Education

KwaZulu-Natal Department of Education

REPUBLIC OF SOUTH AFRICA

MATHEMATICS

COMMON TEST

SEPTEMBER 2017

NATIONAL SENIOR CERTIFICATE

GRADE 11

MARKS: 75

TIME: 1½ hours

This question paper consists of 6 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of FIVE questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to **TWO** decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number your answers correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 Simplify without the use of a calculator:

1.1.1 $\sin^2 300^\circ + \frac{1}{2}\tan 315^\circ$ (3)

1.1.2 $\frac{\cos(-\theta).\cos(90^\circ+\theta)}{\tan(180^\circ-\theta)}$ (5)

1.2 If $\sin 29^\circ = \sqrt{t}$, determine the following, in terms of t, without the use of a calculator.

1.2.1 $\cos 61^\circ$ (1)

1.2.2 $\tan 209^\circ$ (3)

1.2.3 $\sin(-601^\circ)$ (2)

1.3 Prove the identity: $1-\frac{\cos^2 x}{1+\sin x} = \sin x$ (3)

1.4 Solve the equation $\tan x = -\frac{1}{\sqrt{3}}$ for the interval $0^\circ < x < 360^\circ$, without the use of a calculator. (2)
[19]

QUESTION 2

2.1 On the same set of axes, sketch the graphs of $f(x) = \sin(x-60^\circ)$ and $g(x) = \cos 2x$ for the interval $-180^\circ \le x \le 180^\circ$.
Indicate clearly the intercepts with the axes and the turning points. (6)

2.2 Write down the range of f. (1)

2.3 Write down the period of g. (1)

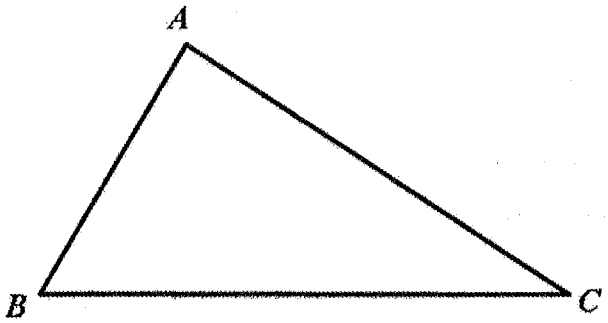
2.4 Determine the general solution of $\sin(x-60^\circ) = \cos 2x$. (5)

2.5 Hence, for which values of x in the interval $-180^\circ \le x \le 0^\circ$ will $f(x) \ge g(x)$? (3)

2.6 If $p(x) = -\cos(2x+20^\circ)$, describe fully, in words, the transformation from g to p. (3)
[19]

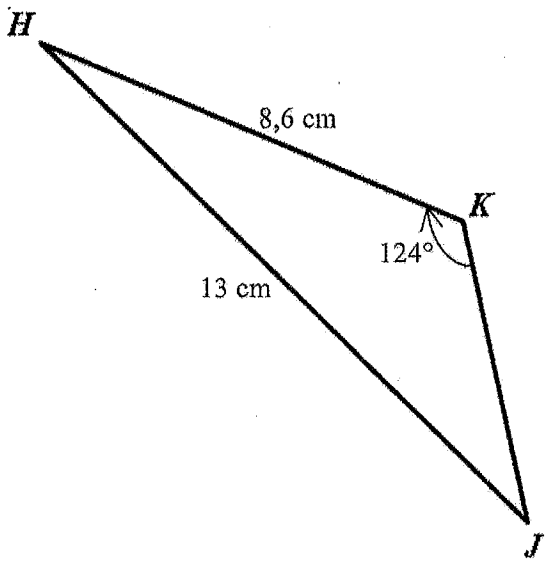
QUESTION 3

3.1 Redraw the sketch below in your answer book and use it to prove that $\frac{\sin B}{b} = \frac{\sin C}{c}$.



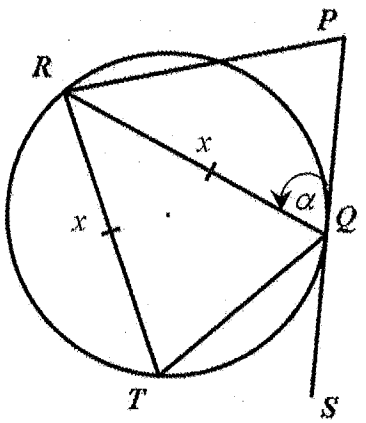
(4)

3.2 ΔHJK has $\hat{K} = 124°$, $HK = 8{,}6\,cm$ and $HJ = 13\,cm$.



Calculate the size of angle H. (4)

3.3 In the figure below $RT = RQ = x$, $P\hat{Q}R = \alpha$ and PQS is a tangent to the circle.



3.3.1 Prove that $TQ = x\sqrt{2(1+\cos 2\alpha)}$. (4)

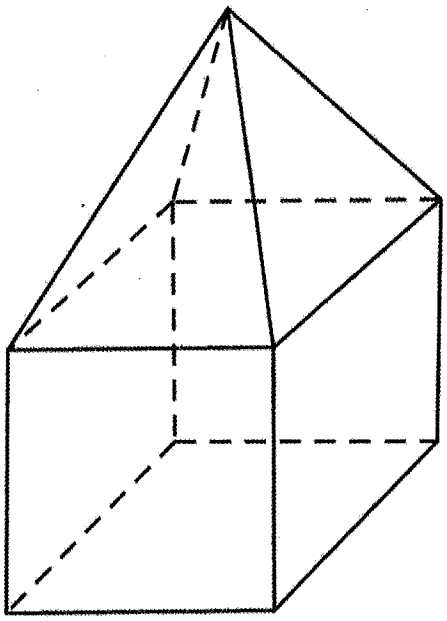
3.3.2 If $\alpha = 63°$ and $x = 17\,cm$, calculate the area of ΔRQT. (2)

[14]

QUESTION 4

The sketch below shows a wooden building block made in the shape of a pyramid mounted on a cube. The height of the pyramid is equal to the length of a side of the cube, namely 8 cm.

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$
$$\text{Volume} = \text{area of base} \times \text{perpendicular height}$$
$$\text{Volume} = \frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$$
$$\text{Volume} = \frac{4}{3}\pi r^3$$



4.1 Calculate the volume of the building block. (3)

4.2 Calculate the total surface area of the building block. (4)
[7]

QUESTION 5

5.1 A coin is tossed and a dice is rolled.

5.1.1 Draw a tree diagram and indicate all the possible outcomes. (3)

5.1.2 Determine the probability that a tail and a factor of 6 will be obtained. (2)

5.2 Given: $P(A) = 0,55$
$P(B) = 0,35$
$P(A \text{ or } B) = 0,8$
Determine whether events A and B are independent or not. Show all relevant calculations used in determining the answer. (4)

5.3 In a particular class at Nqutu High School all learners take Mathematics and Physical Sciences.
The probability that a randomly selected learner from that class will pass....

- Mathematics is 0,6;
- Physical Sciences is 0,7;
- both of these subjects is x; and
- neither of these subjects is 0,2.

5.3.1 Draw a Venn diagram to illustrate the above information. (2)

5.3.2 Calculate x. (2)

5.3.3 Determine the probability that a randomly selected learner from that class (1)

(a) will pass at least one of the two subjects. (2)
(b) will pass exactly one of the two subjects.

[16]

TOTAL MARKS: 75

GR 11 - MATHS + MLIT

Education

KwaZulu-Natal Department of Education

REPUBLIC OF SOUTH AFRICA

MATHEMATICS

MARKING GUIDELINE

COMMON TEST

SEPTEMBER 2017

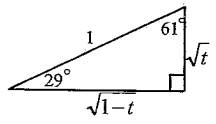
NATIONAL SENIOR CERTIFICATE

GRADE 11

MARKS: 75

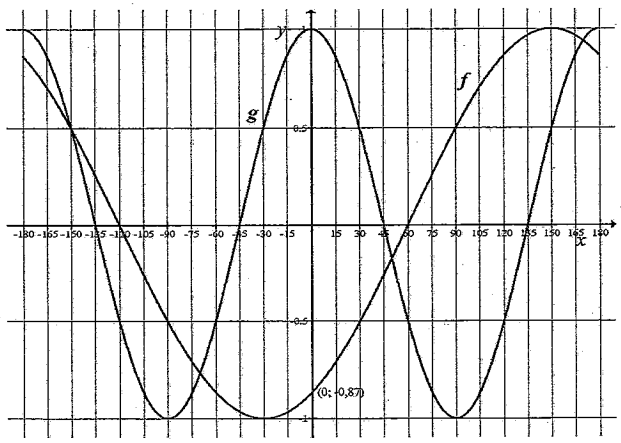
N.B. This marking guideline consists of 8 pages.

QUESTION 1

1.1.1	$(-\sin 60^\circ)^2 + \frac{1}{2}(-\tan 45^\circ)$ $= \sin^2 60^\circ - \frac{1}{2}\tan 45^\circ$ $= \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{1}{2}(1)$ $= \frac{3}{4} - \frac{1}{2}$ $= \frac{1}{4}$	1A for $(-\sin 60^\circ)^2$ 1A for $-\tan 45^\circ$ 1A for answer (3)
1.1.2	$\frac{\cos(-\theta).\cos(90^\circ+\theta)}{\tan(180^\circ-\theta)}$ $= \frac{\cos\theta.-\sin\theta}{-\tan\theta}$ $= \frac{\cos\theta.\sin\theta}{\frac{\sin\theta}{\cos\theta}}$ $= \frac{\cos\theta.\sin\theta}{1} \times \frac{\cos\theta}{\sin\theta}$ $= \cos^2\theta$	1A for $\cos\theta$ 1A for $-\sin\theta$ 1A for $-\tan\theta$ 1A for $\frac{\sin\theta}{\cos\theta}$ 1A for answer (5)
1.2.1	$\cos 61^\circ = \sin 29^\circ = \sqrt{t}$	1A for answer (1)
1.2.2	 $\tan 209^\circ = \tan(180^\circ + 29^\circ)$ $= \tan 29^\circ$ $= \frac{\sqrt{t}}{\sqrt{1-t}}$	1A for calculating $\sqrt{1-t}$ using Pythagoras 1A for $\tan 29^\circ$ 1CA for answer (3)
1.2.3	$\sin(-601^\circ) = -\sin 241^\circ$ $= \sin 61^\circ$ $= \sqrt{1-t}$	1A for reduction to $\sin 61^\circ$ 1CA for answer (2)

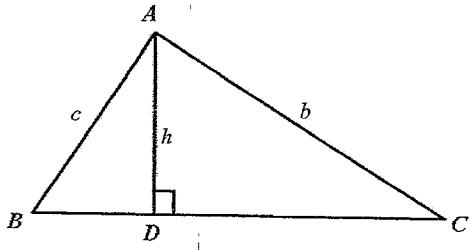
1.3	LHS = $1-\frac{\cos^2 x}{1+\sin x}$ $=\frac{1+\sin x-\cos^2 x}{1+\sin x}$ $=\frac{\sin x+\sin^2 x}{1+\sin x}$ $=\frac{\sin x(1+\sin x)}{1+\sin x}$ $=\sin x$ = RHS	1 A for numerator/ denominator 1A for applying $1-\cos^2 x=\sin^2 x$ 1A for factorisation (3)
1.5	$\tan x = -\frac{1}{\sqrt{3}}$ ref $\angle$: 30° x = 150° or 330°	1A for 150° 1A for 330° (2) **[19]**

QUESTION 2

2.1		$f(x)=\sin(x-60^\circ)$ and $g(x)=\cos 2x$: 2A shape 2A correct intercepts 2A correct turning points (6)
2.2	$[-1;\ 1]$ or $-1\le y\le 1$	1A for correct answer (1)
2.3	180°	1A for correct answer (1)

2.4	$\sin(x-60^\circ)=\sin(90^\circ-2x)$ $x-60^\circ=90^\circ-2x+k.360^\circ, k\in Z$ $3x=150^\circ+k.360^\circ$ $x=50^\circ+k.120^\circ$ or $x-60^\circ=180^\circ-(90^\circ-2x)+k.360^\circ, k\in Z$ $x-60^\circ=90^\circ+2x+k.360^\circ$ $-x=150^\circ+k.360^\circ$ $x=-150^\circ+k.360^\circ$ OR $\cos[90^\circ-(x-60^\circ)]=\cos 2x$ $\cos(150^\circ-x)=\cos 2x$ $150^\circ-x=2x+k.360^\circ, k\in Z$ $-3x=-150^\circ+k.360^\circ$ $x=50^\circ-k.120^\circ$ *or* $150^\circ-x=360^\circ-2x+k.360^\circ, k\in Z$ $x=210^\circ+k.360^\circ$	1A for changing $\cos 2x$ to $\sin(90^\circ-2x)$ 1A for $x-60^\circ=90^\circ-2x+k.360^\circ$ 1A for $x=50^\circ+k.120^\circ$ 1A for $-150^\circ+k.360^\circ$ 1A for $k\in Z$ OR 1A for changing $\sin(x-60^\circ)$ to $\cos(150^\circ-x)$ 1A for $150^\circ-x=2x+k.360^\circ$ 1A for $x=50^\circ-k.120^\circ$ 1A for $210^\circ+k.360^\circ$ 1A for $k\in Z$ (5)
2.5	Subst. $k=-1$ in $x=50^\circ+k.120^\circ$ yields $x=-70^\circ$ Subst. $k=0$ in $x=-150^\circ+k.360^\circ$ (or: $k=-1$ in $x=210^\circ+k.360^\circ$) yields $x=-150^\circ$. The solution therefore is: $-150^\circ\le x\le -70^\circ$	1CA for -70° 1CA for -150° 1A for inequality (3)
2.6	The graph of g has shifted horizontally to the left by 10° and is reflected about the x axis.	1A for horizontal shift to the left 1A for by 10° 1A for reflected about the x-axis (3) **[19]**

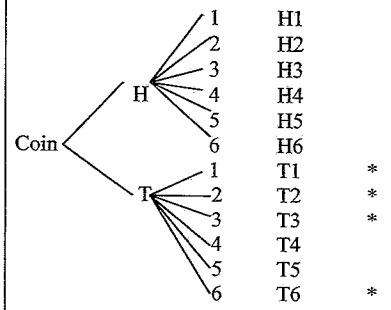
QUESTION 3

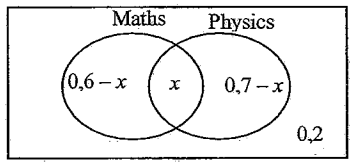
3.1	 Construction: Draw AD ⊥ BC with D on BC. Let AD = h. Proof: $\frac{h}{c}=\sin B$ $h=c\sin B$ $\frac{h}{b}=\sin C$ $h=b\sin C$ $c\sin B=b\sin C$ $\frac{\sin B}{b}=\frac{\sin C}{c}$	1A for construction 1A for $\frac{h}{c}=\sin B$ and $\frac{h}{b}=\sin C$ 1A for $h=c\sin B$ and $h=b\sin C$ 1A for $c\sin B=b\sin C$ (4)
3.2	$\frac{\sin\hat{J}}{j}=\frac{\sin\hat{K}}{k}$ $\frac{\sin\hat{J}}{8,6}=\frac{\sin 124^\circ}{13}$ $\sin\hat{J}=\frac{\sin 124^\circ.8,6}{13}$ $=0,5484....$ $\hat{J}=33,26^\circ$ $\hat{H}=180^\circ-(124^\circ+33,26^\circ)$ [sum of ∠'s of Δ] $=22,74^\circ$	1A for using sine rule 1A for making $\sin\hat{J}$ the subject of the formula 1CA for value of $\hat{J}$ 1CA for value of $\hat{H}$ (4)
3.3.1	$TQ=\sqrt{x^2+x^2-2x^2\cos(180^\circ-2\alpha)}$ $=\sqrt{2x^2+2x^2\cos 2\alpha}$ $=\sqrt{2x^2(1+\cos 2\alpha)}$ $=x\sqrt{2(1+\cos 2\alpha)}$	1A for using cosine rule 1A for $(180^\circ-2\alpha)$ 1CA for use of reduction formula 1CA for factorising (4)
3.3.2	area of $\Delta RQT=\frac{1}{2}x.x.\sin(180^\circ-2\alpha)$ $=\frac{1}{2}.17^2.\sin 54^\circ$ $=116,90\,cm^2$	1A for correct use of area rule 1CA for answer (2) **[14]**

QUESTION 4

4.1	Volume of building block = volume of cube + volume of pyramid = side³ + ⅓(area of base).height $=8^3+\frac{1}{3}(8\times 8)(8)$ $=512+170,67$ $=682,67\,cm^3$	1A for 8^3 1A for $\frac{1}{3}(8\times 8)(8)$ 1CA for answer (3)
4.2	Height of triangle of pyramid $=\sqrt{8^2+4^2}$ $=\sqrt{80}$ Total surface area = 5(area of side of cube) + 4(area of triangle of pyramid) $=5(8\times 8)+4\left(\frac{1}{2}.8.\sqrt{80}\right)$ $=320+143,11$ $=463,11\,cm^2$	1A for calculating height of triangle 1A for $5(8\times 8)$ 1A for $4\left(\frac{1}{2}.8.\sqrt{80}\right)$ 1CA for answer (4) [7]

QUESTION 5

5.1.1		1A for the tree diagram of tossing the coin 1A for adding the tree diagram of rolling the dice 1A for the outcomes (3)

5.1.2	P(tail and factor of 6) $= \frac{4}{12}$ $= \frac{1}{3}$ OR P(tail and factor of 6) = P(tail) x P(factor of 6) $= \frac{1}{2} \times \frac{4}{6}$ $= \frac{4}{12}$ $= \frac{1}{3}$	2A for correct answer OR 2A for correct answer (2)
5.2	P(A or B) = P(A) + P(B) – P(A and B) 0,8 = 0,55 + 0,35 – P(A and B) P(A and B) = 0,1 $P(A) \times P(B) = 0{,}55 \times 0{,}35$ $= 0{,}19$ $\therefore P(A \text{ and } B) \neq P(A) \times P(B)$ And A and B are not independent	1A correct substitution 1A correct answer for P(A and B) 1A calculating $P(A) \times P(B)$ 1A concluding (4)
5.3.1		2 A for correct diagram (2)
5.3.2	$0{,}6 - x + x + 0{,}7 - x + 0{,}2 = 1$ $-x = -0{,}5$ $\therefore x = 0{,}5$	1A correct statement 1A correct answer (2)
5.3.3 (a)	P(passes at least one of two subjects) = 0,8	1A correct answer (1)
5.3.3 (b)	P(passes exactly one of the two subjects) = 0,1 + 0,2 = 0,3	2AA correct answer (2) **[16]**

TOTAL MARKS: 75