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education

Department:
Education
PROVINCE OF KWAZULU-NATAL

NATIONAL
SENIOR CERTIFICATE

GRADE 10

MATHEMATICS P2

COMMON TEST

JUNE 2019

MARKS: **50**

TIME: **1 hour**

This question paper consists of 6 pages.

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 3 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.

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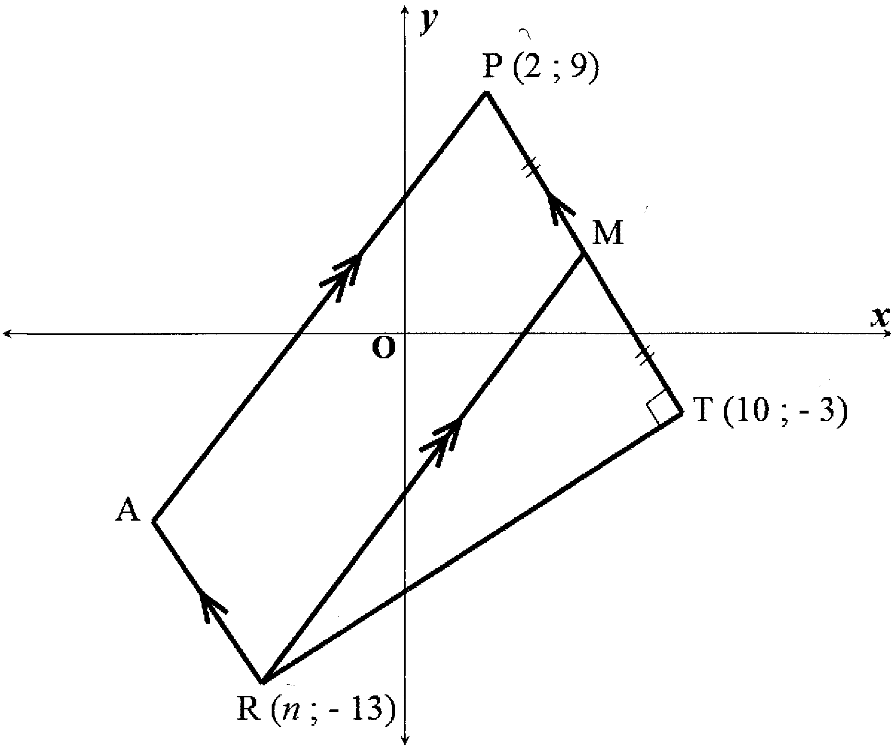
QUESTION 1

In the diagram below P (2 ; 9), A, R (n ; –13) and M are the vertices of parallelogram PARM.

PMT is a straight line such that M is the midpoint of PT.

T (10 ; – 3) is a point such that PT ⊥ RT.



1.1 Determine:

1.1.1 the length of PT. Leave your answer in surd form. (2)

1.1.2 the gradient of PT (2)

1.1.3 the gradient of AR (1)

1.1.4 the coordinates of M (2)

1.2 Determine the equation of PM in the form $y = mx + c$ (3)

1.3 Show that $n = -5$ (4)

1.4 Calculate the area of ΔRMT (4)

[18]

QUESTION 2

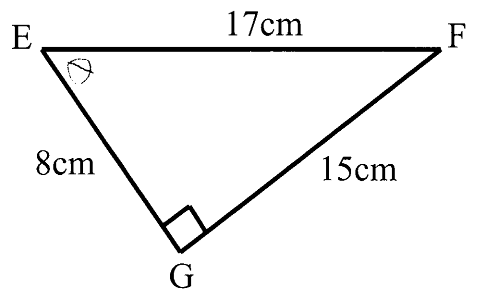
2.1 Given that $\alpha = 24,6°$ and $\beta = 132,7°$ calculate the value of the following (correct to TWO decimal places):

2.1.1 $\frac{1}{2}\cos\alpha$ (1)

2.1.2 $\cos ec 2\beta$ (2)

2.2 Various options are provided as possible answers to the following questions. Choose the correct answer and write only the letter (A – D) next to the question number, for example: **2.2.4 A**

2.2.1 In the diagram below of right angle triangle EFG, EF = 17cm and FG = 15cm.

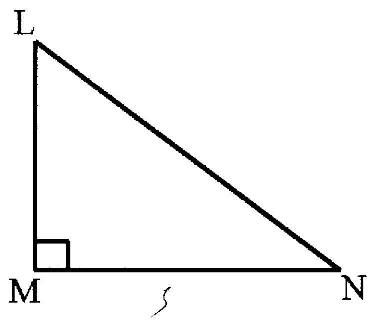


Which trigonometric equation could be used to determine the value of angle E?

A. $\sin E = \frac{17}{15}$ B. $\cos E = \frac{15}{17}$

C. $\tan E = \frac{15}{8}$ D. $\sin E = \frac{8}{17}$ (1)

2.2.2 In the diagram below scalene ΔLMN shown, $\hat{M} = 90°$.



Which of the following statements is always true?

A. $\sin L = \cos L$ B. $\sin L = \cos N$

C. $\cos L = \cos M$ D. $\sin L = \cos M$ (1)

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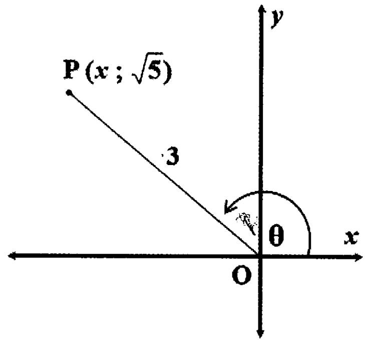
2.3 Simplify the following WITHOUT the use of a calculator:

$\sin^2 45° + \cos^2 45°$ (2)

2.4 Solve for x, correct to ONE decimal place, where $0^o \leq x \leq 90^o$:

$\sin 2x = 0,291$ (2)

2.5 In the diagram $P\left(x\ ;\ \sqrt{5}\right)$, is a point in the Cartesian plane and $P\hat{O}X = \theta$.



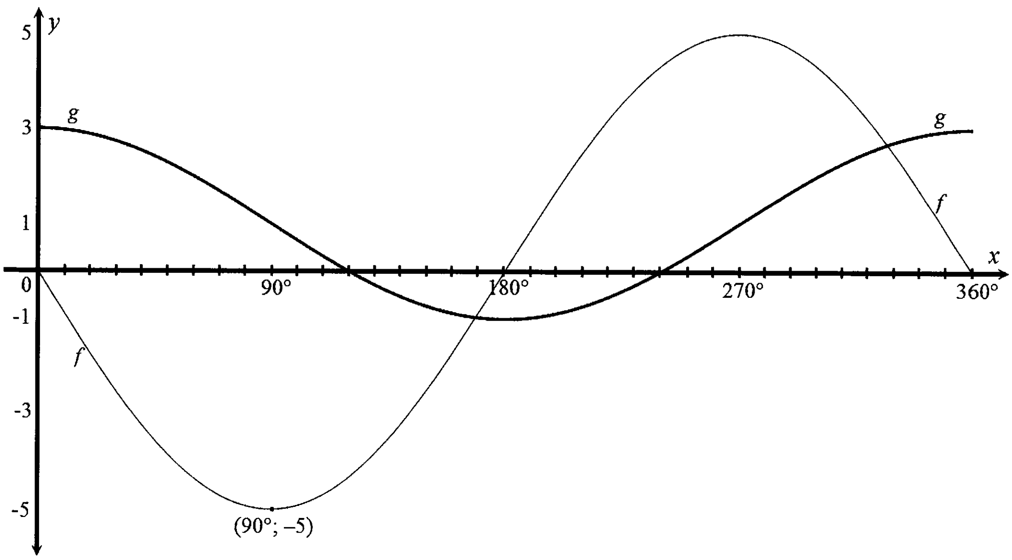
Using the diagram and **without the use of a calculator**, determine:

2.5.1 the value of x (2)

2.5.2 $\cos\theta$ (2)

2.5.3 $1-\sin^2\theta$ (2)

2.6 Sketched below are the graphs of $f(x) = a\sin x$ and $g(x) = \cos x + b$ for $x \in [0°;360°]$



2.6.1 Write down the values of a and b (2)

2.6.2 Write down the period of f (1)

2.6.3 Determine the range of g (1)

2.6.4 For which value(s) of x is $g(x) < 0$? (2)

[21]

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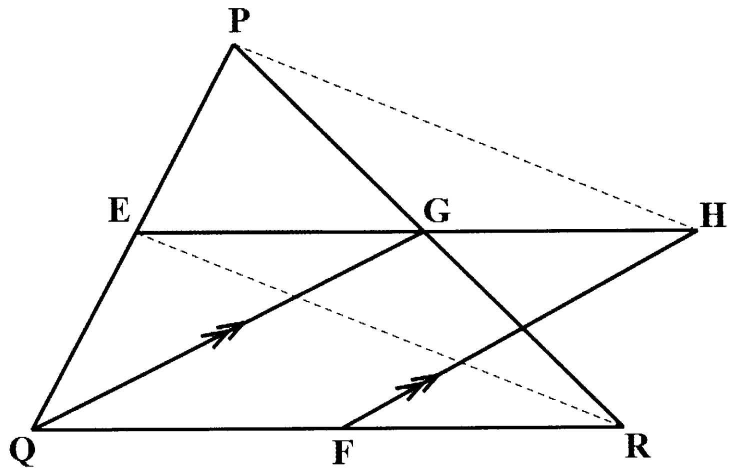
QUESTION 3

3.1 Complete the following:

The line joining the mid-points of two sides of a triangle is ……. to the third side and equal to ……. the length of the third side. (2)

3.2 In the diagram below, ΔPQR has E, F and G the midpoints of PQ, QR and PR respectively. $QG \parallel FH$.



Prove:

3.2.1 $QGHF$ is a parallelogram (3)

3.2.2 $EG = GH$ (3)

3.2.3 $ER \parallel PH$ (3)

[11]

TOTAL: 50

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Department:
Education
PROVINCE OF KWAZULU-NATAL

MATHEMATICS P2

MARKING GUIDELINE

COMMON TEST

JUNE 2019

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MARKS: 150

N.B. This marking guidelines consists of 14 pages.

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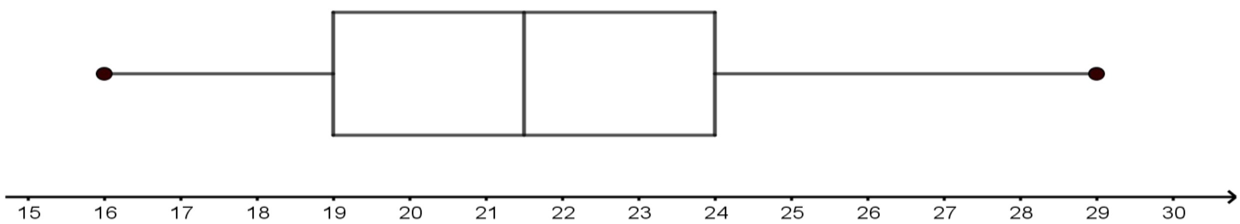
QUESTION 1

1.1	$\bar{x} = \frac{220}{10} = 22$	✓A 220 ✓ CA answer Answer only full marks	(2)
1.2	σ = 3,95	✓✓ AA answer If formula is used 1CA mark for substitution and 1CA mark for answer.	(2)
1.3		✓ A minimum & maximum value ✓ A quartile 1 value ✓ A median value ✓A quartile 3 value If No Diagram No marks	(4)
1.4	$(\bar{x} - \sigma;\ \bar{x} + \sigma)$ (22 – 3,95; 22 + 3,95) (18,05; 25, 95) 6 runners (answer only – full marks)	✓CA 18,05 ✓ CA 25,95 ✓ CA answer	(3)
			[11]

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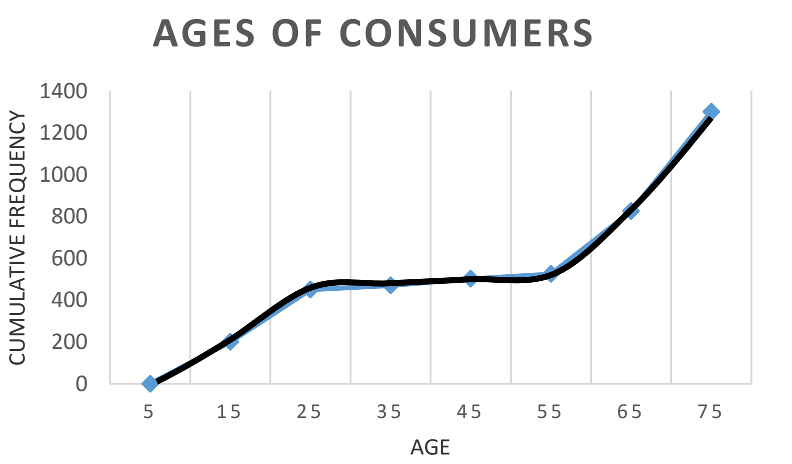
QUESTION 2

2.1	A = 250 B = 502	✓ (A)A ✓ (A) B	(2)
2.2	$\bar{x} = \dfrac{2000 + 5000 + 600 + 1280 + 1150 + 18000 + 33250}{1300}$ $\bar{x} = \dfrac{61280}{1300}$ $\bar{x} = 47{,}14$ (answer only – full marks)	✓CA sum ✓CA 61280 ✓CA answer	(3)
2.3	$65 < x \leq 75$	✓✓AA answer	(2)
2.4	 	✓CA upper limits ✓CA grounding (5; 0) ✓CA joining points with a smooth curve	(3)
2.5	Not a normal distribution. Highest frequency is found between the ages 55 to 75. Mean < median, therefore skewed to the left.	✓A No ✓A Reason	(2)
			[12]

QUESTION 3

3.1	$BC = \sqrt{(x^2 - x^1)^2 + (y^2 - y^1)^2}$ $= \sqrt{(-2-1)^2 + (-2-4)^2}$ $= \sqrt{9+36}$ $= \sqrt{45}$ $= 3\sqrt{5}$	✓A substitution ✓CA answer (2)
3.2	$M\left(\frac{1-2}{2};\ \frac{4-2}{2}\right)$ $M\left(-\frac{1}{2};\ 1\right)$	✓A $\frac{-1}{2}$ ✓A 1 (2)
3.3	$m_{AB} = \frac{-2-4}{-2-1} = \frac{-6}{-3}$ $= 2$ $m_{MD} = -\frac{1}{2}$ (DM ⊥ AB) $y = mx + c$ $1 = -\frac{1}{2}\left(-\frac{1}{2}\right) + c$ $c = 1 - \frac{1}{4}$ $= \frac{3}{4}$ $y = -\frac{1}{2}x + \frac{3}{4}$	✓ A M_{AB} ✓CA gradient of MD ✓CA subst. $\left(-\frac{1}{2};\ 1\right)$ into eq. ✓CA answer (4)

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3.4	E is the midpoint since ME ∥ BC. $\text{E}\left(\frac{1+4}{2};\ \frac{4+1}{2}\right)$ $= \text{E}\left(\frac{5}{2};\ \frac{5}{2}\right)$	✓ A S/R ✓A substitution ✓CA answer (provided coordinates are positive) (3)
3.5	$m_{BE} = \frac{-2-\frac{5}{2}}{-2-\frac{5}{2}}$ $m_{BE} = 1$ $y - y_1 = m(x - x_1)$ OR $y = mx + c$ $y - (-2) = 1(x - (-2))$ $-2 = 1(-2) + c$ $y + 2 = x + 2$ $0 = c$ $y = x$ $y = x$	✓CA m_{BE} (must be positive) ✓CA substitution ✓CA answer (must be positive) (3)
3.6	$m_{BC} = \frac{-2-1}{-2-4} = \frac{1}{2}$ $m_{BC} = \tan\theta = \frac{1}{2}$ $\theta = 26{,}57°$	✓ A Substitution ✓CA $m_{BC} = \frac{1}{2}$ ✓CA $\tan\theta = \frac{1}{2}$ ✓CA answer (4)
		[18]
	QUESTION 4	
4.1	$AQ = \sqrt{(-6-2)^2 + (-7-(-1))^2}$ $AQ = \sqrt{(-8)^2 + (-6)^2}$ $AQ = \sqrt{64+36}$ $AQ = \sqrt{100}$ $\therefore\ AQ = 10$	✓A subst. into dist. formula ✓CA answer (2)

4.2	$(x-a)^2+(y-b)^2=r^2$ $(x-2)^2+(y+1)^2=100$	✓A $(x-2)^2+(y+1)^2$ ✓CA 100 (2)
4.3.1	$m_{AQ}=\frac{-7+1}{-6-2}=\frac{3}{4}$ $\therefore\ m_{QP}=-\frac{4}{3}$ $rad.\perp tan$	✓ A $\frac{3}{4}$ ✓CA $-\frac{4}{3}$ (2)
4.3.2	$m_{AR}=\frac{-7+1}{10-2}=-\frac{3}{4}$ $\therefore\ m_{PR}=\frac{4}{3}$ $rad.\perp tan$	✓A $\frac{-3}{4}$ ✓CA $\frac{4}{3}$ (2)
4.4.1	$m_{QP}=-\frac{4}{3}$ $y-y_1=m(x-x_1)$ $y+7=-\frac{4}{3}(x+6)$ $y=-\frac{4}{3}x-15$	✓CA substitution ✓CA answer (2)
4.4.2	$m_{PR}=\frac{4}{3}$ $y-y_1=m(x-x_1)$ $y+7=\frac{4}{3}(x-10)$ $y=\frac{4x}{3}-\frac{40}{3}-7$ $=\frac{4}{3}x-\frac{61}{3}$	✓CA substitution ✓CA answer (2)

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4.5.1	$\frac{4}{3}x - \frac{61}{3} = -\frac{4}{3}x - 15$ $\frac{8}{3}x = \frac{16}{3}$ $x = 2$ $y = -\frac{53}{3}$ $P(2; = -\frac{53}{3})$ **OR** The x – co-ordinate of P is 2 (ARPQ is a kite) Subst $x=2$ in $y = -\frac{4}{3}x - 15$ $y = -\frac{4}{3}(2) - 15$ $= -\frac{8}{3} - 15$ $= \frac{-8-45}{3}$ $= \frac{-53}{3}$ $P\left(2; -\frac{53}{3}\right)$	✓CA Equating ✓CA x value ✓CA y value ✓CA both co-ordinates ✓A $x = 2$ ✓CA substitution ✓CA y value ✓CA both co-ordinates (4)
4.5.2	In ΔSPR $\alpha = \hat{P} + \beta$ (ext $\angle$ of ΔSPR) $\therefore \hat{P} = \alpha - \beta$	✓A S/R ✓A $\hat{P} = \alpha - \beta$ (2)

4.5.3	$\tan(\alpha-\beta) = \dfrac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$	
	$= \dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$	✓A expansion
	$= \dfrac{\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)-\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)}{\left(-\frac{3}{5}\right)\left(\frac{3}{5}\right)+\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)}$	✓A numerator ✓A denominator
	$= \dfrac{\frac{12}{25}+\frac{12}{25}}{\frac{-9}{25}+\frac{16}{25}}$	✓A Simplification
	$= \dfrac{24}{25}\times\dfrac{25}{7}$	
	$= \dfrac{24}{7}$	
	OR	(4)
	$tan\beta = \dfrac{4}{3} \quad \therefore \beta = 53.13°$	✓A answer
	$tan\alpha = -\frac{4}{3} \;\therefore \alpha = 126{,}87°$	✓A answer
	$\tan(\alpha-\beta) = \dfrac{\sin(126.87-53.13)}{\cos(126.87-53.13)}$	✓A simplification
	$= 3{,}42857\ldots\ldots$ $\cong \frac{24}{7}$	✓A Answer (4)
		[22]

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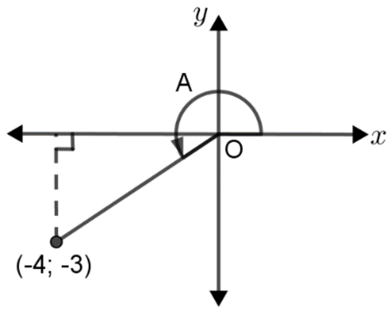
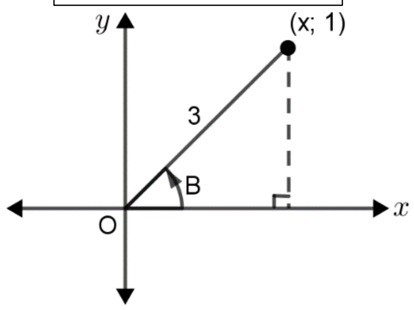
QUESTION 5

5.1	$\tan A = \frac{3}{4}$; $\sin B = \frac{1}{3}$ Diagram 1  Diagram 2  $r^2 = (-4)^2 + (-3)^2$ …. $Pyth$ $r^2 = 25$ $r = 5$ $x^2 = 3^2 - 1^2$ … $Pyth$ $x^2 = 8$ $x = 2\sqrt{2}$	✓A diagram 1	
5.1.1	$\cos 2A$ $= 2cos^2 A - 1$ $= 2\left(\frac{-4}{5}\right)^2 - 1$ $= \frac{7}{25}$	✓A identity ✓CA answer	(3)
5.1.2	$\sin(A + B)$ $= \sin A \cos B + \cos A \sin B$ $= \left(\frac{-3}{5}\right)\left(\frac{2\sqrt{2}}{3}\right) + \left(\frac{-4}{5}\right)\left(\frac{1}{3}\right)$ $= -\frac{4 + 6\sqrt{2}}{15}$	✓A Diagram 2 ✓A expansion ✓CA answer	(3)
5.2	$\sin 20^o \cos 320^o + \cos(-20^o) \sin 400^o$ $= \sin 20^o \cos 40^o + \cos 20^o \sin 40^o$ $= \sin(20^o + 40^o)$ $= \sin 60^o$ $= \frac{\sqrt{3}}{2}$	✓A reduction ✓CA simplify ✓CA answer (provided special angle ratio) ANSWER ONLY = 0	(3)

5.3	$\dfrac{\cos^2(90^\circ+\theta)}{\cos(-\theta)+\sin(90^\circ-\theta)\cos\theta}$ $= \dfrac{\sin^2\theta}{\cos\theta+\cos^2\theta}$ $= \dfrac{\sin^2\theta}{\cos\theta\,(1+\cos\theta)}$ $= \dfrac{1-\cos^2\theta}{\cos\theta\,(1+\cos\theta)}$ $= \dfrac{1-\cos\theta}{\cos\theta}$ $= \dfrac{1}{\cos\theta}-1$ $=$ RHS	✓A numerator ✓A denominator ✓A common factor ✓ A difference of squares ✓A simplification	(5)
5.4.1	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $= (\cos\alpha + \sin\alpha)(\cos\alpha - \sin\alpha)$ $= p.q$	✓A expansion ✓ A answer	(2)
5.4.2	$\dfrac{1+\sin 2\alpha}{\cos 2\alpha}$ $= \dfrac{\sin^2\alpha + 2\sin\alpha\cos\alpha + \cos^2\alpha}{\cos^2\alpha - \sin^2\alpha}$ $= \dfrac{(\sin\alpha+\cos\alpha)^2}{(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}$ $= \dfrac{p}{q}$	✓A numerator ✓A denominator ✓A factorise ✓A factorise ✓CA answer	(5)
5.5	$6\cos^2 x + \sin x - 5 = 0$ $6(1-\sin^2 x) + \sin x - 5 = 0$ $6 - 6\sin^2 x + \sin x - 5 = 0$ $-6sin^2x + \sin x + 1 = 0$ $(3\sin x + 1)(-2\sin x + 1) = 0$ $\sin x = -\dfrac{1}{3}$; $\sin x = \dfrac{1}{2}$ $x = 199{,}47^\circ + k.360^\circ; k \in \mathbb{Z}$ OR $x = 30^\circ + k.360^\circ; k \in \mathbb{Z}$ $x = 340{,}53^\circ + k.360^\circ; k \in \mathbb{Z}$ OR $x = 150^\circ + k.360^\circ; k \in \mathbb{Z}$	✓A identity ✓CA factors ✓CA both ratios ✓A $k \in \mathbb{Z}$ ✓CA both solutions of $\sin x = -\frac{1}{3}$ ✓CA both solutions of $\sin x = \frac{1}{2}$	(6)
			[27]

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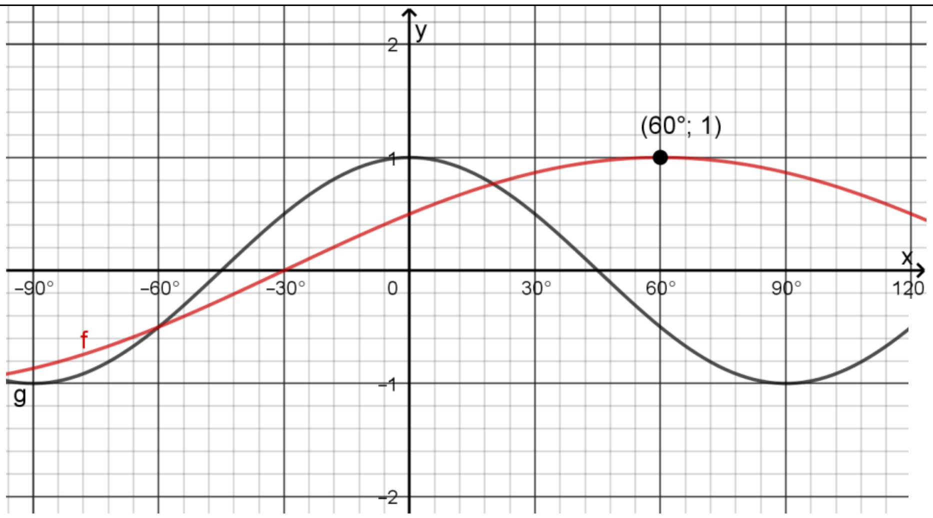
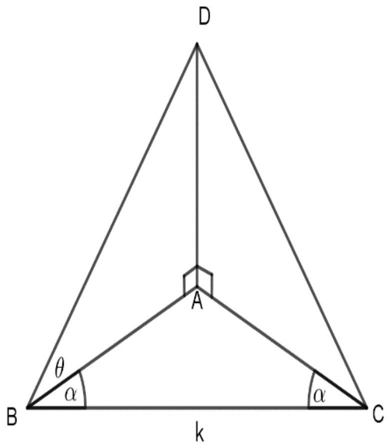
QUESTION 6

6.1		✓A x-intercept ✓A t. pt. $(60^0; 1)$ ✓A shape	(3)
6.2	$60° < x < 90°$	✓CA end values ✓CA notation	(2)
6.3	Graph of f moves 60^0 left.	✓A shifts 60^0 ✓A to the left	(2)
			[7]

QUESTION 7

7	$\tan\theta = \frac{\text{DA}}{\text{AB}}$ $\therefore \text{AD} = \text{AB}\tan\theta$ Also $\frac{\text{AB}}{\sin\alpha} = \frac{k}{\sin(180° - 2\alpha)}$ $\text{AB}\ \sin 2\alpha = k\ \sin\alpha$ $\text{AB} = \frac{k\ \sin\alpha}{2\sin\alpha\cos\alpha}$ $\text{AB} = \frac{k}{\cos\alpha}$ $\therefore \text{AD} = \frac{k.\tan\theta}{2\cos\alpha}$		✓A trig ratio ✓ AAD value ✓A substitute into sine rule ✓A $\sin 2\alpha = 2\sin\alpha\cos\alpha$ ✓A $AB = \frac{k.\sin\alpha}{2\sin\alpha\cos\alpha}$ ✓A making AB value - simplified	
				[6]

QUESTION 8

8.1	$\hat{Q}_4 = \hat{W}_2 = x$ … (tan chord theorem) $\hat{W}_2 = \hat{W}_1 = x$ … (WQ bisects PWR.) $\hat{Q}_1 = \hat{W}_1 = x$ … (tan – chord theorem) $\hat{T}_2 = \hat{W}_2 = x$ … ($\angle$s in same segment) $\hat{S}_2 = \hat{W}_1 = x$ … $\angle$s in same segment)	A✓S/R A✓S/R A✓S/R A✓S/R A✓S/R	(5)
8.2.1	$\hat{T}_2 = \hat{Q}_1 = x$ $\therefore$ TS // PR … (alternate $\angle$s equal)	A✓S A✓R	(2)
8.2.2	$\hat{T}_3 = \hat{P}$ … (corresponding $\angle s$; TS//PR) $\hat{T}_3 = \hat{Q}_3$ … ($\angle$s in same segment) $\therefore \hat{P} = \hat{Q}_3$	AA✓S✓R A✓S/R	(3)
8.2.3	In Δ TQS $\hat{T}_2 = x$ $\hat{S}_2 = x$ $\therefore \hat{T}_2 = \hat{S}_2 = x$ $\therefore$ Δ TQS isosceles … ($\angle$s opposite equal sides)	A✓S A✓S A✓S A✓R	(4)
8.2.4	$W\hat{Q}P = W\hat{S}Q$… (tan – chord theorem) $\hat{T}_1 = W\hat{S}Q$ … (ext $\angle$ of cyclic quad)	AA✓S✓R A✓S/R	(3)
			[17]

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QUESTION 9

9.1.1	$\hat{P}_1 = \hat{Q}_1$ given $\hat{P}_1 = \hat{R}$ tan-chord theorem $\therefore \hat{Q}_1 = \hat{R}$ $\therefore$ TQ ∥ SR (corr $\angle^s$ *are equal*)	AA✓S✓R A✓R	(3)
9.1.2	$\hat{P}_1 = \hat{Q}_1$ given TS = TP tan from same point $\hat{P}_1 = \hat{S}_1$ equal $\angle^s$ opp equal sides $\therefore \hat{Q}_1 = \hat{S}_1$ ∴QPTS is a cyclic quad converse equal $\angle^s$ subtended by same chord	A✓S/R A✓S/R A✓R	(3)
9.1.3	QPTS is a cyclic quad $\therefore \hat{P}_1 = \hat{Q}_2$ $\angle^s$in same ⊙ segm but $\hat{P}_1 = \hat{Q}_1$ given $\therefore \hat{Q}_1 = \hat{Q}_2$ ∴TQ bisect $S\hat{Q}P$	AA✓S✓R A✓S	(3)
9.2.1	In Δ LPK and Δ NPL $K\hat{L}P = L\hat{N}P$ … tan chord theorem $\hat{P}_2 = 90°$ … … ∠ in semi ⊙ $\hat{P}_1 = \hat{P}_2$ … both = 90° $P\hat{K}L = N\hat{L}P$ … remaining angle $\therefore$ ΔLPK///ΔNPL …∠∠∠	AA✓S✓R A✓S/R A✓R	(4)
9.2.2	$\frac{PL}{NP} = \frac{KL}{NL} = \frac{PK}{PL}$… Δ LPK /// Δ NPL $\frac{PL}{NP} = \frac{PK}{PL}$ ……. ΔLPK///ΔNPL $\therefore PL^2 = NP.PK$	AA✓S✓R A✓ proportionality	(3)
9.2.3	Δ NLK	A✓ answer	(1)
9.2.4	ΔNLK///ΔNPL $\therefore \frac{KN}{LN} = \frac{LN}{NP}$ [///Δ's] $LN^2 = KN.NP$ = 16 x 10 = 160 LN = $\sqrt{160}$ Radius = $\frac{1}{2}\sqrt{160}$ Area of Circle = πr^2 = $\pi(\frac{1}{2}\sqrt{160})^2$ = 125.66cm² **OR**	A✓ S/R A ✓ Substitution ✓CA NL value ✓CA radius = $\frac{1}{2}\sqrt{160}$ ✓CA Substitution ✓CA Answer	(6)

	From Question No. 9.2.2: $PL^2 = NP.PK$ $= 10 \text{ cm} \times 6 \text{ cm}$ $= 60 \text{ cm}^2$ $NL^2 = PL^2 + PN^2$ … Pythagoras $= 60 + 100$ (Δ LPN) $= 160 \text{ cm}^2$ $\therefore NL = \sqrt{160} cm$ $\frac{1}{2} NL = \frac{1}{2}$ diameter $= \frac{1}{2}\sqrt{160}\ cm$ Area of circle $= \pi r^2$ $= \pi \times \left(\frac{1}{2}\sqrt{160}\right)^2\ cm^2$ $= 125,66 \text{ cm}^2$ OR $40\pi\ cm^2$	✓A $PL^2 = 60 \text{ cm}^2$ ✓A Pythagoras ✓CA NL value ✓CA radius = $\frac{1}{2}\sqrt{160}$ ✓CA substitution ✓CA answer	
			[23]

QUESTION 10

10.1	$\frac{AE}{EF} = \frac{4}{6}$ …… prop theorem ; EB//FC $\frac{AE}{EF} = \frac{AC}{CD}$ …… prop theorem; EC//FD $\frac{4}{6} = \frac{10}{CD}$ $CD = 15\ units$	A✓ S/R A✓ S/R ✓CA answer	(3)
10.2	$\frac{\Delta FEC}{\Delta CFA} = \frac{3}{5}$ same height $\frac{\text{Area } \Delta CFA}{\text{Area } \Delta FAD} = \frac{10}{25} = \frac{2}{5}$ same height $\frac{\text{Area } \Delta FEC}{\text{Area } \Delta FAD} = \frac{\text{Area } \Delta FEC}{\text{Area } \Delta CFA} \times \frac{\text{Area } \Delta CFA}{\text{Area } \Delta FAD}$ $= \frac{3}{5} \times \frac{2}{5}$ $= \frac{6}{25}$	A✓ S/R A✓ S/R ✓CA simplify ✓CA answer	(4)
			[7]

TOTAL MARKS: 150

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