



Basic Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS

COMMON TEST

SEPTEMBER 2015

**NATIONAL
SENIOR CERTIFICATE**

GRADE 11

MARKS: 75

TIME: 1½ hours

This question paper consists of 6 pages and a diagram sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 5 questions.
2. Answer **ALL** the questions.
3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non graphical) may be used, unless stated otherwise.
6. If necessary, round off answers to **TWO** decimal places, unless stated otherwise.
7. Diagrams are not necessarily drawn to scale.
8. Diagram sheet for answering **QUESTION 3.3** is attached at the end of this question paper. Write your name and insert the diagram sheet inside your **ANSWER BOOK**.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

- 1.1 Samuel bought three shirts: blue, yellow and green. He also bought two ties: red and orange. A shirt and a tie forms one outfit.
- 1.1.1 Draw a tree diagram to represent Samuel's choices. (4)
- 1.1.2 Determine the probability of Samuel not wearing a red tie. (1)
- 1.2 Given $P(A) = 0,2$; $P(B) = 0,35$ and $P(A \text{ or } B) = 0,55$. Explain whether A and B are mutually exclusive events. (Show all working) (2)
- [7]

QUESTION 2

In a University where 36 students study Foreign languages, 26 study French(F), 18 study German(G), 11 study French and German, 10 study French and Latin(L), 6 study German and Latin and 4 study French, German and Latin.

- 2.1 Represent this information in a Venn Diagram. (7)
- 2.2 Use the Venn Diagram to write down the number of students who study:
- (a) Latin. (1)
- (b) French and German but not Latin. (1)
- (c) Latin only. (1)
- [10]

QUESTION 3

3.1 If $\tan \theta = \frac{3}{4}$, and $180^\circ < \theta < 270^\circ$, determine with an aid of a sketch the value of:

3.1.1 $\sin \theta$ (3)

3.1.2 $\cos(180^\circ + \theta)$ (2)

3.1.3 θ (answer to 2 decimal places). (2)

3.2 Simplify, without the use of a calculator:

$$\sqrt{\frac{1 - \tan^2 \theta \cdot \cos(-\theta) \cdot \cos(360^\circ - \theta) \cdot \tan 225^\circ}{[\sin 90^\circ - \sin(180^\circ + \theta)][\sin 90^\circ - \cos(90^\circ - \theta)]}}$$
 (6)

3.3 Consider the functions below:

$$f(x) = \sin 2x \text{ and } g(x) = \cos(x + 60^\circ)$$

3.3.1 Draw a neat sketch of the curves of f and g for $-180^\circ \leq \theta \leq 180^\circ$ on the axes provided on the diagram sheet. Clearly indicate the intercepts with the axes. (6)

3.3.2 Write down the range for g (1)

3.3.3 Write down the period of f (1)

3.3.4 For which value(s) of x is the graph of g decreasing. (2)

3.4 Determine the general solution of the equation:

$$2 \sin^2 x - 5 \sin x = 3$$
 (5)

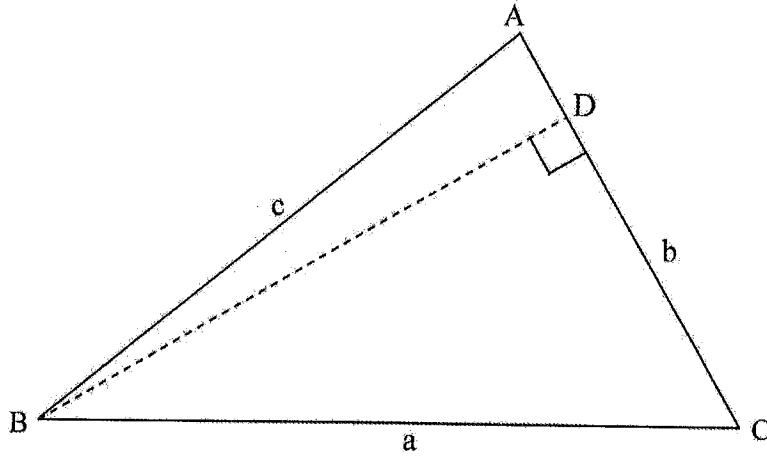
3.5 Prove the following identity:

$$\frac{1 + \cos A}{\sin A} + \frac{\sin A}{1 + \cos A} = \frac{2}{\sin A}$$
 (5)

[33]

QUESTION FOUR

4.1 Prove for any acute angled ΔABC that: $\frac{c}{\sin C} = \frac{a}{\sin A}$

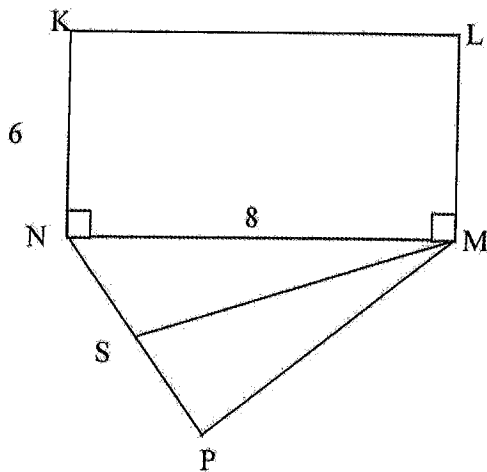


(4)

4.2 Complete: In any ΔABC : Area of $\Delta ABC = \frac{1}{2} ab \dots\dots\dots$

(1)

4.3 The figure consists of a rectangle KLMN and ΔNMP . The area of pentagon KLMPN is 63 square units. $KN = 6$ units, $NM = 8$ units and $NP = 7$ units.



4.3.1 Determine the area of ΔNMP (2)

4.3.2 Calculate the size of acute \hat{MNP} . (3)

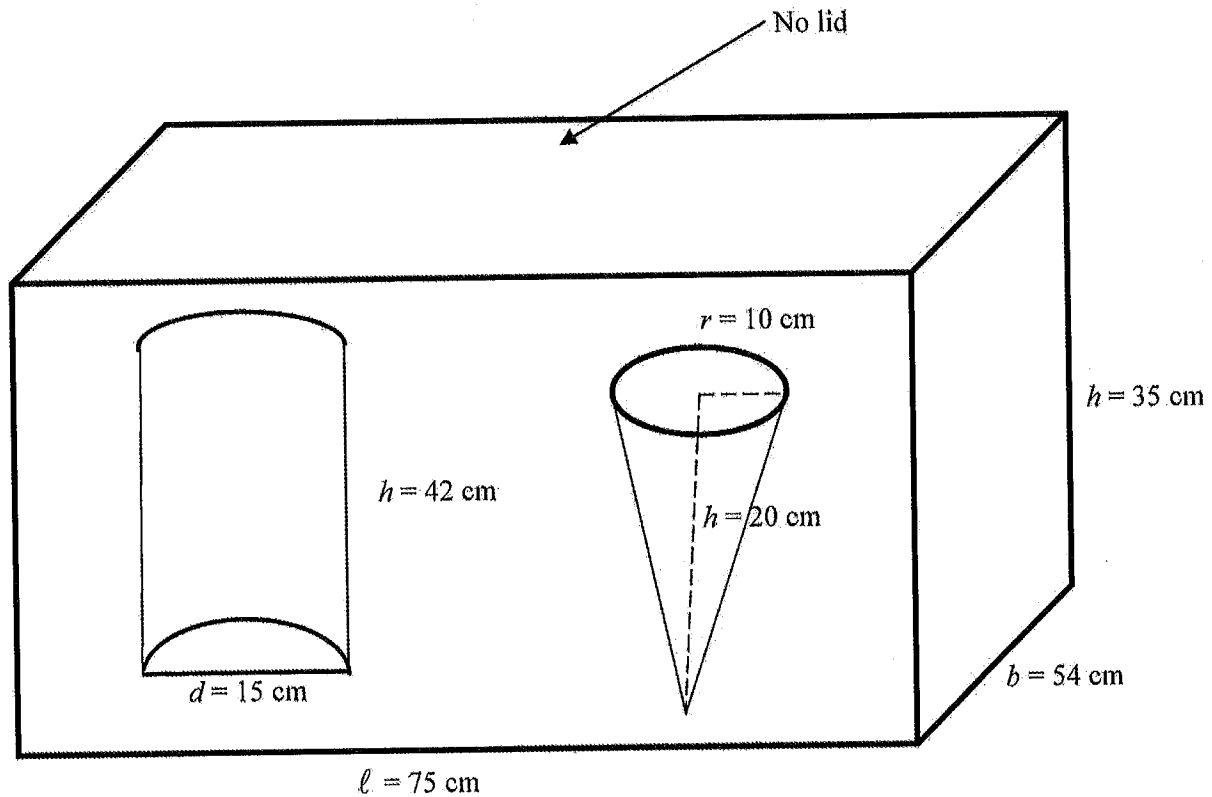
4.3.3 If MS is drawn to meet NP at S and $SM = 5,2$ units, calculate \hat{NSM} . (3)

4.4 Given ΔPQR , with $PQ = 140$ mm, $QR = 123,4$ mm and $PR = 199,2$ mm. Calculate the size of the largest angle of ΔPQR . (4)

[17]

QUESTION 5

The diagram below represents a rectangular prism with no lid. The dimensions are 75 cm by 54 cm by 35 cm. Two solids, a semi-circular rod with diameter of 15 cm, height of 42 cm and a cone with radius 10 cm, height 20 cm are placed inside the box.



- 5.1 Calculate the Volume of each shape inside the box. (4)
- 5.2 Determine the remaining Volume of the box. (2)
- 5.3 Calculate the Total Surface Area of the open box. (2)

[8]

Formulae:

$$\text{Surface area of cylinder} = 2\pi r^2 + 2\pi r h$$

$$\text{Surface area of rectangular prism} = 2\ell b + 2\ell h + 2bh$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of cylinder} = \pi r^2 h$$

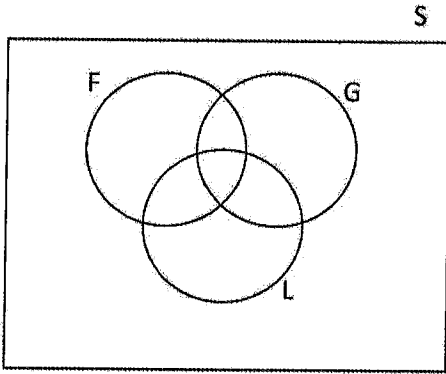
TOTAL [75]

DIAGRAM SHEET

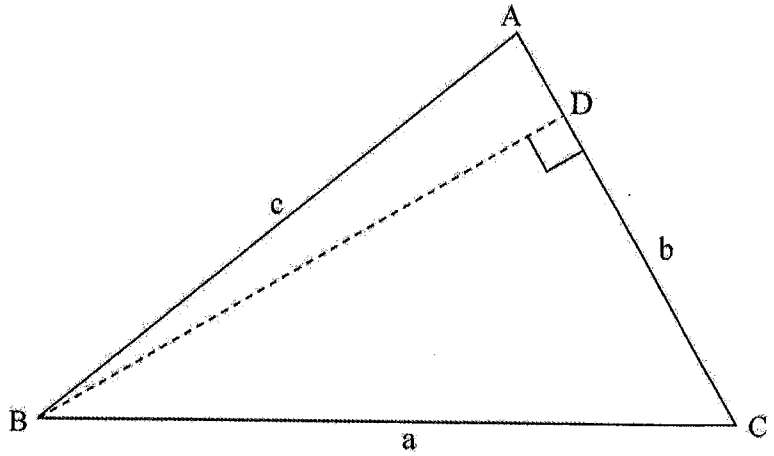
NAME: _____

GRADE: _____

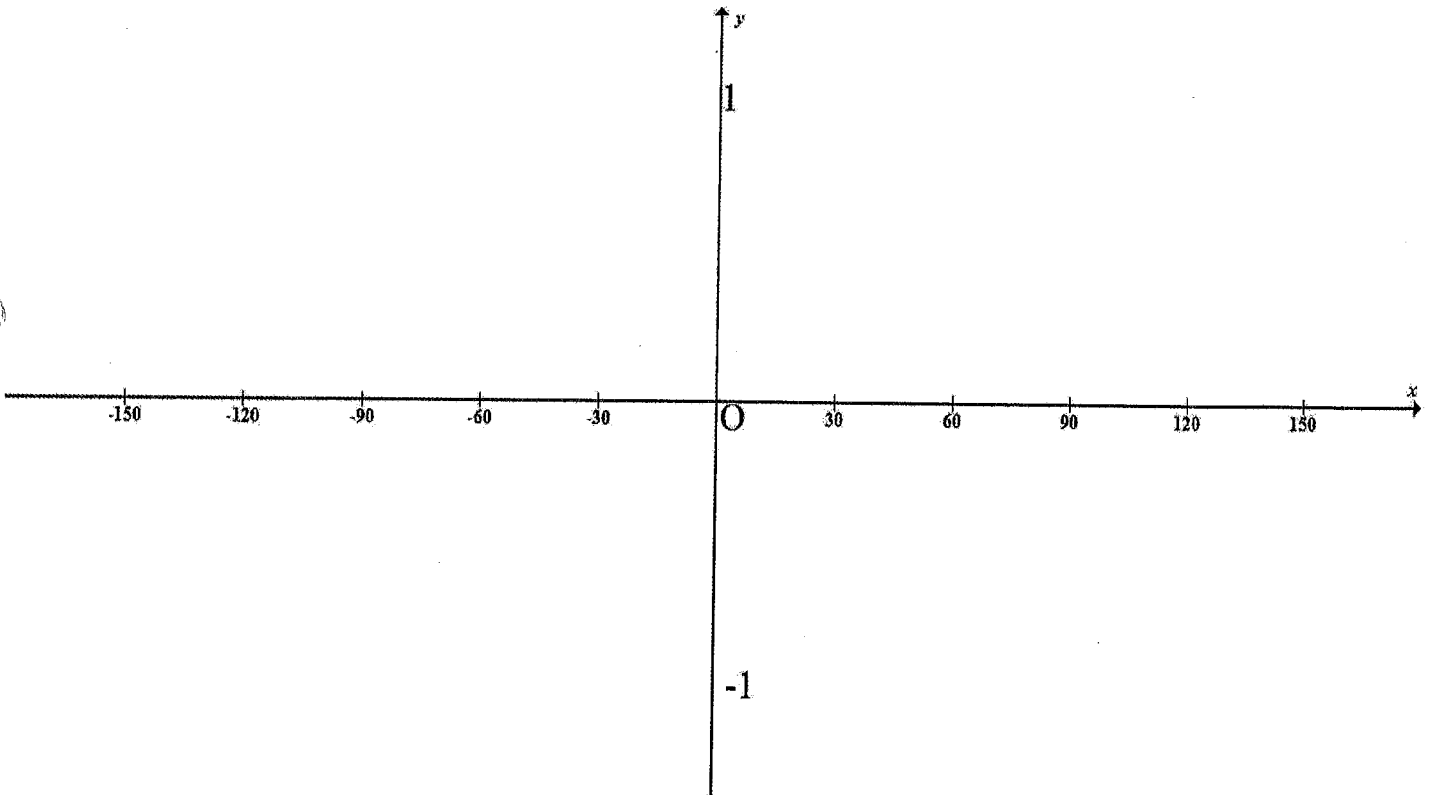
Question 2.1



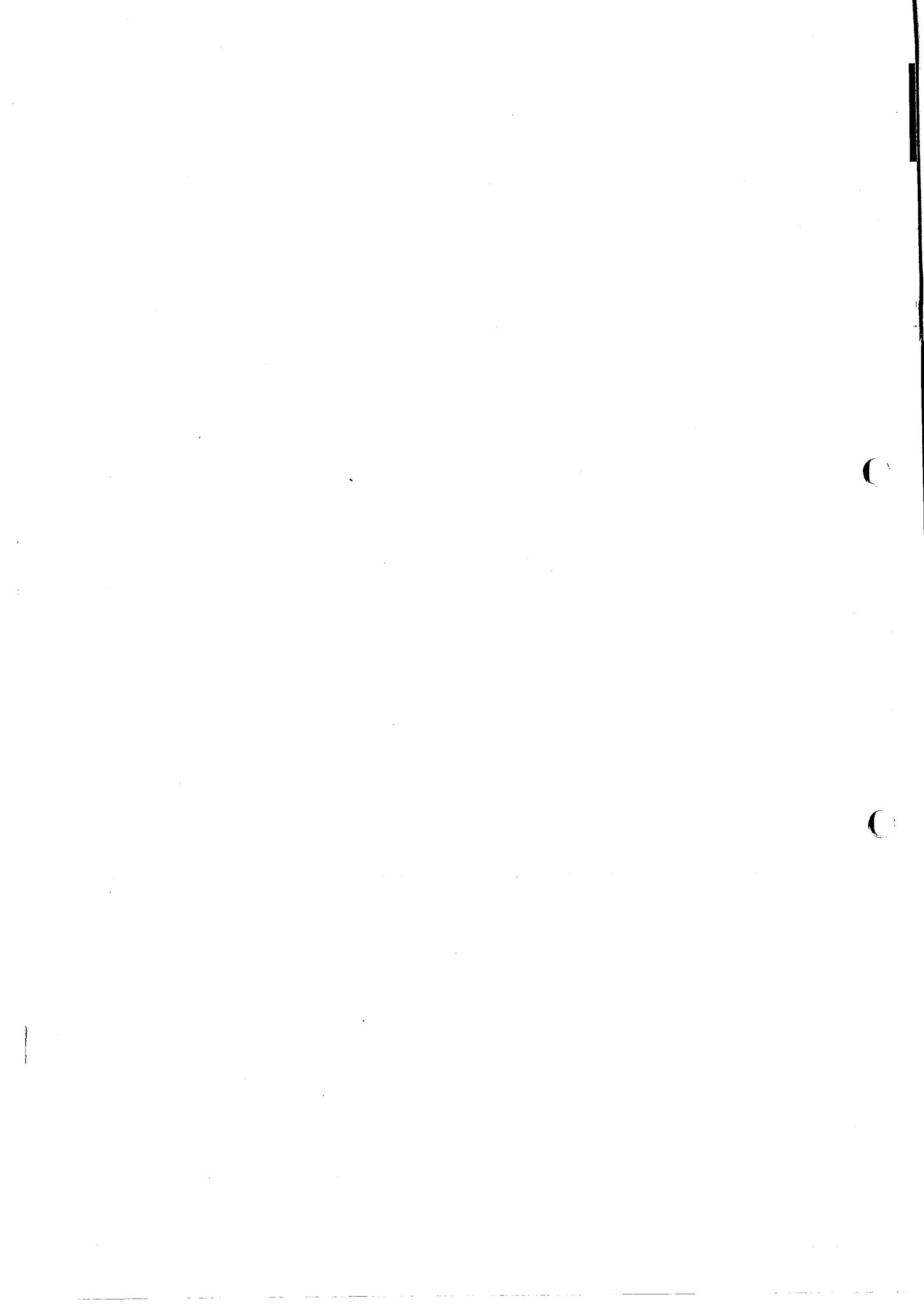
Question 4.1



Question 3.3.1



TEAR-OFF SHEET



(incl Grades 10 + 11)



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This memorandum consists of 8 pages.

QUESTION 1	
<p>1.1.1</p>	<p>1A for 1st strand 1A for second strand 2A marks for final outcomes (4)</p>
<p>1.1.2 $P(\text{orange tie}) = \frac{3}{6} = \frac{1}{2}$</p> <p>OR</p> <p>$P(\text{not red tie}) = 1 - P(\text{red tie})$ $= 1 - \frac{1}{2} = \frac{1}{2}$</p>	<p>1A for answer 1A for answer (1)</p>
<p>1.2 $P(A \text{ or } B) = P(A) + P(B)$ $= 0,2 + 0,35$ $= 0,55 \therefore$ Mutually exclusive ✓</p> <p>OR</p> <p>$P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B)$ $= 0,2 + 0,35 - 0,55$ $= 0 \therefore$ Mutually exclusive ✓</p>	<p>1A for correct substitution 1A for conclusion 1A for correct substitution 1A for conclusion (2) [7]</p>

QUESTION 2

2.1

1 A for each correct answer. (7)

2.2 (a) 15 ✓ (1)
 (b) 7 ✓ (1)
 (c) 3 ✓ (1)

1 CA for correct answer (1)
 1 CA for correct answer (1)
 1 CA for correct answer (1)

[10]

3.1.2 $-\cos \theta \checkmark$
 $= -\left(\frac{-4}{5}\right)$
 $= \frac{4}{5} \checkmark$

1A for $-\cos \theta$
 1CA for finding the ratio $\frac{4}{5}$ (2)

3.1.3 $\tan \theta = \frac{3}{4}$
 $\theta = 180^\circ + 36,87^\circ \checkmark$
 $= 216,87^\circ \checkmark$

OR

$\sin \theta = \frac{-3}{5}$
 $\theta = 180^\circ + 36,87^\circ \checkmark$
 $= 216,87^\circ \checkmark$

OR

$\cos \theta = \frac{-4}{5}$
 $\theta = 180^\circ + 36,87^\circ \checkmark$
 $= 216,87^\circ \checkmark$

1CA for reference angle
 1CA for correct answer, provided angle is reflex (2)

1CA for reference angle
 1CA for correct answer, provided angle is reflex (2)

1CA for reference angle
 1CA for correct answer, provided angle is reflex (2)

3.2

$$\frac{1 - \tan^2 \theta \cos(-\theta) \cos(560^\circ - \theta) \tan 22,5^\circ}{\sqrt{[\sin 90^\circ - \sin(180^\circ + \theta)] [\sin 90^\circ - \cos(90^\circ - \theta)]}}$$

$$= \frac{1 - \frac{\sin^2 \theta}{\cos^2 \theta} \cos \theta \cos \theta \tan 45^\circ}{(1 + \sin \theta)(1 - \sin \theta) \checkmark}$$

$$= \frac{1 - \frac{\sin^2 \theta}{\cos^2 \theta} \cos^2 \theta \cos \theta \cos \theta (1)}{(1 - \sin^2 \theta) \checkmark}$$

$$= \frac{1 - \sin^2 \theta \checkmark}{\sqrt{1 - \sin^2 \theta}}$$

$$= 1 \checkmark$$

1A for $\frac{\sin^2 \theta}{\cos^2 \theta}$ 1A for $\cos \theta \cos \theta$
 1A for simplifying denominator
 1A for $(1 - \sin^2 \theta)$ in the denominator
 1CA for $(1 - \sin^2 \theta)$ in the numerator
 1CA for correct answer (6)

QUESTION 3

3.1.1

1 A for correct diagram

1A for calculating the hypotenuse
 1CA for finding the ratio $\frac{-3}{5}$ (3)

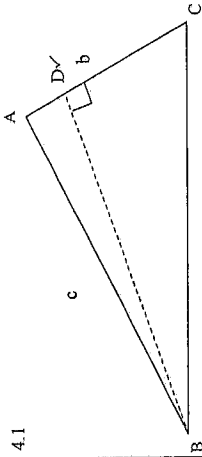
hypotenuse = $\sqrt{(-3)^2 + (-4)^2} = 5 \checkmark$
 $\sin \theta = \frac{-3}{5} \checkmark$

3.3.1		
See last page		
3.3.2	$[-1;1]$ ✓ OR $-1 \leq y \leq 1$	1 A for correct answer (1)
3.3.3	180° ✓	1 A for correct answer (1)
3.3.4	$[-60^\circ; 120^\circ]$ ✓✓ or $-60^\circ \leq x \leq 120^\circ$ ✓	1 CA for end points (2)
		1 A for Notation (2)

3.4	$2 \sin^2 x - 5 \sin x = 3$ $2 \sin^2 x - 5 \sin x - 3 = 0$ $(2 \sin x + 1)(\sin x - 3) = 0$ ✓ $\sin x = \frac{-1}{2}$ or $\sin x = 3$ ✓ N/A Ref angle = 30° ✓ $x = [210^\circ \text{ or } 330^\circ] + k \cdot 360^\circ, k \in \mathbb{Z}$ ✓	1 A for factorizing 1 CA for correct values of $\sin x$ 1 CA for stating the ref angle 1 CA for both angles and 1 M for writing $k \cdot 360^\circ, k \in \mathbb{Z}$ (5)
3.5	$\frac{1 + \cos A}{\sin A} + \frac{\sin A}{1 + \cos A}$ $= \frac{\sin A(1 + \cos A)^2 + \sin^2 A}{\sin A(1 + \cos A)}$ $= \frac{1 + 2 \cos A + \cos^2 A + \sin^2 A}{\sin A(1 + \cos A)}$ $= \frac{2 + 2 \cos A}{\sin A(1 + \cos A)}$ $= \frac{2(1 + \cos A)}{\sin A(1 + \cos A)}$ $= \frac{2}{\sin A}$	1 A for finding numerator 1 A for correct denominator 1 CA for simplifying numerator 1 CA for simplifying numerator 1 CA for factorising numerator (5)

[32]

QUESTION 4



1 M for diagram and construction

Construction: Draw $BD \perp AC$

In $\triangle ABD$, $\sin \hat{A} = \frac{BD}{c}$

$\therefore BD = c \sin \hat{A}$ ✓

In $\triangle CBD$, $\sin C = \frac{BD}{a}$

$\therefore BD = a \sin \hat{C}$ ✓

$c \sin \hat{A} = a \sin \hat{C}$ ✓

$\frac{c}{\sin C} = \frac{a}{\sin A}$

1 A for correct statement (4)

1 A for correct statement

1 A for correct statement

1 A for correct conclusion

1A for $\sin \hat{C}$ (1)

4.2. $\frac{1}{2} ab \sin \hat{C}$ ✓ / $\sin \hat{C}$

4.3.1 Area of $\triangle KLMN = 48$ sq. units
 Area of $\triangle NMP = 63$ sq. units - 48 sq units. ✓
 $= 15$ sq units ✓

1 A for 48 sq. units
 1 CA for correct answer (2)

4.3.2 Area of $\triangle NMP = \frac{1}{2} NP \cdot NM \cdot \sin \hat{MNP}$ ✓

$\frac{15}{\frac{1}{2}(8)(7)} = \sin \hat{MNP}$ ✓

$\hat{MNP} = 32,39^\circ$ ✓

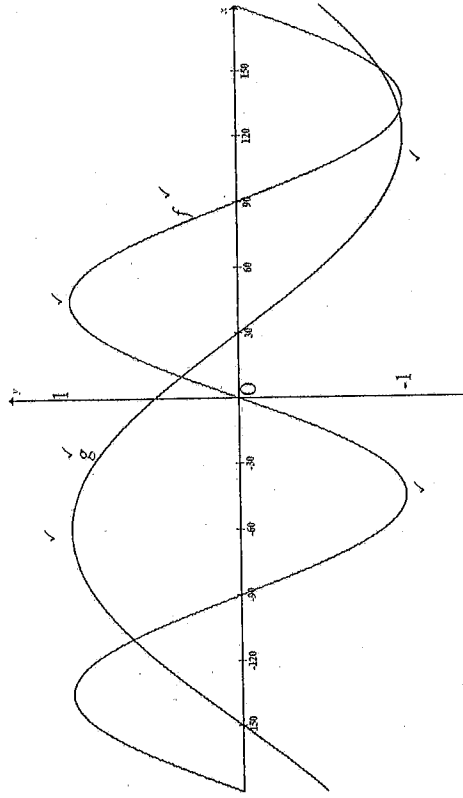
1 A for correct formula

1 CA for correct substitution

1 CA for correct answer (3)

<p>4.3.3 $\frac{\sin \text{NSM}}{8} = \frac{\sin 32,39^\circ}{5,2} \checkmark$ $\sin \text{NSM} = \frac{8 \sin 32,39^\circ}{5,2} \checkmark$ $\text{NSM} = 55,5^\circ \checkmark$</p>	<p>1 A for correct substitution 1 CA for correct simplification 1 CA for correct answer (3)</p> <p>1 M for correct formula 1 A for correct substitution 1 CA for changing the subject of the formula 1 CA for correct answer (4)</p> <p>[17]</p>
<p>4.4 $q^2 = p^2 + r^2 - 2pr \cos Q \checkmark$ $(199,2)^2 = (123,4)^2 + (140)^2 - 2(123,4)(140) \cos Q \checkmark$ $\cos Q = \frac{123,4^2 + 140^2 - 199,2^2}{2(123,4)(140)} \checkmark$ $Q = 98,07^\circ \checkmark$</p>	

QUESTION 3.3.1 NAME: _____ GRADE: _____



- 1A for correct shape of $\sin 2x$
- 2A for correct turning points/intercepts of $\sin 2x$
- 1A for correct shape of $\cos(x+60^\circ)$
- 2A for correct turning points/ intercepts of $\cos(x+60^\circ)$

(6)

<p>QUESTION 5</p> <p>5.1 Volume of semi-circular rod = $\frac{\pi r^2 h}{2}$ $= \frac{\pi(7,5)^2(42)}{2} \checkmark$ $= 3711,01 \text{ cm}^3 \checkmark$ Volume of Cone = $\frac{1}{3} \pi r^2 h$ $= \frac{1}{3} \pi (10)^2 (20) \checkmark$ $= 2094,39 \text{ cm}^3 \checkmark$</p>	<p>1A for correct substitution 1A for correct answer</p> <p>1A for correct substitution 1 CA for correct answer (4)</p> <p>1 CA for total volume of 2 shapes 1A for correct answer (2)</p>
<p>5.2 Remaining Volume of box = $(75 \times 54 \times 35) - 5805,4 \checkmark$ $= 135944,6 \text{ cm}^3 \checkmark$</p> <p>5.3 Total surface Area of open box $= 2LH + 2BH + LB$ $= 2(75 \times 35) + 2(54 \times 35) + (75 \times 54) \checkmark$ $= 13080 \text{ cm}^2 \checkmark$</p>	<p>1 CA for correct substitution 1 CA for correct answer (2)</p> <p>1A for correct substitution 1 CA for correct answer (2)</p> <p>[8]</p>