MATHEMATICS

MATERIAL FOR GRADE 12

ANALYTICAL GEOMETRY

MEMORANDA

QUESTION 1

1.1	$m_{PR} = \frac{2+4}{9+4} = \frac{6}{13}$	✓✓sub. into the formula answer ✓ (3)
1.2	$\tan\theta = \frac{6}{13}$ $\theta = 24.78°$ $\alpha = 90° - 24.78°$ $= 65.22°$	✓ $\tan\theta = \frac{6}{13}$ ✓$\theta = 24.78°$ ✓$\alpha = 90° - 24.78°$ ✓answer (4)
1.3	$(a-9)^2 + (10-2)^2 = (4\sqrt{5})^2$ $a^2 - 18a + 81 + 64 = 80$ $a^2 - 18a + 65 = 0$ $(a-13)(a-5) = 0$ $a = 13_{N/A}$ or $\therefore a = 5$	✓sub. into the formula ✓$4\sqrt{5}$ ✓$a^2 - 18a + 65 = 0$ ✓$(a-13)(a-5)$ ✓$a = 13_{N/A}$ ✓answer (6)
1.4	$m_{PR} = \frac{6}{13}$ and Q (5 ; 10) $y - y_1 = m(x - x_1)$ $y - 10 = \frac{6}{13}(x-5)$ $y = \frac{6}{13}x + \frac{100}{13}$	✓$m_{PR} = \frac{6}{13}$ ✓Sub. into the correct formula ✓answer (3)

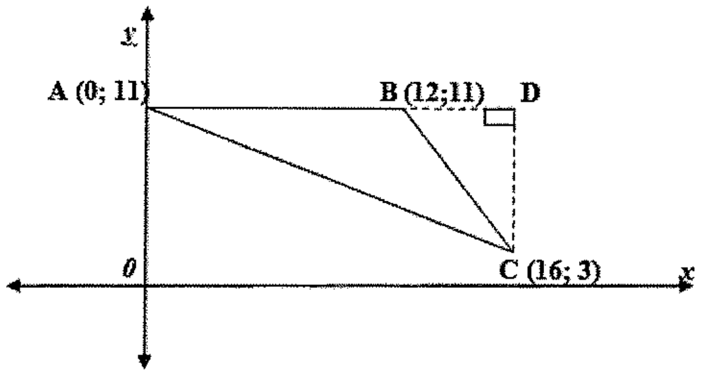
1.5	S (- 8; 4) **OR** Mdpt QR ($\frac{5-4}{2};\frac{10-4}{2}$) $(\frac{1}{2};3)$ $\frac{x+9}{2}=\frac{1}{2}$ $x=-8$ $\frac{y+2}{2}=3$ $y=4$ $\therefore$ S(−8; 4)	✓✓ x co-ordinate ✓✓ y co-ordinate **OR** ✓midpoint of QR ✓sub in formula ✓sub in formula ✓x *and* y coordinate (4)
		[20]

QUESTION 2

2.1	$y=-x+1$ $a=-(-3)+1$ $=4$	✓correct subst ✓answer (2)
2.2	$x^2+y^2=r^2$ $(4)^2+(-3)^2=r^2$ $\therefore r^2=25$	✓equation of circle ✓subst for pt.D ✓$r^2=25$ (3)
2.3	$x^2+(-x+1)^2=25$ $x^2-x-12=0$ $(x-4)(x+3)=0$ $x=4$ or $x=-3$ $y=-4+1$ $=-3$ $\therefore C(4;-3)$	✓subst$(-x+1)$ ✓simplification ✓factors ✓for both values of x ✓for choosing correct x value ✓for finding y (6)

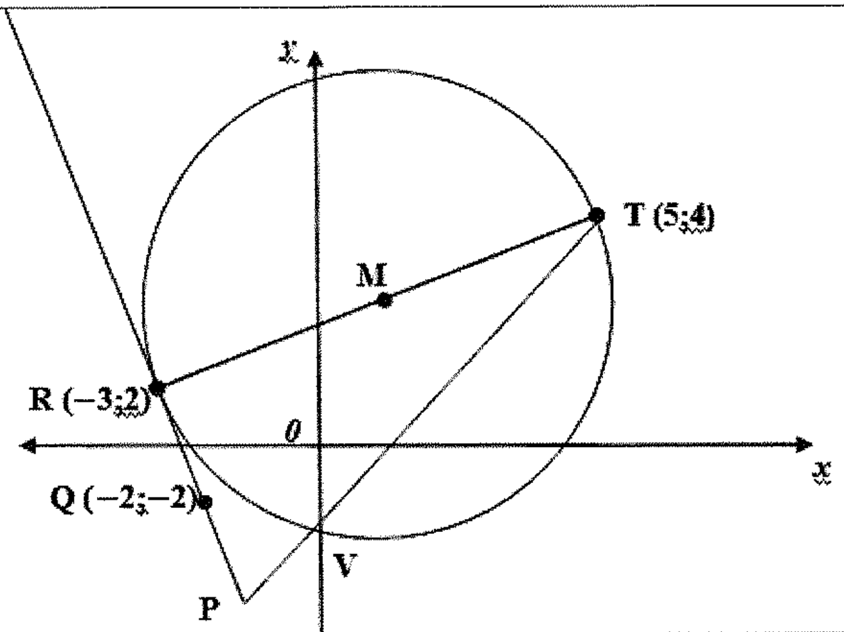
2.4.1	$m_{OD} = \frac{4-0}{-3-0}$ $= \frac{-4}{3}$	✓for subst ✓$\frac{-4}{3}$ (2)
2.4.2	$m_{tan} = \frac{3}{4}$ $y - 4 = \frac{3}{4}(x+3)$ $y = \frac{3}{4}x + \frac{9}{4} + 4$ $y = \frac{3}{4}x + \frac{25}{4}$	✓$\frac{3}{4}$ ✓for subst ✓for correct equation (3)
		[16]

QUESTION 3



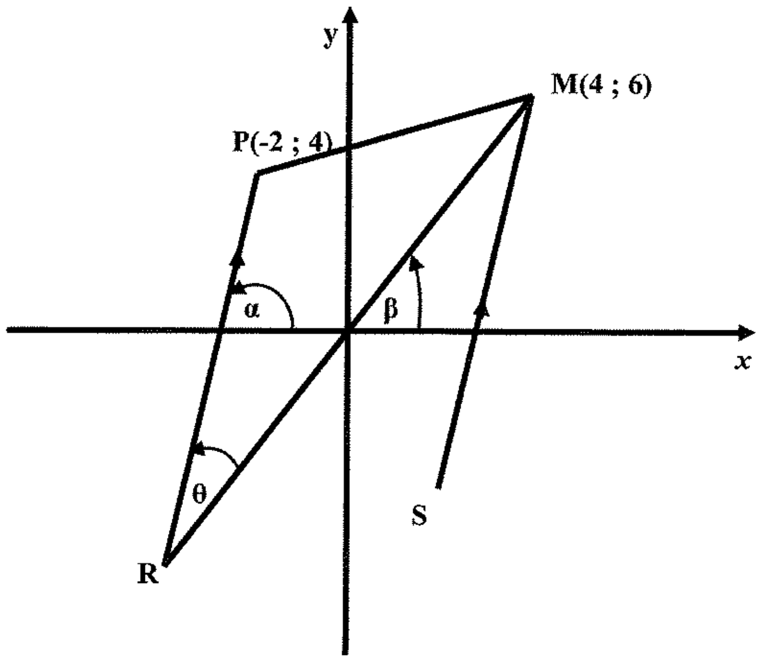
3.1	$y = 11$ AB = 12	✓✓y = 11 ✓ AB = 12 (3)
3.2	D(16;11)	✓✓ (2)
3.3	M(8; 7)	✓✓ (2)
3.4	$m_{AC} = \frac{3-11}{16} = -\frac{8}{16} = -\frac{1}{2}$ $m_{line} = 2$ $y - 7 = 2(x-8)$ $y = 2x - 9$	✓$-\frac{1}{2}$ ✓$m_{line} = 2$ ✓substitution ✓ equation (4)
3.5	$y = 2(12) - 9$ $= 15$ $\neq 11$ No, it does not pass through B	✓substitute ✓ $\neq 11$ No, it does not pass through B (2)
3.6	$\tan\theta = m_{BC} = \frac{11-3}{12-16}$ $\tan\theta = -2$ $\theta = 116{,}57°$	✓tan θ ✓-2 ✓116,57° (3)
3.7	$m_{new\ line} = -\frac{1}{2}$ $y - 11 = -\frac{1}{2}(x - 16)$ $y = -\frac{1}{2}x + 19$	✓ $-\frac{8}{13}$ ✓substitute ✓equation (3)
3.8	Area $\Delta\, ABC = \frac{1}{2}\, base\ height$ $= \frac{1}{2} \times 12 \times 8$ = 48 sq units	✓h=8 ✓ substitution ✓answer (3)
		[22]

QUESTION 4



4.1	M (1; 3) $r^2 = (5-1)^2 + (4-3)^2$ $r^2 = 16 + 1 = 17$ $(x-1)^2 + (y-3)^2 = 17$	✓✓M ✓substitute ✓$r^2 = 17$ ✓$(x-1)^2 + (y-3)^2 = 17$ (5)
4.2	$m_{PR} = \frac{-2-2}{-2+3} = -4$ $m_{RT} = \frac{4-2}{5+3} = \frac{1}{4}$ $m_{PR} \times m_{RT} - 1$ PR is a tangent	✓m_{PR} ✓m_{RT} ✓product = −1 (3)
4.3	Y int: $(0-1)^2 + (y-3)^2 = 17$ $1 + y^2 - 6y + 9 = 17$ $y^2 - 6y - 7 = 0$ $(y-7)(y+1) = 0$ $y = -1$ or $y = 7$ V(0; −1)	✓let $x = 0$ ✓standard form ✓$y = -1$ or $= 7$ ✓V(0; −1) (4)
4.4	$m_{PT} = \frac{4+1}{5-0} = 1$ $\tan\alpha = 1$ $\alpha = 45°$ $\tan\beta = -4$ $\beta = 104°$ $\theta = 59°$	✓m_{PT} ✓$\tan\alpha = 1$ ✓$\alpha = 45°$ ✓$\tan\beta = -4$ ✓$\beta = 104°$✓ $\theta = 59°$ (6)
		[18]

QUESTION 5



5.1	$m_{MR} = \frac{6}{4} = \frac{3}{2}$ Equation of MR is $y = \frac{3}{2}x$	✓ sub into the formula ✓ $\frac{3}{2}$ ✓ equation (3)
5.2	$y - 5x + 14 = 0$ $y = 5x - 14$ $m_{MS} = 5$ $m_{PR} = 5$ Equation of PR is $y - y_1 = m(x - x_1)$ $y - 4 = 5(x + 2)$ $y = 5x + 14$	✓ $m_{MS} = 5$ ✓ $m_{PR} = 5$ ✓sub (– 2; 4) into the formula ✓answer (4)
5.3	$m_{PR} = 5$ $\tan\alpha = 5$ $\alpha = 78{,}69°$ $m_{MR} = \frac{3}{2}$ $\tan\beta = \left(\frac{3}{2}\right)$ $\beta = 56{,}31°$ $\therefore \theta = \alpha - \beta$ $= 22{,}38°$	✓ $\tan\alpha = 5$ ✓ 78,69° ✓56,31° ✓$\therefore \theta = \alpha - \beta$ ✓22,38° (5)

5.4	$y = \frac{2}{3}x$ and $y = 5x + 14$ $5x + 14 = \frac{3}{2}x$ $10x + 28 = 3x$ $7x = -28$ $x = -4$ $y = -6$ $R(-4\,;-6)$	✓equating $5x + 14 = \frac{3}{2}x$ ✓$10x + 28 = 3x$ ✓ $x = -4$ ✓ $y = -6$ (4)
5.5	$MR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(4+4)^2 + (6+6)^2}$ $= \sqrt{64 + 144}$ $= 4\sqrt{13}$	✓sub into the formula ✓answer (2)
5.6	$PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(-4+2)^2 + (-6-4)^2}$ $= \sqrt{104}$ or $2\sqrt{26}$ Area of Δ PMR $= \frac{1}{2} PR.MR.\sin\theta$ $= \frac{1}{2} 2\sqrt{26}.4\sqrt{13}.\sin 22{,}38°$ or $\frac{1}{2}\sqrt{104}.4\sqrt{13}.\sin 22,$ = 28 square units	✓sub.into formula ✓ $\sqrt{104}$ or $2\sqrt{26}$ ✓$\frac{1}{2} PR.QR.\sin\theta$ ✓ $\frac{1}{2} 2\sqrt{26}.4\sqrt{13}.\sin 22{,}38°$ or $\frac{1}{2}\sqrt{104}.4\sqrt{13}.\sin 22{,}38°$ ✓answer (5)
5.7	S (2 ; – 4)	✓✓(2; – 4) (2) **[25]**

QUESTION 7.

7.1. $m_{BC} = \frac{0+8}{-5+1}$ ✓

$= \frac{8}{-6} = \frac{-4}{3}$ ✓ (2)

7.2. $E = \left(\frac{-5+1}{2} ; \frac{0-8}{2}\right)$ (2)

$E = (-2 ; -4)$ ✓✓

7.3. $m_{DE} = \frac{3}{4}$ ✓ ... $DE \perp BC$

$y - (-4) = \frac{3}{4}(x - (-2))$ ✓ (3)

$y = \frac{3}{4}x - \frac{5}{2}$ ✓

7.4 $\tan\theta = m_{AD}$

$\tan\theta = \frac{-4}{3}$ ✓

$\theta = 180° - 53,13°$ ✓ (3)

$= 126,87°$ ✓

7.5. $O\hat{F}D = 126,87° - 90°$ (Ext. ∠ of Δ) ✓

$= 36,87°$ ✓ (2)

7.6. ~~$\sqrt{2}$~~ $(5\sqrt{2})^2 = (-5-x)^2 + (0-7)^2$ ✓

50 ✓ $= x^2 + 10x + 25 + 49$

$0 = x^2 + 10x + 24$ ✓ (5)

$0 = (x+6)(x+4)$ ✓

$\therefore x \neq -6$ or $x = -4$ ✓.

7.7. $(x-a)^2 + (y-b)^2 = r^2$ ✓

$(x-(-2))^2 + (y-(-4))^2 = (-5-(-2))^2 + (0-(-4))^2$ ✓

$(x+2)^2 + (y+4)^2 = (-5+2)^2 + (0+4)^2 = 25.$ (3)

$(x+2)^2 (y+4)^2 = 25.$ ✓

OR $(x+2)^2+(y+4)^2=r^2$ ✓

$(1+2)^2+(-8+4)^2=r^2$ ✓ Subst: $(1;-8)$ (3)

$(x+2)^2+(y+4)^2=25$ ✓

OR $(x+2)^2+(y+4)^2=r^2$ ✓

$(-5+2)^2+(0+4)^2=r^2$ ✓ Subst $(-5;0)$ (3)

$(x+2)^2+(y+4)^2=25$ ✓

QUESTION 8.

8.1.1. $x^2 + y^2 + 4x - 4y - 12 = 0$

$(x+2)^2 + (y-2)^2 = 12 + 4 + 4 = 20$ ✓ (2)

$M(-2; 2)$ ✓ ✓

OR $M\left(\frac{4}{-2}; \frac{-4}{-2}\right)$

$M(-2; 2)$ ✓ ✓ (2)

8.1.2. $x^2 + 0^2 + 4x - 4(0) - 12 = 0$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$ ✓ (3)

$x \neq -6$ and $x = 2$

$C[2; 0]$ ✓ ✓

OR $\frac{x_c - 6}{2} = -2$ ✓ (3)

$x_c = 2$

$C[2; 0]$ ✓ ✓

8.2. $m_{MC} = -\frac{1}{2}$

$m_{tan} = 2$ ✓ [tan ⊥ radius] ✓

$y - y_1 = m(x - x_1)$

$y - 4 = 2(x - (-6))$ ✓ (4)

$y = 2x + 16$ ✓

8.3 $B(x; 0)$

$y = 2x + 16$

$\therefore 0 = 2x + 16$ ✓ (5)

$-8 = x$ ✓

Area $\triangle ABC = \frac{1}{2} BC \times h$

$= \frac{1}{2}(10)(4)$ ✓ ✓

$= 20$ sq units ✓

OR $B(x; 0)$

$y = 2x + 16$

$0 = 2x + 16$ ✓ (5)

$-8 = x$ ✓

$AB^2 = (-6+8)^2 + (4-0)^2$

$AB = \sqrt{20}$ ✓

Area $\triangle ABC = \frac{1}{2} AB.AC$

$= \frac{1}{2}(\sqrt{20})(2\sqrt{20})$ ✓

$= 20$ unit^2 ✓

3.4 Eq of tangent parallel to AB through C.

$y - y_1 = m(x - x_1)$

$y - 0 = 2(x - 2)$ ✓

$y = 2x - 4$ ✓ ⑤

$\therefore -4 < k < 16$ ✓

OR $x^2 + (2x + k)^2 + 4x - 4(2x + k) - 12 = 0$.

$5x^2 + 4xk - 4x + k^2 - 4k - 12 = 0$.

$5x^2 + (4k - 4)x + (k^2 - 4k - 12) = 0$.

$\therefore \Delta > 0$ ✓

$(4k - 4)^2 - 4(5)(k^2 - 4k - 12) > 0$

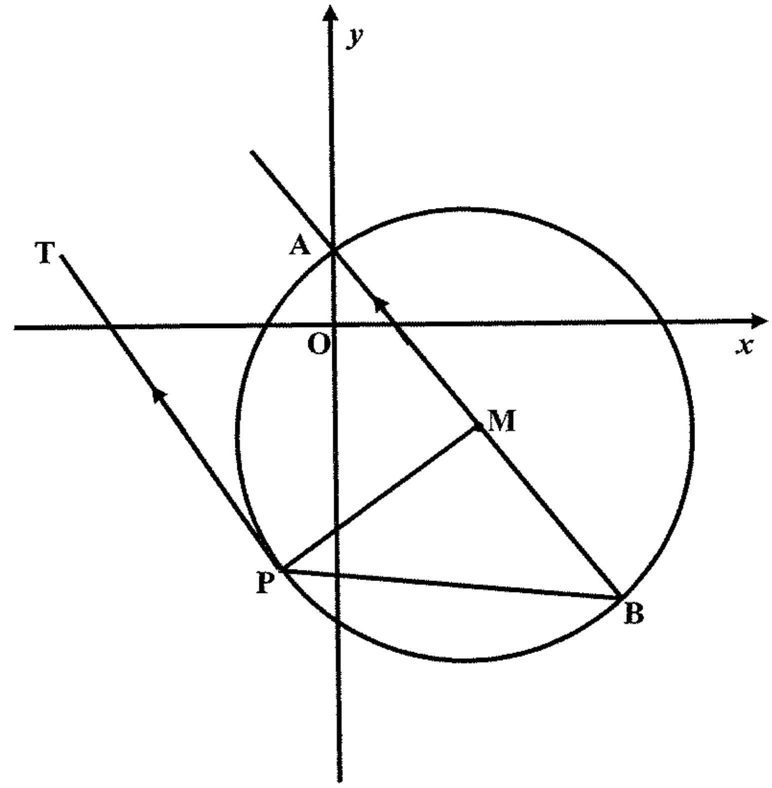
$-4k^2 + 64k + 256 > 0$ ⑤

$k^2 - 16k - 64 > 0$ ✓

$(k - 16)(k + 4) > 0$

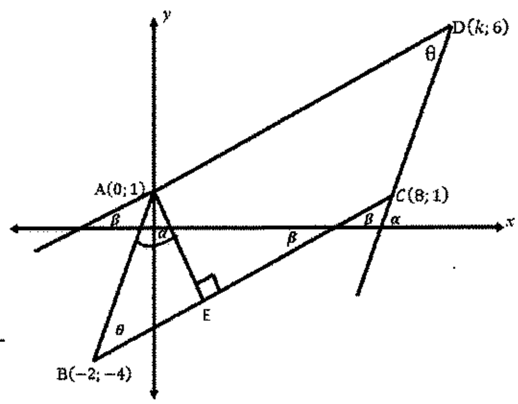
$\therefore -4 < k < 16$ ✓

QUESTION 6



6.1	$x^2 - 2x + y^2 + 4y - 5 = 0$ $x^2 - 2x + (1)^2 + y^2 + 4y + (2)^2 = 5 + 1 + 4$ $(x-1)^2 + (y+2)^2 = 10$ M (1 ; − 2)	✓$(x-1)^2 + (y+2)^2$ ✓10 ✓✓answer (4)
6.2	$T\hat{P}M = 90°$ (radius ⊥ tangent) $P\hat{M}B = T\hat{P}M$ (alternate $\angle s$) $= 90°$	✓$T\hat{P}M = 90°$ ✓ radius ⊥ tangent ✓ answer (3)
6.3	PM: $3y - x + 7 = 0$ $y = \frac{1}{3}x - \frac{7}{3}$ $m_{AB} = -3$ Equation of AB: $y - y_1 = m(x - x_1)$ $y + 2 = -3(x - 1)$ $y = -3x + 1$	✓ $m_{PM} = \frac{1}{3}$ ✓ $m_{AB} = -3$ ✓ sub. (1; − 2) ✓answer (4)
6.4	A (0; 1)	✓x- coordinate ✓y-coordinate (2)
6.5	$TM = \sqrt{80}$; $PM = \sqrt{10}$ $PT = \sqrt{TM^2 - PM^2}$ (Pythagoras thm) $= \sqrt{80 - 10}$ $= \sqrt{70}$ or 8,37	✓$\sqrt{10}$ ✓$\sqrt{80-10}$ ✓$\sqrt{70}$ or 8,37 (3) **[16]**

QUESTION 9

9.1	$BC = \sqrt{(8+2)^2 + (1+4)^2}$ $= \sqrt{(10)^2 + (5)^2}$ $= \sqrt{100 + 25}$ $= \sqrt{125}$ $= 5\sqrt{5}$	✓correct sub into formula ✓simplification ✓answer (3)
9.2	AD= $BC = 5\sqrt{5}$ opp sides of a parm are equal $5\sqrt{5} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $(5\sqrt{5})^2 = (k-0)^2 + (6-1)^2$ $125 = k^2 + 25$ $k^2 = 100$ $\therefore k = \pm 10$ $k = 10$ AB // DC x moved 2 places to the right. From A →B	✓S ✓substitution ✓answer (3)
9.3	$m_{BC} = \frac{-4-1}{-2-8}$ $= \frac{-5}{-10}$ $= \frac{1}{2}$ $\therefore m_{AE} = -2$ Equation of AE: $y - y_1 = m(x - x_1)$ $y - 1 = -2(x - 0)$ $y = -2x + 1$	✓$\frac{1}{2}$ ✓$m_{AE} = -2$ ✓ subs (0;1) ✓ answer (4)
9.4	$mAD = \frac{6-1}{10-0}$ $= \frac{1}{2}$ $\tan\beta = m$ $= \frac{1}{2}$ $\therefore \beta = 26{,}57°$  	✓$\frac{1}{2}$ ✓ $\beta = 26{,}57°$

	$mDC = \frac{6-1}{10-8}$ $= \frac{5}{2}$ $\tan\alpha = \frac{5}{2}$ $\alpha = 68{,}20°$ $\therefore \theta = \alpha - \beta$ $\theta = 68{,}20° - 26{,}57°$ (ext. angle of Δ) $= 41{,}43°$	✓ $\frac{5}{2}$ ✓ $\alpha = 68{,}20°$ ✓ $\theta = \alpha - \beta$ ✓ $41{,}43°$ (6)
		[16]

QUESTION 10

10.1	$\frac{x_1+x_2}{2} = -4$; $\frac{y_1+y_2}{2} = q$ Q is the midpoint $\frac{-12+a}{2} = -4$; $\frac{2+11}{2} = q$ AQ = QB $-12 + a = -8$; $13 = 2q$ $a = 4$ $q = \frac{13}{2}$	✓ subst ✓ $a = 4$ ✓ $q = \frac{13}{2}$ (3)
10.2	MQ ⊥ AB $M_{AB} = \frac{11-2}{4+12}$ $= \frac{9}{16}$ $\therefore M_{MQ} = \frac{-16}{9}$ $\frac{-16}{9} = \frac{3-\frac{13}{2}}{m+4}$ $-16m - 64 = \frac{-63}{2}$ $-32m = 128 - 63$ $\therefore m = \frac{65}{-32}$	✓ $\frac{-16}{9}$ ✓ $-32m = 128 - 63$ ✓ $m = \frac{65}{-32}$ (3)
10.3	$(x+2)^2 + (y-3)^2 = 100$	✓ subst ✓ answer (2)
10.4	$m_{MA} = \frac{3-2}{-2+12}$	✓ $\frac{-4}{3}$

	$= \frac{1}{10}$ $\therefore m_{tan} = -10$ Equation: $y - 2 = -10(x + 12)$ $\therefore y = -10x - 118$	✓$m_{tan} = \frac{3}{4}$ ✓equation (3)
		[11]

QUESTION 11

11.1	$m_{AB} = m_{BC}$ $\therefore \frac{2-5}{3-6} = \frac{k+4-2}{2k-3}$ $\therefore 1 = \frac{k+2}{2k-3}$ $\therefore 2k-3 = k+2$ $\therefore k = 5$	✓ $m_{AB} = m_{BC}$ ✓ working out gradients ✓ $k = 5$ (3)
11.2.1	$x^2 - 4x + (-2)^2 + y^2 + 4y + (2)^2 = -3 + (-2)^2 + (2)^2$ $\therefore (x-2)^2 + (y+2)^2 = 5$ Centre is $(2\,;-2)$ radius $= \sqrt{5}$	✓ $(x-2)^2$ ✓ $(y+2)^2$ ✓ $(2\,;-2)$ ✓ $\sqrt{5}$ (4)
11.2.2	LHS: $(3-2)^2 + (-3+2)^2 = 5$ RHS: 5 $\therefore$ LHS = RHS and $\therefore$ T lies on the circle	✓ substitution ✓ LHS = RHS T lies on the circle (2)
		[9]

QUESTION 12

12.1 AC $= \sqrt{(-5-7)^2 + (1-(2))}^2$ $= \sqrt{(12)^2 + (3)^2}$ $= \sqrt{144+9}$ $= \sqrt{153}$ $= 12{,}37$	A✓ correct Subst CA ✓ answer (2)
12.2 $M_{BC} = \frac{6-(2)}{1-7}$ $= \frac{8}{-6}$ $= \frac{-4}{3}$ $y - y_1 = m(x - x_1)$ $y - 6 = -\frac{4}{3}(x-1)$ $3y - 18 = -4x + 4$ $3y = -4x + 22$	A✓ $\frac{-4}{3}$ CA✓ correct subst. of (1;6) And (7; -2) CA✓ equation in any form (3)

12.3 $\hat{B} = \theta = \alpha - \beta$ .... Ext $\angle$	
$\tan\alpha = m_{BC} = -\frac{4}{3}$	CA✓ $\tan\alpha = -\frac{4}{3}$
$\therefore \quad \alpha = 126{,}9^\circ$	CA✓ $\alpha = 126{,}9^\circ$
$\tan_{\beta} = m_{AB} = \frac{5}{6}$	A ✓ $\tan\beta = \frac{5}{6}$
$\therefore \quad \beta = 39{,}8^\circ$	CA✓ $\beta = 39{,}8^\circ$
$\theta = \alpha - \beta$	
$\theta = \alpha - \beta$ $= 126{,}9^\circ - 39{,}8^\circ$ $= 87{,}1^\circ$	
$\therefore A\hat{B}C = 87{,}1^\circ$	CA ✓ $A\hat{B}C = 87{,}1^\circ$
OR	(5)
Distance AB $= \sqrt{(1+5)^2 + (6-1)^2}$ $= \sqrt{61}$	A ✓Distance AB
Distance BC $= \sqrt{(1-7)^2 + (6+2)^2}$ $= \sqrt{100}$ $= 10$	A ✓Distance BC
Distance AC $= \sqrt{(-5-7)^2 + (1+2)^2}$ $= \sqrt{153}$	A ✓Distance AC

$\cos\hat{B} = \frac{a^2+c^2-b^2}{2ac}$ $= \frac{10^2+(\sqrt{61})^2-(\sqrt{153})^2}{2(10)(\sqrt{61})}$ $= 0{,}051$ $\hat{B} = 87{,}1^\circ$	CA ✓substitution in cosine rule CA ✓answer (5)
12.4 $P\left(\frac{-5+1}{2};\frac{1+6}{2}\right)$ $P\left(-2;\ \frac{7}{2}\right)$	AA✓✓both co-ordinates (2)
12.5 $m_{AC} = \frac{-2-1}{7+5}$ $= \frac{-3}{12}$ $= \frac{-1}{4}$ through =(–1; 3) equation: $y-3 = -\frac{1}{4}(x+1)$ $y-3 = -\frac{1}{4}x-\frac{1}{4}$	A ✓ $\frac{-1}{4}$ CA ✓ subst. (-1;3)

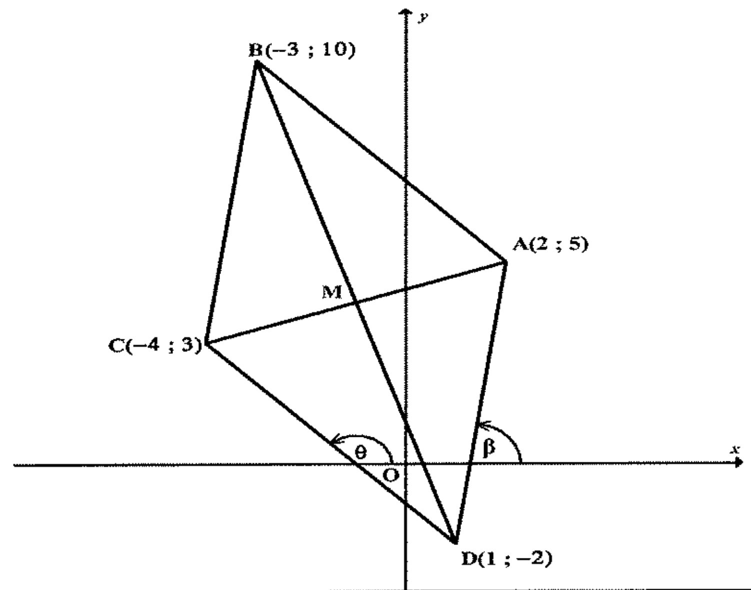
$\therefore \quad y = \frac{-1}{4}x + 2\frac{3}{4}$ or $y = -\frac{1}{4}x + \frac{11}{4}$ or $4y + x - 11 = 0$	CA✓equation.in any form (3)
12.6 $m_{AB} = \frac{5}{6}$; $6x + 5y = 18$ $5y = -6x + 18$ $y = \frac{-6}{5}x + \frac{18}{5}$ $\therefore m_1 = \frac{-6}{5}$ $m_{AB}\ m_1 = -1$ $\therefore \quad m_{AB} \perp 6x + 5y = 18$	A✓ $m_1 = -\frac{6}{5}$ A✓ $m_{AB} \cdot m_1$ A ✓= -1 (3)
	[18]

QUESTION 13	
13.1.1 At W, y = 2 $3x + 4(2) + 7 = 0$ $3x = -15$ $x = -5$ W (–5;2) r = 5 $(x+5)^2 + (y-2)^2 = 25$	A✓ subst y = 2 CA ✓$x = -5$ CA✓ co -ordinates of W CA✓ r = 5 CA✓equation of the circle. (5)
13.1.2 VZ = 2r = 2 x 5 = 10 units	CA✓answer (1)
13.1.3 $m_{GZ} = \frac{2+1}{0+1}$ $= 3$	A✓substitution into formula CA✓answer (2)
13.1.4 Midpoint of GZ is $\left(-\frac{1}{2}; \frac{1}{2}\right)$	A✓ coordinates (1)
13.1.5 $m_{GZ} = 3$ $m_{\perp} = -\frac{1}{3}$ $y - \frac{1}{2} = -\frac{1}{3}\left(x + \frac{1}{2}\right)$ $y = -\frac{1}{3}x + \frac{1}{3}$	CA✓ gradient of perpendicular bisector CA✓substitution into formula CA ✓ answer (3)

13.1.6 W (–5; 2) into $x + 3y - 1 = 0$ LHS $= (2(-5) + 6(2) - 2$ $= -10 + 12 - 2$ $= 0$ $= \text{RHS}$ W is on the line that bisects GZ perpendicularly and W on GZ. $\therefore$ lines intersect at W.	A✓ substitution A✓ $= 0$ (2)
OR $-\frac{1}{3}x + \frac{1}{3} = 2$ $-x + 1 = 6$ $x = -5$ This is the x - value of the coordinate of W.	A✓equating eq. of perpendicular bisector to the horizontal line $y = 2$ A✓ $x = -5$ (2)
OR Equation of WZ: $y - y_1 = m(x - x_1)$ $y + 1 = \frac{2+1}{-5+1}(x+1)$ $y + 1 = -\frac{3}{4}(x+1)$ $y = -\frac{3}{4}x - \frac{7}{4}$ $\therefore -\frac{3}{4}x - \frac{7}{4} = -\frac{1}{3}x + \frac{1}{3}$ $-9x - 21 = -4x + 4$ $-5x = 25$ $x = -5$ $\therefore y = -\frac{1}{3}(-5) + \frac{1}{3} = 2$ This is the coordinate of W.	A✓equation of WZ A✓ $x = -5$ (2)

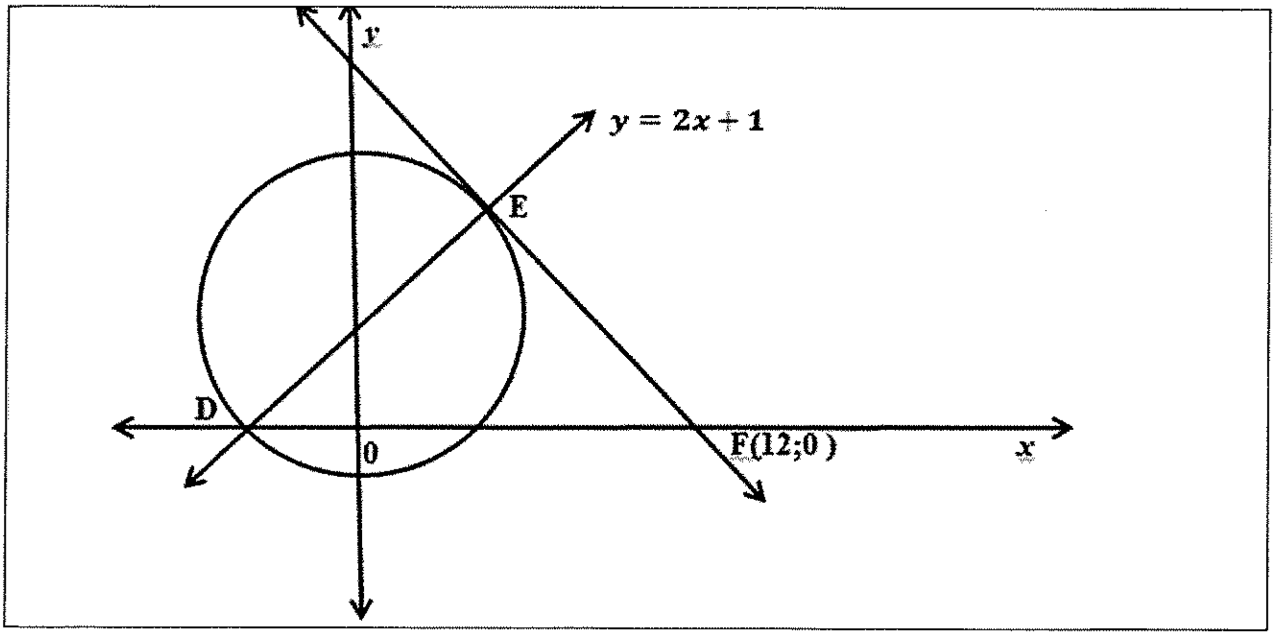
13.2.1 circle M: $M(-2; 1)$; $r_1 = 5$ circle N: $N(1;3)$; $r_2 = 3$ $\therefore \quad r_1 + r_2 = 8$ and $r_1 - r_2 = 2$ $MN = \sqrt{(1-(-2))^2 + (3-1)^2}$ $= \sqrt{3^2 + 2^2}$ $= \sqrt{9+4}$ $= \sqrt{13}$ or 3,6 $\therefore \quad r_1 + r_2 > MN > r_1 - r_2$ $\therefore$ The two circles intersect at two distinct points.	A✓ $r_1 = 5$ A✓ $r_2 = 3$ A✓ $r_1 + r_2 = 8$ A✓ $MN = \sqrt{13}$ $= 3,6$ A✓comparing A✓ conclusion(6)
13.2.2 circle M = circle N $(x+2)^2 + (y-1)^2 - 25 = (x-1)^2 + (y-3)^2 - 9$ $x^2 + 4x + 4 + y^2 - 2y + 1 - 25 = x^2 - 2x + 1 + y^2 - 6y + 9 - 9$ $6x + 4y = 21$ $\therefore$ The equation of the common chord is: $6x + 4y = 21$	M✓ equating A✓ simplifying CA✓ equation of the chord (3) **[23]**

QUESTION 14



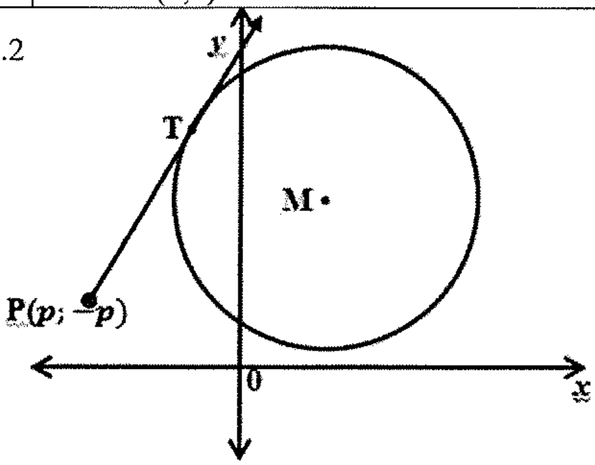
14.1	AC = $\sqrt{(2+4)^2+(5-3)^2}$ = $\sqrt{40}$ =$2\sqrt{10}$	✓subst in distance formula ✓$2\sqrt{10}$ (2)
14.2	M($\frac{2-4}{2}$;$\frac{5+3}{2}$) M(-1;4)	✓x value ✓y value (2)
14.3	Midpoint of BD: ($\frac{-3+1}{2}$;$\frac{10-2}{2}$) = (-1;4) $m_{BD} \times m_{AC}$ = $-3 \times \frac{1}{3}$ = -1 Diagonals bisect at 90°	✓midpoint of BD : ($\frac{-3+1}{2}$;$\frac{10-2}{2}$) = (-1;4) ✓m_{BD} =-3✓m_{AC} =$\frac{1}{3}$ ✓product = -1✓(5)
3.4	BM = $\sqrt{(-3+1)^2+(10-4)^2} = \sqrt{40}$ area$\Delta ABC = \frac{1}{2} \times AC \times BM$ = $\frac{1}{2} \times \sqrt{40} \times \sqrt{40}$ =20 sq units	✓BM = $\sqrt{40}$ ✓areaΔABC $\frac{1}{2} \times AC \times BM$ ✓substitute ✓20 (4)
14.5	$m_{CD} = \frac{3+2}{-4-1} = -1$ y−3 = −1(x+4) $y = -x - 1$	✓m= −1 ✓sub m and point ✓equation (3)
14.6	$\tan\theta = -1$ θ= 180° − 45° θ = 135°	✓$\tan\theta = -1$ ✓✓135° (3)
14.7	Let the inclination angle of AD be β ,$\tan\beta = \frac{5+2}{2-1} = 7$ β= 81,87° $A\hat{D}C$ = 135° − 81,87° = 53,13°	✓$\tan\beta$ = 7 ✓β= 81,87° ✓$A\hat{D}C$ = 53,13° (3) **[22]**

QUESTION 15



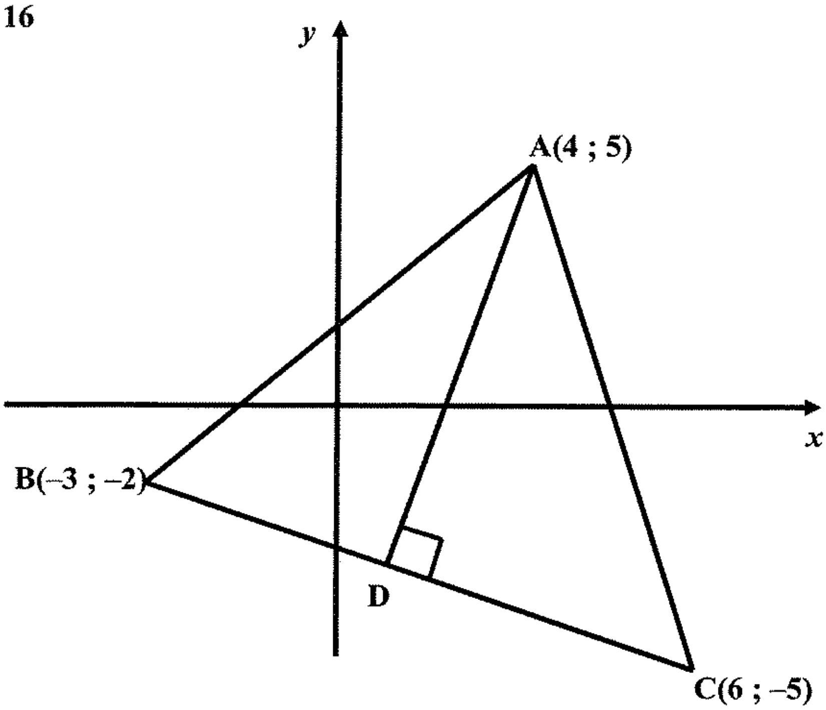
15.1	$mDE = 2$ $mEF = -\frac{1}{2}$ (rad ⊥tan) $y - 0 = -\frac{1}{2}(x - 12)$ $y = -\frac{1}{2}x + 6$ at/by E: $2x + 1 = -\frac{1}{2}x + 6$ $2x + 2 = -x + 12$ $5x = 10$ $x = 2$ $y = 2(2) + 1 = 5$ E(2;5)	✓$mEF = -\frac{1}{2}$ ✓reason ✓$y = -\frac{1}{2}x + 6$ ✓equating ✓x-value ✓y-value (6)

15.2



15.2.1	$x^2 - 2x + y^2 - 4y + 1 = 0$ $x^2 - 2x + 1 + y^2 - 4y + 4 = 4$ $(x-1)^2 + (y-2)^2 = 4$ M (1;2) and r = 2	✓$x^2 - 2x + 1 + y^2 - 4y + 4 = 4$ ✓LHS RHS✓ $(x-1)^2 + (y-2)^2 = 4$ (3)
15.2.2	$PT^2 = PM^2 - TM^2$ $= (p-1)^2 + (-p-2)^2 - 4$ $= p^2 - 2p + 1 + p^2 + 4p + 4 - 4$ $= 2p^2 + 2p + 1$ $= \sqrt{2p^2 + 2p + 1}$	✓$PT^2 = PM^2 - TM^2$ ✓substitute ✓$2p^2 + 2p + 1$ (3)
15.2.3	Dp$(2p^2 + 2p + 1) = 0$ $4p + 2 = 0$ $p = -\frac{1}{2}$ $P(-\frac{1}{2}; \frac{1}{2})$ PT = $\sqrt{2(-\frac{1}{2})^2 + 2(-\frac{1}{2}) + 1}$ $= \sqrt{\frac{1}{2}}$ or $\frac{\sqrt{2}}{2}$ or 0,707	✓$4p + 2 = 0$ ✓$p = -\frac{1}{2}$ ✓$(-\frac{1}{2}; \frac{1}{2})$ ✓substitute into PT ✓answer (5)
		[17]

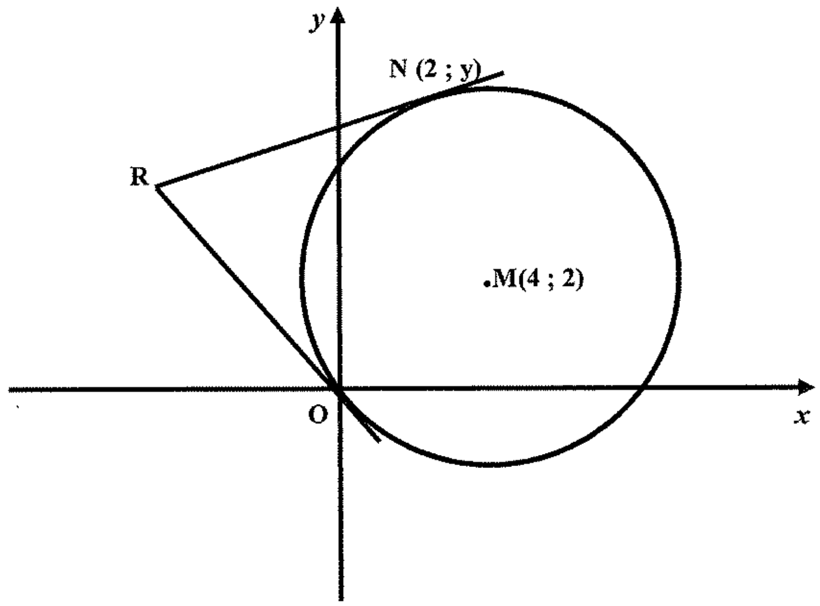
QUESTION 16



16.1	$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{(6+3)^2+(-5+2)^2}$ $= \sqrt{90}$ or $3\sqrt{10}$ or 9,49	✓sub. into the distance formula ✓ answer (2)
16.2	$m_{BC} = \dfrac{y_2-y_1}{x_2-x_1}$ $= \dfrac{-5+2}{6+3}$ $= -\dfrac{1}{3}$ $y+5 = -\dfrac{1}{3}(x-6)$ or $y+2 = -\dfrac{1}{3}(x+3)$ $y = -\dfrac{1}{3}x-3$ or $3y = -x-9$	✓ sub. into gradient formula ✓ $-\dfrac{1}{3}$ ✓ equation (3)
16.3	$m_{AD} = 3$ $y-5 = 3(x-4)$ $y = 3x-7$	✓ $m_{AD} = 3$ ✓sub. of the point ✓equation (3)

16.4	$-\frac{1}{3}x-3 = 3x-7$ $-x-9 = 9x-21$ $-10x = -12$ $x = \frac{6}{5}$ $y = 3(\frac{6}{5})-7$ $= -\frac{17}{5}$ $\therefore D(\frac{6}{5};-\frac{17}{5})$	✓ equating the two equations ✓ x- value ✓y-value (3)
16.5	$m_{AB} = \frac{5+2}{4+3}$ $= 1$ $\tan\alpha = 1$ $\alpha = 45°$ $\tan\beta = 3$ $\beta = 71,57°$ $B\hat{A}D = 71,57° - 45°$ $= 26,57°$	✓ $m_{AB} = 1$ ✓ $\tan\alpha = 1$ ✓ $\alpha = 45°$ ✓ $\beta = 71,57°$ ✓answer (5)
16.6	Equation of a line AE ‖ BC $y-5 = -\frac{1}{3}(x-4)$ $3y-15 = -x+4$ AE: $3y+x = 19$ x-intercept is $3(0)+x = 19$ $x = 19$ E (19: 0	✓ sub. of $-\frac{1}{3}$ and point into the equation ✓equation of AE ✓ x –intercepts ✓answer (4) **[20]**

QUESTION 17



17.1	$(x-4)^2+(y-2)^2=r^2$ $(0-4)^2+(0-2)^2=r^2$ $20=r^2$ $(x-4)^2+(y-2)^2=20$	✓sub. of M into equation of a circle ✓sub. of O (0; 0) ✓ value of r^2 (3)
17.2	$(x-4)^2+(y-2)^2=20$ Subst $(2; y)$ $(2-4)^2+(y-2)^2=20$ $4+y^2-4y+4=20$ $y^2-4y-12=0$ $(y-6)(y+2)=0$ $y=6$ or $y=-2$ N/A **OR** $(x-4)^2+(y-2)^2=20$ Subst $(2; y)$ $(2-4)^2+(y-2)^2=20$ $(y-2)^2=16$ $y-2=\pm 4$ $y=6$ or $y=-2$ N/A	✓sub of N $(2; y)$ ✓ $y^2-4y-12=0$ ✓$(y-6)(y+2)=0$ ✓ $y=6$ (4) **OR** ✓sub of N $(2; y)$ ✓ $(y-2)^2=16$ ✓ $y-2=\pm 4$ ✓ $y=6$ (4)

17.3	$m_{OM} = \frac{2}{4} = \frac{1}{2}$ $m_{OR} = -2$ Equation of OR is: $y = -2x$	✓ $m_{OM} = \frac{1}{2}$ ✓ $m_{OR} = -2$ ✓ $y = -2x$ (3)
17.4	$m_{MN} = \frac{6-2}{2-4} = -2$ $m_{NR} = \frac{1}{2}$ $y - 6 = \frac{1}{2}(x-2)$ $2y - 12 = x - 2$ NR: $2y - x - 10 = 0$ **OR:** $y = -2x$ $2(-2x) - x - 10 = 0$ $-5x = 10$ $x = -2$ $y = -2(-2) = 4$ R $(-2; 4)$ **OR** $RO^2 = NR^2$ $x^2 + y^2 = (x-2)^2 + (y-6)^2$ $x^2 + y^2 = x^2 - 4x + 4 + y^2 - 12y + 36$ $4x + 12y = 40$ $x + 3y = 10$ and $y = -2x$ $x + 3(-2x) = 10$ $x - 6x = 10$ $-5x = 10$ $x = -2$ $y = -2(-2) = 4$ R $(-2; 4)$	✓ $m_{MN} = -2$ ✓ $m_{NR} = \frac{1}{2}$ ✓ $y - 6 = \frac{1}{2}(x-2)$ ✓ $2(-2x) - x - 10 = 0$ ✓ $x = -2$ ✓ $y = 4$ (6) ✓ $x^2 + y^2 = (x-2)^2 + (y-6)^2$ ✓ $4x + 12y = 40$ ✓ $x + 3y = 10$ ✓ $x + 3(-2x) = 10$ ✓ $x = -2$ ✓ $y = 4$ (6)

17.5	MNRO is a kite because OR = RN and MN = OM	✓Kite ✓ adjacent sides equal (2) **[18]**

QUESTION 18

Q18	SUGGESTED ANSWER	DESCRIPTORS	MA K
18.1	$AB = \sqrt{(4+2)^2 + (9-1)^2}$ $= \sqrt{6^2 + 8^2}$ $= \sqrt{100}$ $= 10$	✓ Subst in correct formula ✓ Ans	(2
18.2	$AB = BC = 10$ $y\ value = 9 + 10$ BC ∥ $y - axis$ $C(4; 19)$	✓x value ✓y – value	(2
18.3	$K\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ $K\left(\frac{4-2}{2}; \frac{19+1}{2}\right)$ $K(1; 10)$	✓Subst in correct formula ✓Ans	(2
18.4	$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{19-1}{4+2} = \frac{18}{6} = 3$ $y - y_1 = m(x - x_1)$ $y - 1 = 3(x + 2)$ $y = 3x + 6 + 1$ $y = 3x + 7$	✓m ✓Subst A or C point into correct formula ✓Ans	(3
18.5	$m_{AC} = 3$ $Angle\ of\ inclination = \tan^{-1}(3)$ $= 71{,}57°$ $\therefore \theta = 90° - 71{,}57°$ $= 18{,}43°$ OR In the right-angled triangle: $\tan\theta = \frac{6}{18}$ $\theta = 18.43°$	✓ tan ∠ = m ✓∠ of inclination ✓ Comple ∠'s ✓Answer OR ✓correct trig ratio ✓✓ correct values ✓Ans	(4
18.6	Area ΔABC = $\frac{1}{2}$. BC. ⊥h $= \frac{1}{2}(10)(6)$ = 30 sq units	✓correct formula ✓base = 10 ✓ height = 6 ✓ ans	(4
			[1

QUESTION 19

Q19	SUGGESTED ANSWER	DESCRIPTORS	MA
19.1	OC ⊥ tangent [radius ⊥ tangent] CE ⊥ tangent [r ⊥ tangent] ∴ O, C and E straight line [adjacent ∠s = 180°]	✓ both S ✓ R	
19.2	$m_{OC} = \frac{-2-0}{1-0} = -2$	✓ Substitute ✓Ans	
19.3	$m_{CD} = -2$ $\frac{y_2 - y_1}{x_2 - x_1} = -2$ $\frac{-6+2}{t-1} = -2$ $-2(t-1) = -4$ $t - 1 = 2$ $t = 3$	✓ m_{OCD} *from* 4.1 ✓ Substitute (any point) correct equation	
19.4	$m_{tangent} = \frac{1}{2}$ $AC:\ y - y_1 = m(x - x_1)$ $y + 2 = \frac{1}{2}(x-1)$ $y = \frac{1}{2}x - \frac{1}{2} - 2$ $y = \frac{1}{2}x - \frac{5}{2}$	✓ radius ⊥ tangent ✓ Substitute (any point) correct equation ✓ Answer	
19.5	D is the midpoint of circle $x - coordinate: \frac{1+x}{2} = 3$ $1 + x = 6$ $x = 5$ $y - coordinate: \frac{y-2}{2} - -6$ $y - 2 = -12$ $y = -10$ $E(5;\ -10)$	✓ x value ✓ y value	
19.6	$A\hat{C}D = 90°$ [*radius* ⊥ *tangent*] ∴ *AE is the diameter* ⊙ ACE $AE = \sqrt{(5-5)^2 + (0+10)^2}$ $AE = \sqrt{0 + 100}$ $AE = 10$ $\therefore radius = \frac{1}{2}(10) = 5$ $Midpnt_{AE}\ \left(\frac{5+5}{2}; \frac{-10}{2}\right)$ ∴ *Centre of circle* (5; -5) Equation of circle ACE: $(x-5)^2 + (y+5)^2 = 25$	✓AE diameter ✓AE = 10 ✓Radius = 5 ✓Midpnt Equation of Circle ✓$(x-5)^2 + (y+5)^2$ ✓$r^2 = 25$	(6

19.7	Circle centre $O: x^2 + y^2 = 5$ $Diameter = 2\sqrt{5}$ $\sqrt{20} < r < \sqrt{80}$ $2\sqrt{5} < r < 4\sqrt{5}$	✓ $r^2 = 5$ ✓Diameter ✓✓Endpoints	
			[