETWEEN QUANTITIES

Consider the following seven situations. There are two quantities in each situation. For each quantity, state whether it is constant (always the same number) or whether it changes. Also state, in each case, whether one quantity has an influence on the other. If it has, try to say how the one quantity will influence the other quantity.

1. Your age and the number of fingers on your hands
2. The number of calls you make and the airtime left on your cell phone
3. The length of your arm and your ability to finish Mathematics tests quickly
4. The number of identical houses to be built and the number of bricks required
5. The number of learners at a school and the length of the school day
6. The number of learners at a school and the number of classrooms needed
7. The number of matches in each arrangement, and the number of triangles in the arrangement:

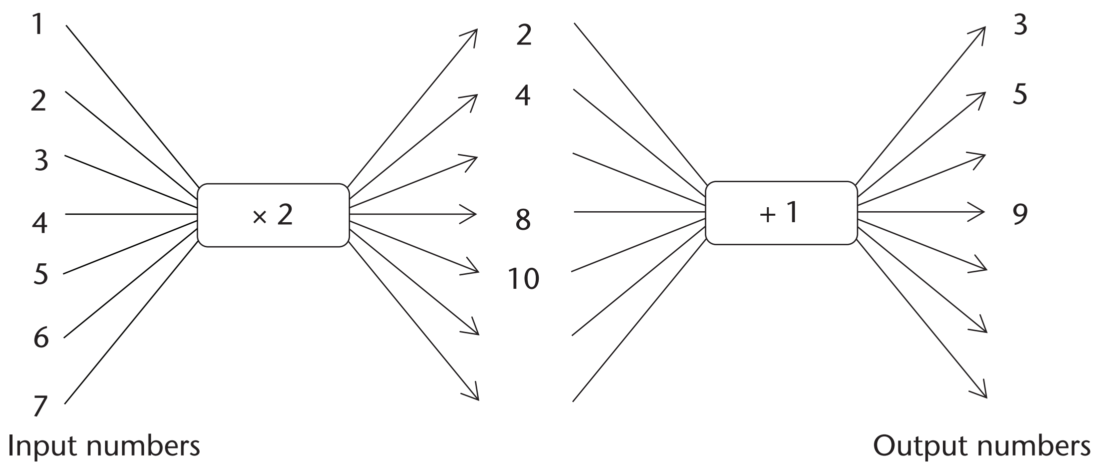


If one variable quantity is influenced by another, we say there is a **relationship** between the two variables. It is sometimes possible to find out what value of the one quantity, in other words what number, is linked to a specific value of the other quantity.

A quantity that changes is called a **variable quantity**, or just a **variable**.

8. (a) Look at the match arrangements in question 7. If you know that there are three triangles in an arrangement, can you say with certainty how many matches there are in that specific arrangement?
   (b) How many matches are there in the arrangement with ten triangles?
   (c) Is there another possible answer for question (b)?

9. Copy and complete the following flow diagrams by filling in all the missing numbers. Do you see any connections between the situation in question 7 on page 59 and this flow diagram? If so, describe the connections.

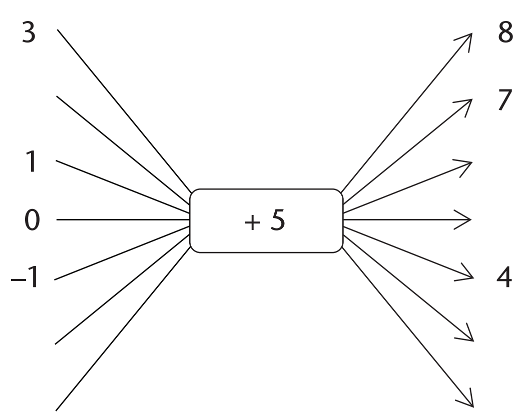


COMPLETING SOME FLOW DIAGRAMS

A relationship between two quantities can be shown with a flow diagram, such as those below. Unfortunately, only some of the numbers can be shown on a flow diagram.

1. Copy the following flow diagram. Calculate the output numbers. Some input numbers are missing. Choose and insert your own input numbers.

(a)



Each input number in a flow diagram has a corresponding **output number**. The first (top) **input number** corresponds to the first output number. The second input number corresponds to the second output number and so on. We call + 5 the **operator**.

(b) What type of numbers are the given input numbers?

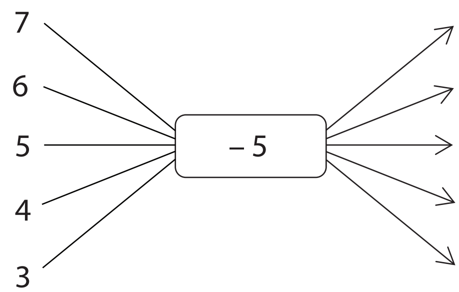
(c) In the above flow diagram, the output number 8 corresponds to the input number 3. Copy and complete the following sentences:

In the relationship shown in the above flow diagram, the output number ..... corresponds to the input number –1.

The input number ..... corresponds to the output number 7.

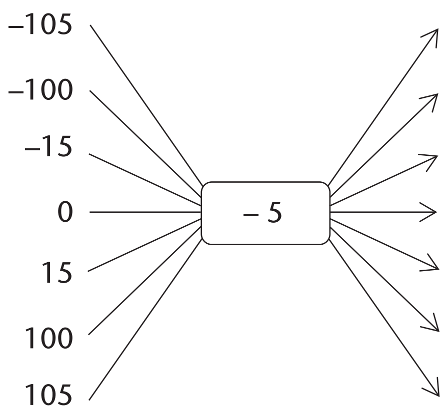
If more places are added to the flow diagram, the input number ..... will correspond to the output number 31.

2. (a) Copy and complete this flow diagram.
   (b) Compare this flow diagram to the flow diagram in question 1. What link do you find between the two?

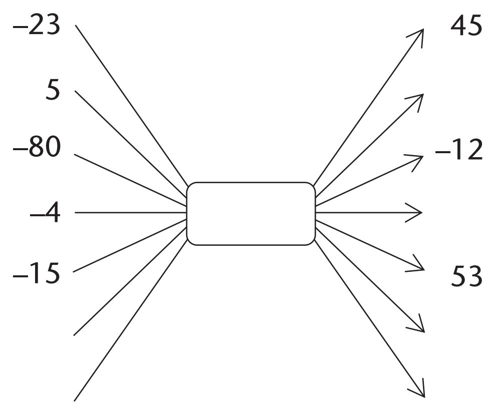


3. Copy and complete the following flow diagrams. You have to find out what the **operator** for (b) is, and fill it in yourself.

(a)

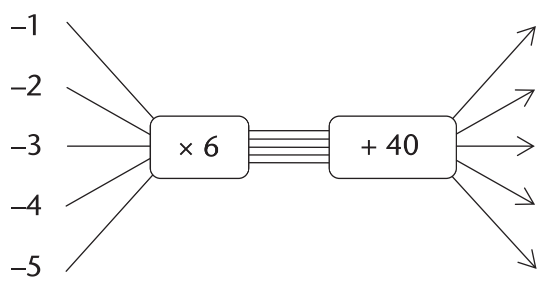


(b)



   (c) What number can you add in (a), instead of subtracting 5, that will produce the same output numbers?
   (d) What number can you subtract in (b), instead of adding a number, that will produce the same output numbers?

4. Copy and complete the following flow diagram:



A completed flow diagram shows two kinds of information:

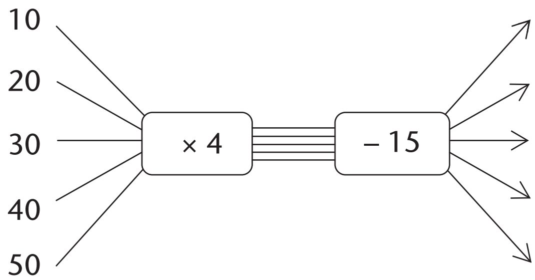
- It shows what calculations are done to produce the output numbers.
- It shows which output number is connected to which input number.

The flow diagram that you completed in question 4 shows the following information:

- Each input number is multiplied by 6, then 40 is added to produce the output numbers.
- The input and output numbers are connected, as shown in the table on page 62.

Input numbers	–1	–2	–3	–4	–5
Output numbers	34	28	22	16	10

5. (a) Describe, in words, how the following output numbers can be calculated:

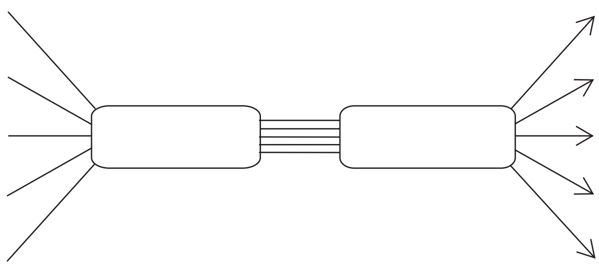


(b) Copy the following table and use it to show which output numbers are connected to which input numbers in the above flow diagram.

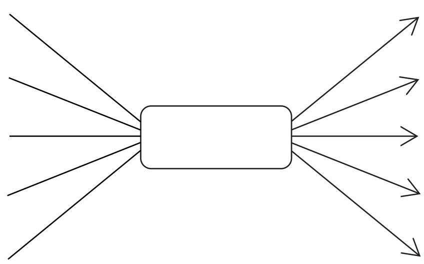
6. The following information is available about the floor space and cost of houses in a new development. The cost of an empty stand is R180 000.

Floor space in square metres (m^2)	90	120	150	180	210
Cost of house and stand	540 000	660 000	780 000	900 000	1 020 000

(a) Represent the above information in the following flow diagram:



(b) Show what the houses only will cost, if you get the stand for free.

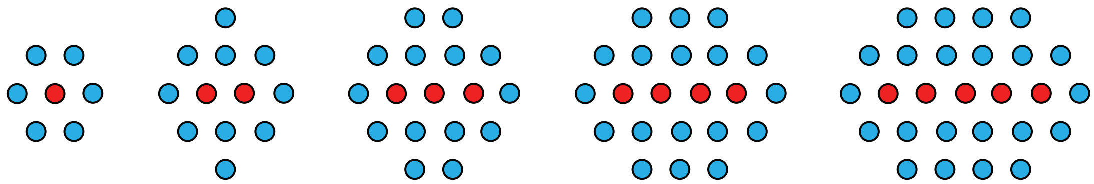


(c) Try to figure out what the cost of a house and stand will be, if there are exactly 100 m^2 of floor space in the house.

5.2 Different ways to describe relationships

A RELATIONSHIP BETWEEN RED DOTS AND BLUE DOTS

Here is an example of a relationship between two quantities:



In each arrangement there are some red dots and some blue dots.

1. How many blue dots are there if there is one red dot?
2. How many blue dots are there if there are two red dots?
3. How many blue dots are there if there are three red dots?
4. How many blue dots are there if there are four red dots?
5. How many blue dots are there if there are five red dots?
6. How many blue dots are there if there are six red dots?
7. How many blue dots are there if there are seven red dots?
8. How many blue dots are there if there are ten red dots?
9. How many blue dots are there if there are 20 red dots?
10. How many blue dots are there if there are 100 red dots?
11. Which of the following descriptions correctly describe the relationship between the number of blue dots and the number of red dots in the above arrangements? Test each description thoroughly for all the above arrangements.

    (a) The number of red dots → $\times 4$ → $+ 2$ → the number of blue dots

    (b) To calculate the number of blue dots you multiply the number of red dots by 2, add 1 and multiply the answer by 2

    (c) The number of blue dots = 2 × the number of red dots + 4

Something to think about

Are there different possibilities for the number of blue dots if there are three red dots in the arrangement?

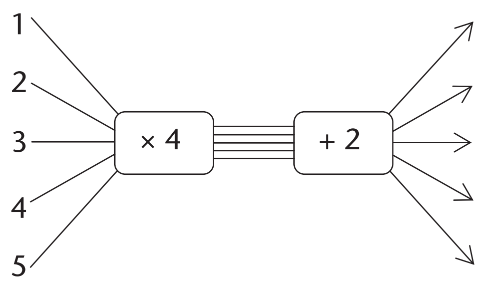
Are there different possibilities for the number of blue dots if there are two red dots in the arrangement?

Are there different possibilities for the number of blue dots if there are 20 red dots in the arrangement?

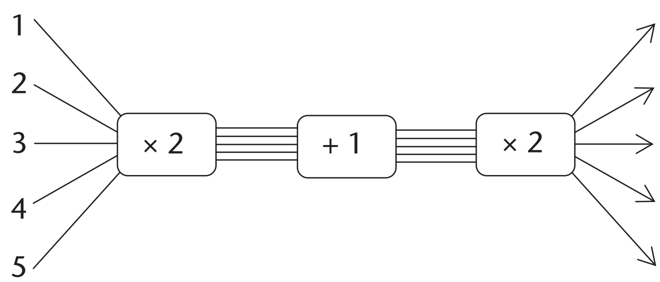
(d)

Number of red dots	1	2	3	4	5	6
Number of blue dots	6	10	14	18	22	26

(e)



(f)



(g) The number of blue dots = 4 × the number of red dots + 2

(h) The number of blue dots = 2 × (2 × the number of red dots + 1)
(Remember that the calculations inside the brackets are done first.)

The descriptions in (c), (g) and (h) above are called **word formulae**.

TRANSLATING BETWEEN DIFFERENT LANGUAGES OF DESCRIPTION

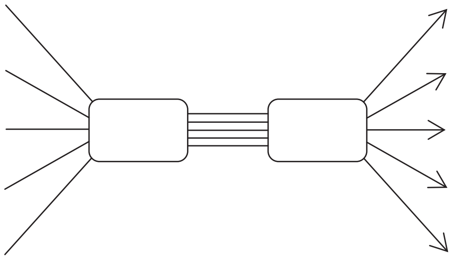
A relationship between two quantities can be described in different ways, including:

- a table of values of the two quantities
- a flow diagram
- a word formula
- a symbol formula (or symbolic formula).

You will learn about symbolic formulae in Section 5.3.

1. The relationship between two quantities is described as follows:
   The second quantity is always three times the first quantity plus 8.
   The first quantity varies between 1 and 5, and it is always a whole number.

(a) Describe this relationship using the flow diagram.



(b) Describe the relationship between the two quantities using this table:

(c) Describe the relationship between the two quantities using a word formula.

2. The relationship between two quantities is described as follows:
The input numbers are the first five odd numbers.

value of the one quantity → + 5 → × 3 → *the corresponding value of the other quantity*

(a) Describe this relationship using a table.

(b) Describe the relationship using a word formula.

5.3 Algebraic symbols for variables and relationships

DESCRIBING PROCEDURES IN DIFFERENT WAYS

1. In each case, do four things:
   - Complete the table.
   - Describe the relationship with a word formula.
   - Describe the input numbers in words.
   - Describe the output numbers in words.

(a) input number → × 10 → + 15 → output number

Input number	5	10	15	20	25	30
Output number						

output number = ...

(b) input number → + 15 → × 10 → output number

Input number	5	10	15	20	25	30
Output number						

(c) input number → × 2 → + 3 → × 5 → output number

Input number	5	10	15	20	25	30
Output number						

Formulae with symbols

Instead of writing "input number" and "output number" in formulae, you may just write a single letter symbol as an abbreviation.

Many years ago, mathematicians adopted the convention of using the letter symbol x as an abbreviation for the "input number", and the letter symbol y as an abbreviation for the "output number".

Letter symbols other than x and y are also used to indicate variable quantities.

The word formula you wrote for question 1(a) can be written more shortly as:

$$y = 10 \times x + 15$$

Mathematicians have also agreed that one may leave the ×-sign (multiplication sign) out when writing **symbolic formulae**.

Note that it is not at all wrong to use the multiplication sign in symbolic formulae.

So, instead of $y = 10 \times x + 15$, we may write $y = 10x + 15$.

2. Rewrite your word formulae in questions 1(b) and 1(c) as symbolic formulae.
3. Write a word formula for each of the following relationships:
   (a) $y = 7x + 10$ (b) $y = 7(x + 10)$ (c) $y = 7(2x + 10)$

WRITING SYMBOLIC FORMULAE

Describe each of the following relationships with a symbolic formula:

1. To calculate the output number, the input number is multiplied by 4 and 7 is subtracted from the answer.
2. To calculate the output number, 7 is subtracted from the input number and the answer is multiplied by 5.
3. To calculate the output number, 7 is subtracted from the input number, the answer is multiplied by 5 and 3 is added to this answer.

Chapter 6
Algebraic expressions 1

6.1 Algebraic language

WORDS, DIAGRAMS AND SYMBOLS

1. Copy and complete the following table:

	Words	Flow diagram	Expression
	Multiply a number by two and add six to the answer.	→ [× 2] → [+ 6] →	$2 \times x + 6$
(a)	Add three to a number and then multiply the answer by two.		
(b)		→ [× 5] → [– 1] →	
(c)			$7 + 4 \times x$
(d)			$10 - 5 \times x$

An **algebraic expression** indicates **a sequence of calculations** that can also be described in words or by means of a flow diagram.

The flow diagram illustrates the **order** in which the calculations must be done.

In algebraic language, the **multiplication sign is usually omitted**. So, we write $2x$ instead of $2 \times x$. We also write $x \times 2$ as $2x$.

2. Write the following expressions in "normal" algebraic language:
   (a) $-2 \times a + b$
   (b) $a2$

LOOKING DIFFERENT BUT YET THE SAME

1. Copy and complete the table by calculating the numerical values of the expressions for the values of x. Some answers for $x = 1$ have been done for you as an example.

	x	1	3	7	10
(a)	$2x + 3x$	$2 \times 1 + 3 \times 1$ $2 + 3 = 5$			
(b)	$5x$				
(c)	$2x + 3$				
(d)	$5x^2$	$5 \times (1)^2$ $5 \times 1 = 5$			

2. Do the expressions $2x + 3x$ and $5x$ in question 1 above, produce different answers or the same answer for:
   (a) $x = 3$?
   (b) $x = 10$?
3. Do the expressions $2x + 3$ and $5x$ produce different answers or the same answer for:
   (a) $x = 3$?
   (b) $x = 10$?
4. Although they may look different, write down all the algebraic expressions in question 1 that have the same numerical value for the same value(s) of x. Justify your answer.

One of the things we do in algebra is to **evaluate** expressions. When we evaluate an expression we choose or are given a value of the variable in the expression. Now that we have an actual value, we can carry out the operations in the expression using this value, as in the examples given in the table.

Algebraic expressions that have the same numerical value for the same value of x but look different, are called **equivalent expressions**.

5. Say whether the following statements are true or false. Explain your answer in each case.
   (a) The expressions $2x + 3x$ and $5x$ are equivalent.
   (b) The expressions $2x + 3$ and $5x$ are equivalent.
6. Consider the expressions $3x + 2z + y$ and $6xyz$.
   (a) What is the value of $3x + 2z + y$ for $x = 4$, $y = 7$ and $z = 10$?
   (b) What is the value of $6xyz$ for $x = 4$, $y = 7$ and $z = 10$?
   (c) Are the expressions $3x + 2z + y$ and $6xyz$ equivalent? Explain.

Remember that $6xyz$ is the same as $6 \times x \times y \times z$.

To show that the two expressions in question 5(a) are equivalent, we write $2x + 3x = 5x$.

We can explain this as:

$2x + 3x = (x + x) + (x + x + x) = 5x$

We say the expression $2x + 3x$ **simplifies** to $5x$.

The term $3x$ is a product. The number 3 is called the **coefficient** of x.

7. In each case below, write down an expression equivalent to the one given:
   (a) $3x + 3x$
   (b) $3x + 8x + 2x$
   (c) $8b + 2b + 2b$
   (d) $7m + 2m + 10m$
   (e) $3x^2 + 3x^2$
   (f) $3x^2 + 8x^2 + 2x^2$

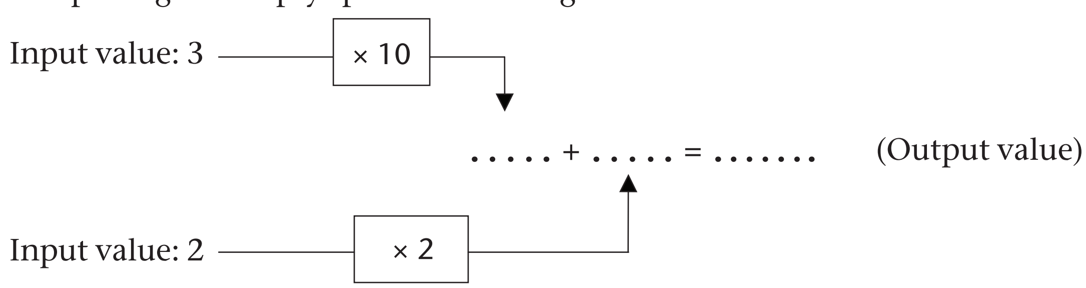
8. What is the coefficient of x^2 for the expression equivalent to $3x^2 + 8x^2 + 2x^2$?

In an expression that can be written as a sum, the different parts of the expression are called the **terms of the expression**. For example, $3x$, $2z$ and y are the terms of the expression $3x + 2z + y$.

An expression can have **like terms** or **unlike terms**, or both.

Like terms are terms that have the **same variable(s) raised to the same power**. The terms $2x$ and $3x$ are examples of like terms.

9. (a) Copy the diagram. Calculate the numerical value of $10x + 2y$ for $x = 3$ and $y = 2$ by completing the empty spaces in the diagram.



   (b) What is the output value for the expression $12xy$ for $x = 3$ and $y = 2$?
   (c) Are the expressions $10x + 2y$ and $12xy$ equivalent? Explain.
   (d) Are the terms $10x$ and $2y$ like or unlike terms? Explain.

10. (a) Which of the following algebraic expressions do you think will give the same results?
    A. $6x + 4x$ B. $10x$ C. $10x^2$ D. $9x + x$
   (b) Test the algebraic expressions you have identified for the following values of x:
    $x = 10$ $x = 17$ $x = 54$
   (c) Are the terms $6x$ and $4x$ like or unlike terms? Explain.
   (d) Are the terms $10x$ and $10x^2$ like or unlike terms? Explain.

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11. Ashraf and Hendrik have a disagreement about whether the terms $7x^2y^3$ and $301y^3x^2$ are like terms or not. Hendrik thinks they are not, because in the first term the x^2 comes before the y^3, whereas in the second term the y^3 comes before the x^2. Explain to Hendrik why his argument is not correct.
12. Explain why the terms $5abc$, $10acb$ and $15cba$ are like terms.

6.2 Add and subtract like terms

REARRANGE TERMS AND THEN COMBINE LIKE TERMS

1. Copy and complete the table by evaluating the expressions for the given values of x:

x	1	2	10
$30x + 80$	$30 \times 1 + 80$ $= 30 + 80 = 110$		
$5x + 20$			
$30x + 80 + 5x + 20$			
$35x + 100$			
$135x$			

2. Write down all the expressions in the table that are equivalent.
3. Tim thinks that the expressions $135x$ and $35x + 100$ are equivalent because for $x = 1$, they both have the same numerical value 135. Explain to Tim why the two expressions are not equivalent.

We have already come across the commutative and associative properties of operations. We will now use these properties to help us form equivalent algebraic expressions.

Commutative property
The order in which we add or multiply numbers does not change the answer: $a + b = b + a$ and $ab = ba$

Associative property
The way in which we group three or more numbers when adding or multiplying does not change the answer: $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$

We can find an equivalent expression by **rearranging** and **combining like terms**, as shown below:

$30x + 80 + 5x + 20$
Hence $30x + (80 + 5x) + 20$
Hence $30x + (5x + 80) + 20$
$= (30x + 5x) + (80 + 20)$
$= 35x + 100$

Like terms are combined to form a single term.

The terms 80 and 20 are called **constants**. The numbers 30 and 5 are called **coefficients**.

Brackets are used in the expression on the left to show how the like terms have been rearranged.

The terms $30x$ and $5x$ are combined to get the new term $35x$, and the terms 80 and 20 are combined to form the new term: 100. We say that the **expression** $30x + 80 + 5x + 20$ is **simplified** to a new expression $35x + 100$.

4. Simplify the following expressions:
   (a) $13x + 7 + 6x - 2$ (b) $21x - 8 + 7x + 15$ (c) $18c - 12d + 5c - 7c$
   (d) $3abc + 4 + 7abc - 6$ (e) $12x^2 + 2x - 2x^2 + 8x$ (f) $7m^3 + 7m^2 + 9m^3 + 1$

When you are not sure about whether or not you correctly simplified an expression, it is always advisable to check your work by evaluating the original expression and the simplified expression for some values.

When we use a value of the variable in the expression, we call it a **substitution**.

5. Make a simpler expression that is equivalent to the given expression. Test your answer for three different values of x, and redo your work until you get it right.
   (a) Simplify $(15x + 7y) + (25x + 3 + 2y)$ (b) Simplify $12mn + 8mn$

In questions 6 to 8 below, write down the letter representing the correct answer. Explain why you think your answer is correct.

6. The sum of $5x^2 + x + 7$ and $x - 9$ is:
   A. $x^2 - 2$ B. $5x^2 + 2x + 16$ C. $5x^2 + 16$ D. $5x^2 + 2x - 2$
7. The sum of $6x^2 - x + 4$ and $x^2 - 5$ is equivalent to:
   A. $7x^2 - x + 9$ B. $7x^2 - x - 1$ C. $6x^4 - x - 9$ D. $7x^4 - x - 1$
8. The sum of $5x^2 + 2x + 4$ and $3x^2 - 5x - 1$ can be expressed as:
   A. $8x^2 + 3x + 3$ B. $8x^2 + 3x - 3$ C. $8x^2 - 3x + 3$ D. $8x^2 - 3x - 3$

Combining like terms is a useful algebraic habit. It allows us to replace an expression with another expression that may be convenient to work with.

Do the questions on page 72 to get a sense of what we are talking about.

CONVENIENT REPLACEMENTS

1. Consider the expression $x + x + x + x + x + x + x + x + x + x$. What is the value of the expression in each of the following cases?
   (a) $x = 2$ (b) $x = 50$
2. Consider the expression $x + x + x + z + z + y$. What is the value of the expression in each of the following cases?
   (a) $x = 4, y = 7, z = 10$ (b) $x = 0, y = 8, z = 22$
3. Suppose you have to evaluate $3x + 7x$ for $x = 20$. Will calculating 10×20 give the correct answer? Explain.

Suppose we evaluate the expression $3x + 7x$ for $x = 20$ without first combining the like terms. We will have to do **three** calculations, namely 3×20, then 7×20 and then find the sum of the two: $3 \times 20 + 7 \times 20 = 60 + 140 = 200$.

But if we first combine the like terms $3x$ and $7x$ into one term $10x$, we only have to do **one calculation**: $10 \times 20 = 200$. This is one way of thinking about the convenience or usefulness of simplifying an algebraic expression.

4. The expression $5x + 3x$ is given and you are required to evaluate it for $x = 8$. Will calculating 8×8 give the correct answer? Explain.
5. Suppose you have to evaluate $7x + 5$ for $x = 10$. Will calculating 12×10 give the correct answer? Explain.
6. The expression $5x + 3$ is given and you have to evaluate it for $x = 8$. Will calculating 8×8 give the correct answer? Explain.

Samantha was asked to evaluate the expression $12x^2 + 2x - 2x^2 + 8x$ for $x = 12$. She thought to herself that just substituting the value of x directly into the terms would require a lot of work. She first combined the like terms as shown below:

$12x^2 \mathbf{- 2x^2 + 2x} + 8x$
$= 10x^2 + 10x$

The terms $+ 2x$ and $- 2x^2$ change positions by the commutative property of operations.

Then for $x = 10$, Samantha found the value of $10x^2 + 10x$ by calculating:

$10 \times 10^2 + 10 \times 10$
$= 1\,000 + 100$
$= 1\,100$

Use Samatha's way of thinking for questions 7 to 9.

7. What is the value of $12x + 25x + 75x + 8x$ when $x = 6$?
8. Evaluate $3x^2 + 7 + 2x^2 + 3$ for $x = 5$.
9. When Zama was asked to evaluate the expression $2n - 1 + 6n$ for $n = 4$, she wrote down the following:

   $2n - 1 + 6n = n + 6n = 6n^2$

   Hence for $n = 4$: $6 \times (4)^2 = 6 \times 8 = 48$. Explain where Zama went wrong and why.

WORKSHEET

1. Copy and complete the following table:

	Words	Flow diagram	Expression
(a)	Multiply a number by three and add two to the answer	→ $\times 3$ → $+ 2$ →	
(b)			$9x - 6$
(c)		→ $\times 7$ → $- 3$ →	$7x - 3$

2. Which of the following pairs consist of like terms? Explain.
   A. $3y; -7y$ B. $14e^2; 5e$ C. $3y^2z; 17y^2z$ D. $-bcd; 5bd$

3. Write the following in the "normal" algebraic way:
   (a) $c2 + d3$ (b) $7 \times d \times e \times f$

4. Consider the expression $12x^2 - 5x + 3$.
   (a) What is the number 12 called? (b) Write down the coefficient of x.
   (c) What name is given to the number 3?

5. Explain why the terms $5pqr$, $-10prq$ and $15qrp$ are like terms.

6. If $y = 7$, what is the value of each of the following?
   (a) $y + 8$ (b) $9y$ (c) $7 - y$

7. Simplify the following expressions:
   (a) $18c + 12d + 5c - 7c$ (b) $3def + 4 + 7def - 6$

8. Evaluate the following expressions for $y = 3$, $z = -1$:
   (a) $2y^2 + 3z$ (b) $(2y)^2 + 3z$

9. Write each of the following algebraic expressions in the simplest form:
   (a) $5y + 15y$ (b) $5c + 6c - 3c + 2c$ (c) $4b + 3 + 16b - 5$
   (d) $7m + 3n + 2 - 6m$ (e) $5h^2 + 17 - 2h^2 + 3$ (f) $7e^2f + 3ef + 2 + 4ef$

10. Evaluate each of the following expressions:
    (a) $3y + 3y + 3y + 3y + 3y + 3y$ for $y = 18$
    (b) $13y + 14 - 3y + 6$ for $y = 200$
    (c) $20 - y^2 + 101y^2 + 80$ for $y = 1$
    (d) $12y^2 + 3yz + 18y^2 + 2yz$ for $y = 3$ and $z = 2$

CHAPTER 7
Algebraic equations 1

7.1 Setting up equations

An **equation** is a mathematical sentence that is true for some numbers but false for other numbers. The following are examples of equations:

$x + 3 = 11$ and $2x = 8$

$x + 3 = 11$ is true if $x = 8$, but false if $x = 3$.

When we look for a number or numbers that make an equation true, we say that we are **solving the equation**. For example, $x = 4$ is the **solution** of $2x = 8$ because it makes $2x = 8$ true. (Check: $2 \times 4 = 8$)

LOOKING FOR NUMBERS TO MAKE STATEMENTS TRUE

1. Are the following statements true or false? Justify your answer.
   (a) $x - 3 = 0$, if $x = -3$ (b) $x^3 = 8$, if $x = -2$ (c) $3x = -6$, if $x = -3$
   (d) $3x = 1$, if $x = 1$ (e) $6x + 5 = 47$, if $x = 7$

2. Find the original number. Show your reasoning.
   (a) A number multiplied by 10 is 80.
   (b) Add 83 to a number and the answer is 100.
   (c) Divide a number by 5 and the answer is 4.
   (d) Multiply a number by 4 and the answer is 20.
   (e) Twice a number is 100.
   (f) A number raised to the power 5 is 32.
   (g) A number raised to the power 4 is –81.
   (h) Fifteen times a number is 90.
   (i) 93 added to a number is –3.
   (j) Half a number is 15.

3. Write the equations below in words using "a number" in place of the letter symbol x. Then write what you think "the number" is in each case.
   Example: $4 + x = 23$. *Four plus a number equals twenty-three. The number is 19.*
   (a) $8x = 72$ (b) $\frac{2x}{5} = 2$ (c) $2x + 5 = 21$
   (d) $12 + 9x = 30$ (e) $30 - 2x = 40$ (f) $5x + 4 = 3x + 10$

7.2 Solving equations by inspection

THE ANSWER IS IN PLAIN SIGHT

1. Seven equations are given in the following table. Use the table to find out for which of the given values of x it will be true that the left-hand side of the equation is equal to the right-hand side.

You can read the solutions of an equation from a table.

x	–3	–2	–1	0	1	2	3	4
$2x + 3$	–3	–1	1	3	5	7	9	11
$x + 4$	1	2	3	4	5	6	7	8
$9 - x$	12	11	10	9	8	7	6	5
$3x - 2$	–11	–8	–5	–2	1	4	7	10
$10x - 7$	–37	–27	–17	–7	3	13	23	33
$5x + 3$	–12	–7	–2	3	8	13	18	23
$10 - 3x$	19	16	13	10	7	4	1	–2

(a) $2x + 3 = 5x + 3$
(b) $5x + 3 = 9 - x$
(c) $2x + 3 = x + 4$
(d) $10x - 7 = 5x + 3$
(e) $3x - 2 = x + 4$
(f) $9 - x = 2x + 3$
(g) $10 - 3x = 3x - 2$

Two or more equations can have the same solution. For example, $5x = 10$ and $x + 2 = 4$ have the same solution; $x = 2$ is the solution for both equations.

Two equations are called **equivalent** if they have the same solution.

2. Which of the equations in question 1 have the same solutions? Explain.

3. Copy and complete the following table. Then answer the questions that follow.

You can also do a search by narrowing down the possible solution to an equation.

x	0	5	10	15	20	25	30	35	40
$2x + 3$									
$3x - 10$									

(a) Can you find a solution for $2x + 3 = 3x - 10$ in the table?
(b) What happens to the values of $2x + 3$ and $3x - 10$ as x increases? Do they become bigger or smaller?
(c) Is there a point where the value of $3x - 10$ becomes bigger or smaller than the value of $2x + 3$ as the value of x increases? If so, between which x-values does this happen?

This point where the two expressions are equal is called the **break-even point**.

(d) Now that you narrowed down where the possible solution can be, try other possible values for x until you find out for what value of x the statement $2x + 3 = 3x - 10$ is true.

"Searching" for the solution of an equation by using tables or by narrowing down to the possible solution is called **solution by inspection**.

7.3 More examples

LOOKING FOR AND CHECKING SOLUTIONS

1. What is the solution for the equations shown below?
   (a) $x - 3 = 4$ (b) $x + 2 = 9$ (c) $3x = 21$ (d) $3x + 1 = 22$

When a certain number is the solution of an equation we say that the number **satisfies** the equation. For example, $x = 4$ satisfies the equation $3x = 12$ because $3 \times 4 = 12$.

2. Choose the number in brackets that satisfies the equation. Explain your choice.
   (a) $12x = 84$ {5; 7; 10; 12}
   (b) $\frac{84}{x} = 12$ {–7; 0; 7; 10}
   (c) $48 = 8k + 8$ {–5; 0; 5; 10}
   (d) $19 - 8m = 3$ {–2; –1; 0; 1; 2}
   (e) $20 = 6y - 4$ {3; 4; 5; 6}
   (f) $x^3 = -64$ {–8; –4; 4; 8}
   (g) $5^x = 125$ {–3; –1; 1; 3}
   (h) $2^x = 8$ {1; 2; 3; 4}
   (i) $x^2 = 9$ {1; 2; 3; 4}
3. What makes the following equations true? Check your answers.
   (a) $m + 8 = 100$ (b) $80 = x + 60$ (c) $26 - k = 0$ (d) $105 \times y = 0$
   (e) $k \times 10 = 10$ (f) $5x = 100$ (g) $\frac{15}{t} = 5$ (h) $3 = \frac{t}{5}$
4. Solve the following equations by inspection. Check your answers.
   (a) $12x + 14 = 50$ (b) $100 = 15m + 25$
   (c) $\frac{100}{x} = 20$ (d) $7m + 5 = 40$
   (e) $2x + 8 = 10$ (f) $3x + 10 = 31$
   (g) $-1 + 2x = -11$ (h) $2 + \frac{x}{7} = 5$
   (i) $100 = 64 + 9x$ (j) $\frac{2x}{6} = 4$