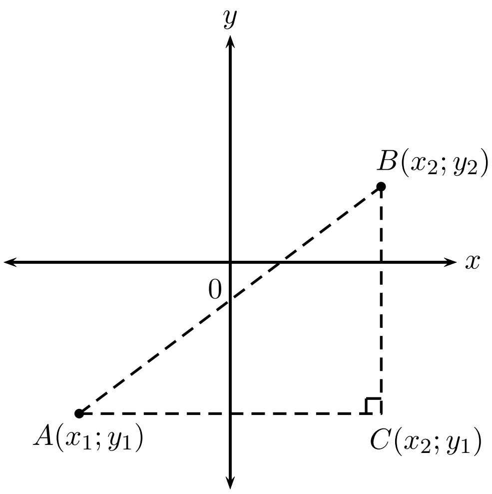
CHAPTER 4

Analytical geometry

Analytical geometry, also referred to as coordinate or Cartesian geometry, is the study of geometric properties and relationships between points, lines and angles in the Cartesian plane. Geometrical shapes are defined using a coordinate system and algebraic principles. In this chapter we deal with the equation of a straight line, parallel and perpendicular lines and inclination of a line.

4.1 Revision EMBG7

Points $A(x_1; y_1)$, $B(x_2; y_2)$ and $C(x_2; y_1)$ are shown in the diagram below:



Theorem of Pythagoras

$$AB^2 = AC^2 + BC^2$$

Distance formula

Distance between two points:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Notice that $(x_1 - x_2)^2 = (x_2 - x_1)^2$.

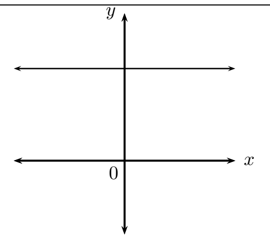
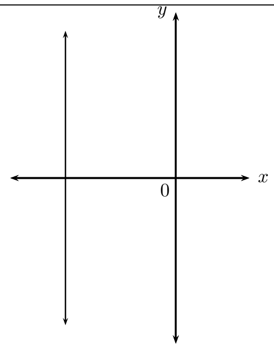
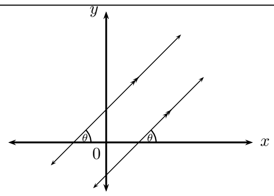
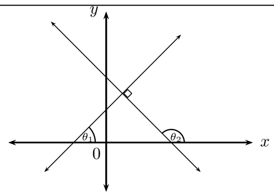
See video: 22JD at www.everythingmaths.co.za

Gradient

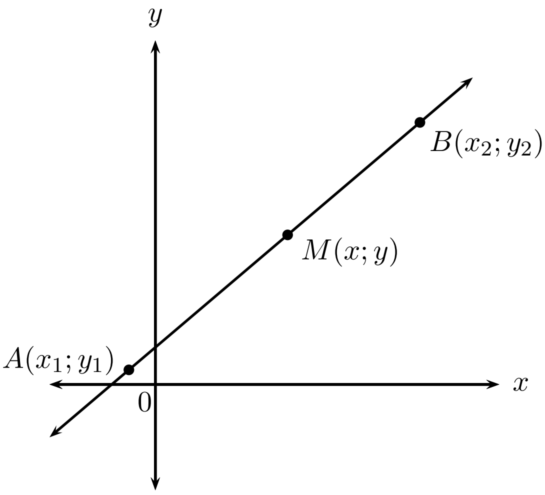
Gradient (m) describes the slope or steepness of the line joining two points. The gradient of a line is determined by the ratio of vertical change to horizontal change.

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad m_{AB} = \frac{y_1 - y_2}{x_1 - x_2}$$

Remember to be consistent: $m \neq \dfrac{y_1 - y_2}{x_2 - x_1}$.

Horizontal lines		$m = 0$
Vertical lines		m is undefined
Parallel lines		$m_1 = m_2$
Perpendicular lines		$m_1 \times m_2 = -1$

Mid-point of a line segment



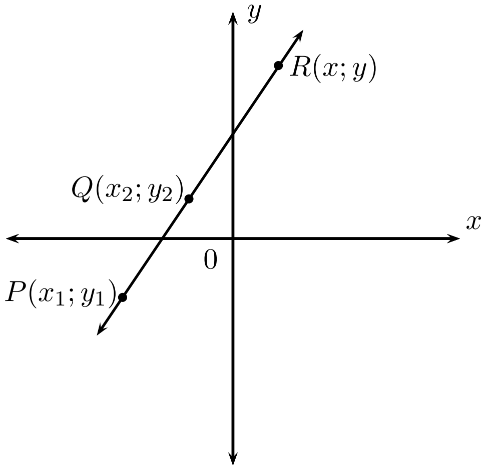
The coordinates of the mid-point $M(x;y)$ of a line between any two points $A(x_1;y_1)$ and $B(x_2;y_2)$:

$$M(x;y) = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

See video: 22JF at www.everythingmaths.co.za

Points on a straight line

The diagram shows points $P(x_1; y_1)$, $Q(x_2; y_2)$ and $R(x; y)$ on a straight line.



We know that $m_{PR} = m_{QR} = m_{PQ}$.

Using $m_{PR} = m_{PQ}$, we obtain the following for any point $(x; y)$ on a straight line

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Worked example 1: Revision

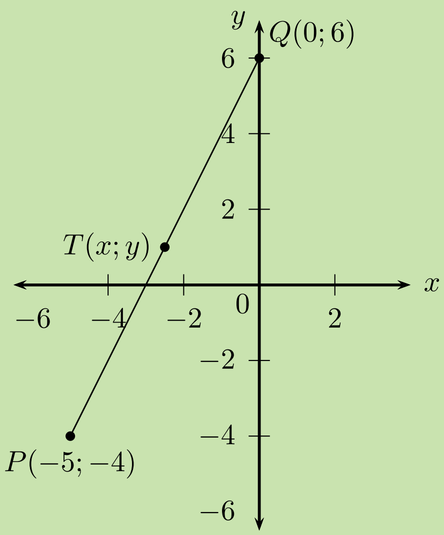
QUESTION

Given the points $P(-5; -4)$ and $Q(0; 6)$:

1. Determine the length of the line segment PQ.
2. Determine the mid-point $T(x; y)$ of the line segment PQ.
3. Show that the line passing through $R(1; -\frac{3}{4})$ and $T(x; y)$ is perpendicular to the line PQ.

SOLUTION

Step 1: Draw a sketch



Step 2: Assign variables to the coordinates of the given points

Let the coordinates of P be $(x_1; y_1)$ and Q $(x_2; y_2)$

$$x_1 = -5; \qquad y_1 = -4; \qquad x_2 = 0; \qquad y_2 = 6$$

Write down the distance formula

$$\begin{aligned} PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0 - (-5))^2 + (6 - (-4))^2} \\ &= \sqrt{25 + 100} \\ &= \sqrt{125} \\ &= 5\sqrt{5} \end{aligned}$$

The length of the line segment PQ is $5\sqrt{5}$ units.

Step 3: Write down the mid-point formula and substitute the values

$$T(x; y) = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$\begin{aligned} x &= \frac{x_1 + x_2}{2} \\ &= \frac{-5 + 0}{2} \\ &= -\frac{5}{2} \\ y &= \frac{y_1 + y_2}{2} \\ &= \frac{-4 + 6}{2} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

The mid-point of PQ is $T(-\frac{5}{2}; 1)$.

Step 4: Determine the gradients of PQ and RT

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\begin{aligned} m_{PQ} &= \frac{6 - (-4)}{0 - (-5)} \\ &= \frac{10}{5} \\ &= 2 \end{aligned}$$

$$
\begin{aligned}
m_{RT} &= \frac{-\frac{3}{4} - 1}{1 - (-\frac{5}{2})} \\
&= \frac{-\frac{7}{4}}{\frac{7}{2}} \\
&= -\frac{7}{4} \times \frac{2}{7} \\
&= -\frac{1}{2}
\end{aligned}
$$

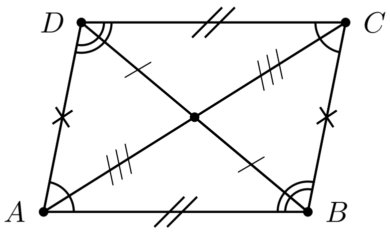
Calculate the product of the two gradients:

$$
\begin{aligned}
m_{RT} \times m_{PQ} &= -\frac{1}{2} \times 2 \\
&= -1
\end{aligned}
$$

Therefore PQ is perpendicular to RT.

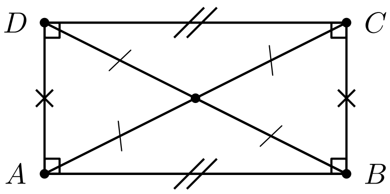
Quadrilaterals

- A quadrilateral is a closed shape consisting of four straight line segments.
- A parallelogram is a quadrilateral with both pairs of opposite sides parallel.



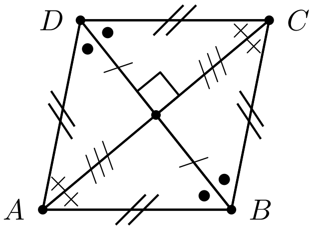
  - Both pairs of opposite sides are equal in length.
  - Both pairs of opposite angles are equal.
  - The diagonals bisect each other.

- A rectangle is a parallelogram that has all four angles equal to 90°.



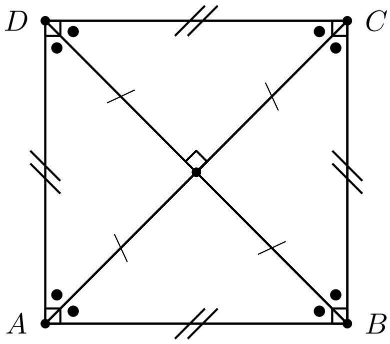
  - Both pairs of opposite sides are equal and parallel.
  - The diagonals bisect each other.
  - The diagonals are equal in length.

- A rhombus is a parallelogram that has all four sides equal in length.



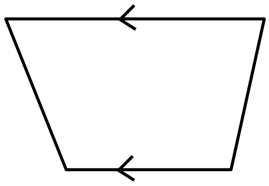
  - Both pairs of opposite sides are equal and parallel.
  - The diagonals bisect each other at 90°.
  - The diagonals of a rhombus bisect both pairs of opposite angles.

- A square is a rhombus that has all four interior angles equal to 90°.

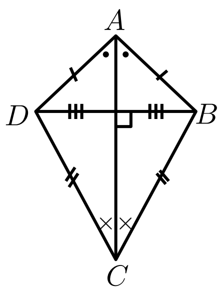


  - Both pairs of opposite sides are equal and parallel.
  - The diagonals bisect each other at 90°.
  - The diagonals are equal in length.
  - The diagonals bisect both pairs of interior opposite angles (that is, all angles are 45°).

- A trapezium is a quadrilateral with one pair of opposite sides parallel.



- A kite is a quadrilateral with two pairs of adjacent sides equal.



  - One pair of opposite angles are equal (the angles are between unequal sides).
  - The diagonal between equal sides bisects the other diagonal.
  - The diagonal between equal sides bisects the interior angles.
  - The diagonals intersect at 90°.

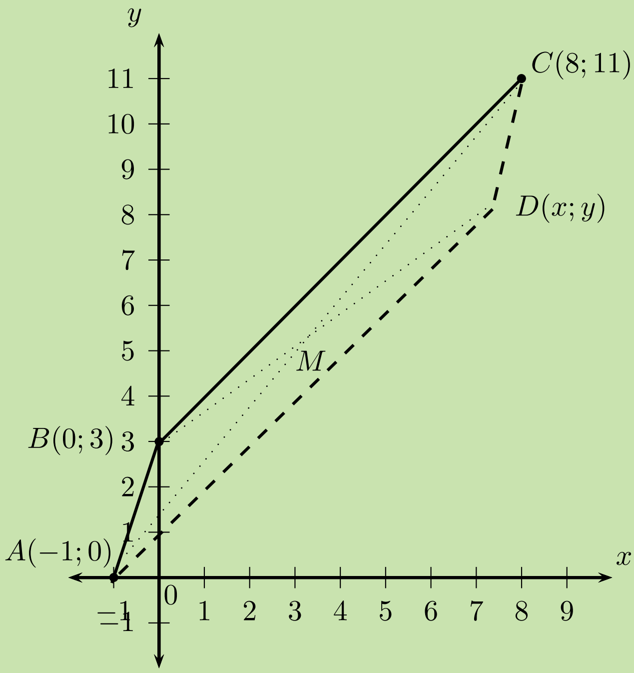
Worked example 2: Quadrilaterals

QUESTION

Points $A\,(-1;0)$, $B\,(0;3)$, $C\,(8;11)$ and $D\,(x;y)$ are points on the Cartesian plane. Determine $D\,(x;y)$ if $ABCD$ is a parallelogram.

SOLUTION

Step 1: Draw a sketch



The mid-point of AC will be the same as the mid-point of BD. We first find the mid-point of AC and then use it to determine the coordinates of point D.

Step 2: Assign values to $(x_1;y_1)$ and $(x_2;y_2)$

Let the mid-point of AC be $M(x;y)$

$$x_1 = -1; \qquad y_1 = 0; \qquad x_2 = 8; \qquad y_2 = 11$$

Step 3: Write down the mid-point formula

$$M(x;y) = \left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$$

Step 4: Substitute the values and calculate the coordinates of M

$$\begin{aligned} M(x;y) &= \left(\frac{-1+8}{2};\frac{0+11}{2}\right) \\ &= \left(\frac{7}{2};\frac{11}{2}\right) \end{aligned}$$

Step 5: Use the coordinates of M to determine D

M is also the mid-point of BD so we use $M\left(\frac{7}{2};\frac{11}{2}\right)$ and $B(0;3)$ to find $D(x;y)$

Step 6: Substitute values and determine x and y

$$M = \left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$$
$$\therefore \left(\frac{7}{2};\frac{11}{2}\right) = \left(\frac{0+x}{2};\frac{3+y}{2}\right)$$

$$\frac{7}{2} = \frac{0+x}{2}$$
$$7 = 0+x$$
$$\therefore x = 7$$

$$\frac{11}{2} = \frac{3+y}{2}$$
$$11 = 3+y$$
$$\therefore y = 8$$

Step 7: Alternative method: inspection

Since we are given that $ABCD$ is a parallelogram, we can use the properties of a parallelogram and the given points to determine the coordinates of D.

From the sketch we expect that point D will lie below C.

Consider the given points A, B and C:

- Opposite sides of a parallelogram are parallel, therefore BC must be parallel to AD and their gradients must be equal.
- The vertical change from B to C is 8 units up.
- Therefore the vertical change from A to D is also 8 units up ($y = 0+8 = 8$).
- The horizontal change from B to C is 8 units to the right.
- Therefore the horizontal change from A to D is also 8 units to the right ($x = -1+8 = 7$).

or

- Opposite sides of a parallelogram are parallel, therefore AB must be parallel to DC and their gradients must be equal.
- The vertical change from A to B is 3 units up.

- Therefore the vertical change from C to D is 3 units down ($y = 11 - 3 = 8$).
- The horizontal change from A to B is 1 unit to the right.
- Therefore the horizontal change from C to D is 1 unit to the left ($x = 8 - 1 = 7$).

Step 8: Write the final answer

The coordinates of D are $(7;8)$.

Exercise 4 – 1: Revision

1. Determine the length of the line segment between the following points:
   a) $P(-3;5)$ and $Q(-1;-5)$
   b) $R(0{,}75;3)$ and $S(0{,}75;-4)$
   c) $T(2x;y-2)$ and $U(3x+1;y-2)$
2. Given $Q(4;1), T(p;3)$ and length $QT = \sqrt{8}$ units, determine the value of p.
3. Determine the gradient of the line AB if:
   a) $A(-5;3)$ and $B(-7;4)$
   b) $A(3;-2)$ and $B(1;-8)$
4. Prove that the line PQ, with $P(0;3)$ and $Q(5;5)$, is parallel to the line $5y+5 = 2x$.
5. Given the points $A(-1;-1)$, $B(2;5)$, $C(-1;-\frac{5}{2})$ and $D(x;-4)$ and $AB \perp CD$, determine the value of x.
6. Calculate the coordinates of the mid-point $P(x;y)$ of the line segment between the points:
   a) $M(3;5)$ and $N(-1;-1)$
   b) $A(-3;-4)$ and $B(2;3)$
7. The line joining $A(-2;4)$ and $B(x;y)$ has the mid-point $C(1;3)$. Determine the values of x and y.
8. Given quadrilateral $ABCD$ with vertices $A(0;3), B(4;3), C(5;-1)$ and $D(1;-1)$.
   a) Determine the equation of the line AD and the line BC.
   b) Show that $AD \parallel BC$.
   c) Calculate the lengths of AD and BC.
   d) Determine the equation of the diagonal BD.
   e) What type of quadrilateral is $ABCD$?

9. $MPQN$ is a parallelogram with points $M(-5;3)$, $P(-1;5)$ and $Q(4;5)$. Draw a sketch and determine the coordinates of $N(x;y)$.

10. $PQRS$ is a quadrilateral with points $P(-3;1)$, $Q(1;3)$, $R(6;1)$ and $S(2;-1)$ in the Cartesian plane.

    a) Determine the lengths of PQ and SR.
    b) Determine the mid-point of PR.
    c) Show that $PQ \parallel SR$.
    d) Determine the equations of the line PS and the line SR.
    e) Is $PS \perp SR$? Explain your answer.
    f) What type of quadrilateral is $PQRS$?

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 22JG 1b. 22JH 1c. 22JJ 2. 22JK 3a. 22JM 3b. 22JN
4. 22JP 5. 22JQ 6a. 22JR 6b. 22JS 7. 22JT 8. 22JV
9. 22JW 10. 22JX

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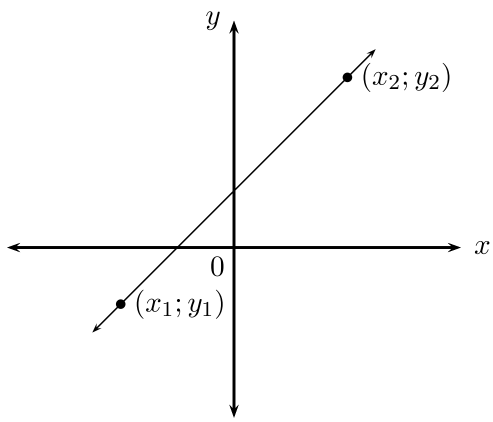
4.2 Equation of a line EMBG8

We can derive different forms of the straight line equation. The different forms are used depending on the information provided in the problem:

- The two-point form of the straight line equation: $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$
- The gradient–point form of the straight line equation: $y - y_1 = m(x - x_1)$
- The gradient–intercept form of the straight line equation: $y = mx + c$

The two-point form of the straight line equation EMBG9



Given any two points $(x_1; y_1)$ and $(x_2; y_2)$, we can determine the equation of the line passing through the two points using the equation:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

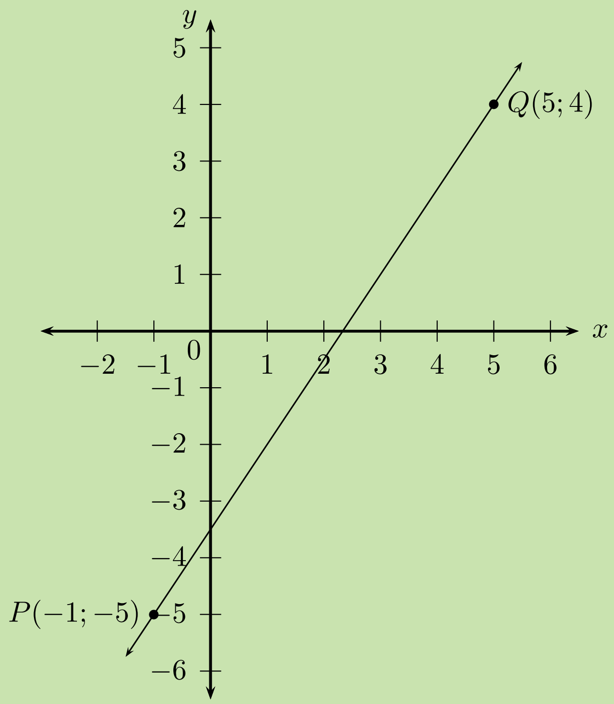
Worked example 3: The two-point form of the straight line equation

QUESTION

Find the equation of the straight line passing through $P(-1; -5)$ and $Q(5; 4)$.

SOLUTION

Step 1: Draw a sketch



Step 2: Assign variables to the coordinates of the given points

Let the coordinates of P be $(x_1; y_1)$ and $Q(x_2; y_2)$

$$x_1 = -1; \qquad y_1 = -5; \qquad x_2 = 5; \qquad y_2 = 4$$

Step 3: Write down the two-point form of the straight line equation

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 4: Substitute the values and make y the subject of the equation

$$\begin{aligned}\frac{y-(-5)}{x-(-1)} &= \frac{4-(-5)}{5-(-1)} \\ \frac{y+5}{x+1} &= \frac{9}{6} \\ y+5 &= \frac{3}{2}(x+1) \\ y+5 &= \frac{3}{2}x+\frac{3}{2} \\ y &= \frac{3}{2}x-\frac{7}{2}\end{aligned}$$

Step 5: Write the final answer

$$y = \frac{3}{2}x - 3\frac{1}{2}$$

Exercise 4 – 2: The two-point form of the straight line equation

Determine the equation of the straight line passing through the points:

1. $(3;7)$ and $(-6;1)$
2. $(1;-\frac{11}{4})$ and $(\frac{2}{3};-\frac{7}{4})$
3. $(-2;1)$ and $(3;6)$
4. $(2;3)$ and $(3;5)$
5. $(1;-5)$ and $(-7;-5)$
6. $(-4;0)$ and $(1;\frac{15}{4})$
7. $(s;t)$ and $(t;s)$
8. $(-2;-8)$ and $(1;7)$
9. $(2p;q)$ and $(0;-q)$

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1. 22JY 2. 22JZ 3. 22K2 4. 22K3 5. 22K4 6. 22K5
7. 22K6 8. 22K7 9. 22K8

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The gradient–point form of the straight line equation EMBGB

We derive the gradient–point form of the straight line equation using the definition of gradient and the two-point form of a straight line equation

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ on the right-hand side of the equation

$$\frac{y - y_1}{x - x_1} = m$$

Multiply both sides of the equation by $(x - x_1)$

$$y - y_1 = m(x - x_1)$$

To use this equation, we need to know the gradient of the line and the coordinates of one point on the line.

See video: 22K9 at www.everythingmaths.co.za

Worked example 4: The gradient–point form of the straight line equation

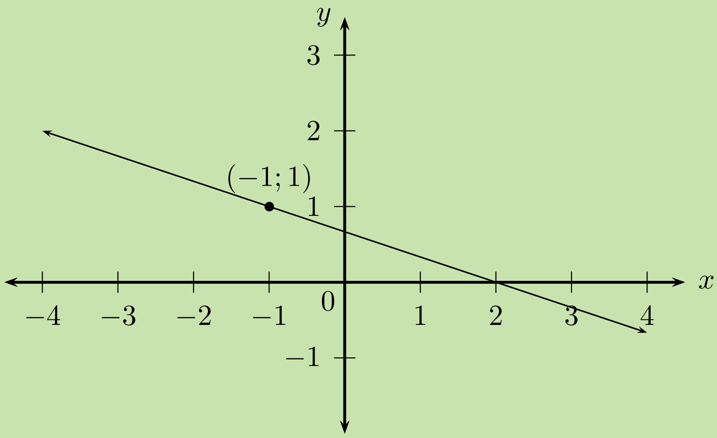
QUESTION

Determine the equation of the straight line with gradient $m = -\frac{1}{3}$ and passing through the point $(-1; 1)$.

SOLUTION

Step 1: Draw a sketch

We notice that $m < 0$, therefore the graph decreases as x increases.



Step 2: Write down the gradient–point form of the straight line equation

$$y - y_1 = m(x - x_1)$$

Substitute the value of the gradient

$$y - y_1 = -\frac{1}{3}(x - x_1)$$

Substitute the coordinates of the given point

$$\begin{aligned} y - 1 &= -\frac{1}{3}(x - (-1)) \\ y - 1 &= -\frac{1}{3}(x + 1) \\ y &= -\frac{1}{3}x - \frac{1}{3} + 1 \\ &= -\frac{1}{3}x + \frac{2}{3} \end{aligned}$$

Step 3: Write the final answer

The equation of the straight line is $y = -\frac{1}{3}x + \frac{2}{3}$.

If we are given two points on a straight line, we can also use the gradient–point form to determine the equation of a straight line. We first calculate the gradient using the two given points and then substitute either of the two points into the gradient–point form of the equation.

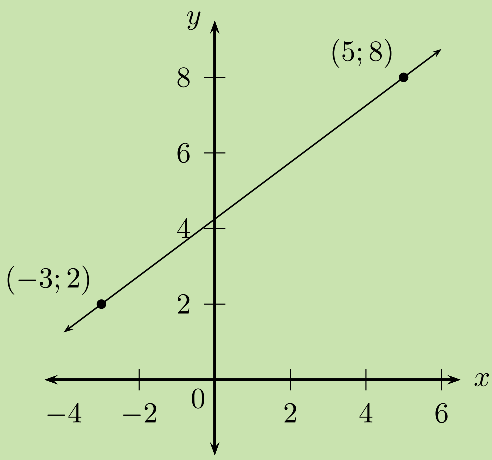
Worked example 5: The gradient–point form of the straight line equation

QUESTION

Determine the equation of the straight line passing through $(-3; 2)$ and $(5; 8)$.

SOLUTION

Step 1: Draw a sketch



Step 2: Assign variables to the coordinates of the given points

$$x_1 = -3; \qquad y_1 = 2; \qquad x_2 = 5; \qquad y_2 = 8$$

Step 3: Calculate the gradient using the two given points

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{8 - 2}{5 - (-3)} \\ &= \frac{6}{8} \\ &= \frac{3}{4} \end{aligned}$$

Step 4: Write down the gradient–point form of the straight line equation

$$y - y_1 = m(x - x_1)$$

Substitute the value of the gradient

$$y - y_1 = \frac{3}{4}(x - x_1)$$

Substitute the coordinates of a given point

$$\begin{aligned} y - y_1 &= \frac{3}{4}(x - x_1) \\ y - 2 &= \frac{3}{4}(x - (-3)) \\ y - 2 &= \frac{3}{4}(x + 3) \\ y &= \frac{3}{4}x + \frac{9}{4} + 2 \\ &= \frac{3}{4}x + \frac{17}{4} \end{aligned}$$

Step 5: Write the final answer

The equation of the straight line is $y = \frac{3}{4}x + 4\frac{1}{4}$.

See video: 22KB at www.everythingmaths.co.za

Exercise 4 – 3: Gradient–point form of a straight line equation

Determine the equation of the straight line:

1. passing through the point $(-1; \frac{10}{3})$ and with $m = \frac{2}{3}$.
2. with $m = -1$ and passing through the point $(-2; 0)$.
3. passing through the point $(3; -1)$ and with $m = -\frac{1}{3}$.
4. parallel to the x-axis and passing through the point $(0; 11)$.
5. passing through the point $(1; 5)$ and with $m = -2$.
6. perpendicular to the x-axis and passing through the point $(-\frac{3}{2}; 0)$.
7. with $m = -0{,}8$ and passing through the point $(10; -7)$.
8. with undefined gradient and passing through the point $(4; 0)$.
9. with $m = 3a$ and passing through the point $(-2; -6a + b)$.

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1\. 22KC 2. 22KD 3. 22KF 4. 22KG 5. 22KH 6. 22KJ
7\. 22KK 8. 22KM 9. 22KN

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The gradient–intercept form of a straight line equation EMBGC

Using the gradient–point form, we can also derive the gradient–intercept form of the straight line equation.

Starting with the equation

$$y - y_1 = m(x - x_1)$$

Expand the brackets and make y the subject of the formula

$$\begin{aligned} y - y_1 &= mx - mx_1 \\ y &= mx - mx_1 + y_1 \\ y &= mx + (y_1 - mx_1) \end{aligned}$$

We define constant c such that $c = y_1 - mx_1$ so that we get the equation

$$y = mx + c$$

This is also called the **standard form** of the straight line equation.

Notice that when $x = 0$, we have

$$\begin{aligned} y &= m(0) + c \\ &= c \end{aligned}$$

Therefore c is the y-intercept of the straight line.

Worked example 6: The gradient–intercept form of straight line equation

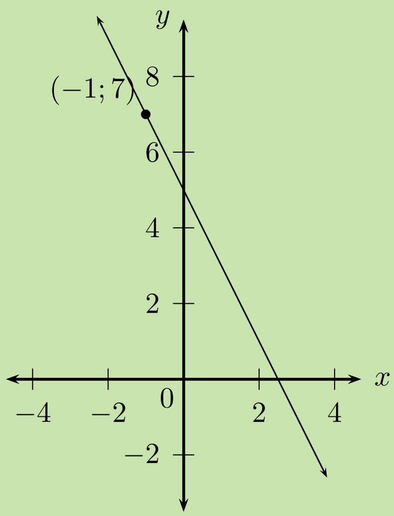
QUESTION

Determine the equation of the straight line with gradient $m = -2$ and passing through the point $(-1; 7)$.

SOLUTION

Step 1: Slope of the line

We notice that $m < 0$, therefore the graph decreases as x increases.



Step 2: Write down the gradient–intercept form of straight line equation

$$y = mx + c$$

Substitute the value of the gradient

$$y = -2x + c$$

Substitute the coordinates of the given point and find c

$$\begin{aligned} y &= -2x + c \\ 7 &= -2(-1) + c \\ 7 - 2 &= c \\ \therefore c &= 5 \end{aligned}$$

This gives the y-intercept $(0;5)$.

Step 3: Write the final answer

The equation of the straight line is $y = -2x + 5$.

If we are given two points on a straight line, we can also use the gradient–intercept form to determine the equation of a straight line. We solve for the two unknowns m and c using simultaneous equations — using the methods of substitution or elimination.

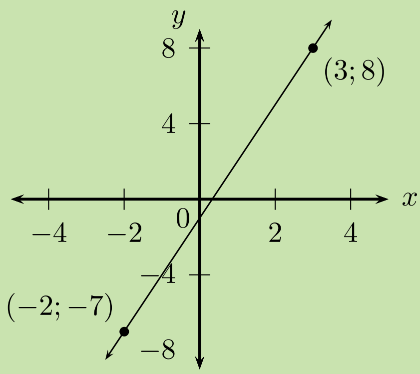
Worked example 7: The gradient–intercept form of straight line equation

QUESTION

Determine the equation of the straight line passing through the points $(-2;-7)$ and $(3;8)$.

SOLUTION

Step 1: Draw a sketch



Step 2: Write down the gradient–intercept form of straight line equation

$$y = mx + c$$

Step 3: Substitute the coordinates of the given points

$$
\begin{aligned}
-7 &= m(-2) + c \\
-7 &= -2m + c \qquad \ldots (1)
\end{aligned}
$$

$$
\begin{aligned}
8 &= m(3) + c \\
8 &= 3m + c \qquad \ldots (2)
\end{aligned}
$$

We have two equations with two unknowns; we can therefore solve using simultaneous equations.

Step 4: Make the coefficient of one of the variables the same in both equations

We notice that the coefficient of c in both equations is 1, therefore we can subtract one equation from the other to eliminate c:

$$
\begin{aligned}
-7 &= -2m + c \\
-(8 &= 3m + c) \\
-15 &= -5m \\
\therefore 3 &= m
\end{aligned}
$$

Substitute $m = 3$ into either of the two equations and determine c:

$$
\begin{aligned}
-7 &= -2m + c \\
-7 &= -2(3) + c \\
\therefore c &= -1
\end{aligned}
$$

or

$$
\begin{aligned}
8 &= 3m + c \\
8 &= 3(3) + c \\
\therefore c &= -1
\end{aligned}
$$

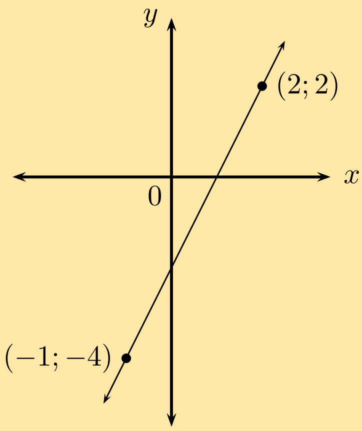
Step 5: Write the final answer

The equation of the straight line is $y = 3x - 1$.

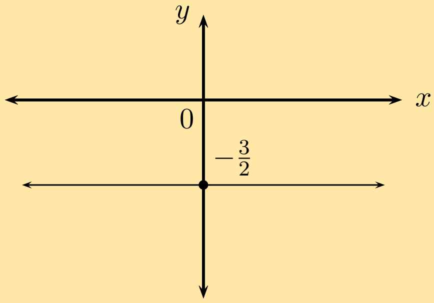
Exercise 4 – 4: The gradient–intercept form of a straight line equation

Determine the equation of the straight line:

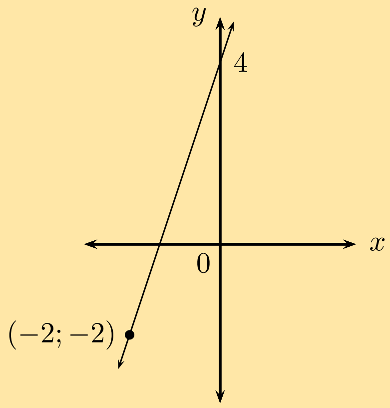
1. passing through the point $(\frac{1}{2};4)$ and with $m = 2$.
2. passing through the points $(\frac{1}{2};-2)$ and $(2;4)$.
3. passing through the points $(2;-3)$ and $(-1;0)$.
4. passing through the point $(2;-\frac{6}{7})$ and with $m = -\frac{3}{7}$.
5. which cuts the y-axis at $y = -\frac{1}{5}$ and with $m = \frac{1}{2}$.
6.



7.



8.



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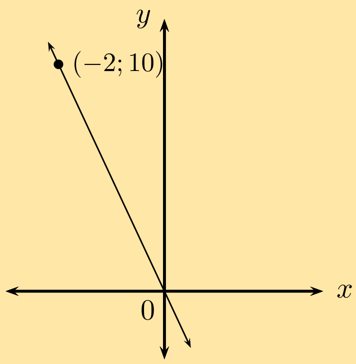

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1. 22KP 2. 22KQ 3. 22KR 4. 22KS 5. 22KT 6. 22KV
7. 22KW 8. 22KX 9. 22KY

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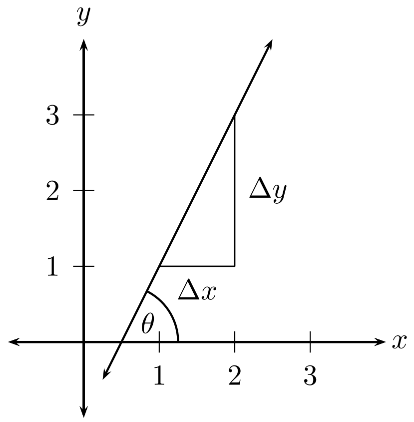


The diagram shows that a straight line makes an angle θ with the positive x-axis. This is called the **angle of inclination** of a straight line.

We notice that if the gradient changes, then the value of θ also changes, therefore the angle of inclination of a line is related to its gradient. We know that gradient is the ratio of a change in the y-direction to a change in the x-direction:

$$m = \frac{\Delta y}{\Delta x}$$

From trigonometry we know that the tangent function is defined as the ratio:

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

And from the diagram we see that

$$\begin{aligned}\tan\theta &= \frac{\Delta y}{\Delta x}\\ \therefore m &= \tan\theta \qquad \text{for } 0^\circ \leq \theta < 180^\circ\end{aligned}$$

Therefore the gradient of a straight line is equal to the tangent of the angle formed between the line and the positive direction of the x-axis.

Vertical lines

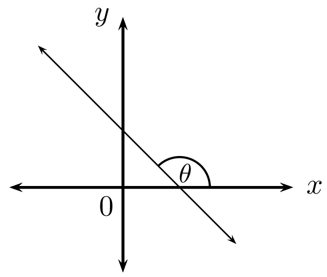
- $\theta = 90^\circ$
- Gradient is undefined since there is no change in the x-values ($\Delta x = 0$).
- Therefore $\tan\theta$ is also undefined (the graph of $\tan\theta$ has an asymptote at $\theta = 90^\circ$).

Horizontal lines

- $\theta = 0°$
- Gradient is equal to 0 since there is no change in the y-values ($\Delta y = 0$).
- Therefore $\tan\theta$ is also equal to 0 (the graph of $\tan\theta$ passes through the origin $(0°; 0)$.

Lines with negative gradients

If a straight line has a negative gradient ($m < 0$, $\tan\theta < 0$), then the angle formed between the line and the positive direction of the x-axis is obtuse.



From the CAST diagram in trigonometry, we know that the tangent function is negative in the second and fourth quadrant. If we are calculating the angle of inclination for a line with a negative gradient, we must add 180° to change the negative angle in the fourth quadrant to an obtuse angle in the second quadrant:

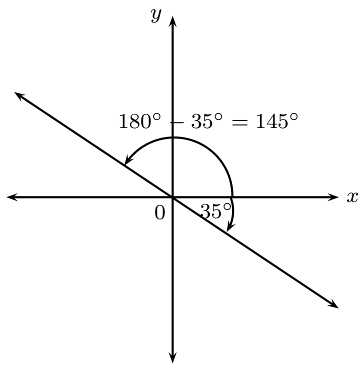
If we are given a straight line with gradient $m = -0{,}7$, then we can determine the angle of inclination using a calculator:

$$\begin{aligned}\tan\theta &= m \\ &= -0{,}7 \\ \therefore \theta &= \tan^{-1}(-0{,}7) \\ &= -35{,}0°\end{aligned}$$

This negative angle lies in the fourth quadrant. We must add 180° to get an obtuse angle in the second quadrant:

$$\begin{aligned}\theta &= -35{,}0° + 180° \\ &= 145°\end{aligned}$$

And we can always use our calculator to check that the obtuse angle $\theta = 145°$ gives a gradient of $m = -0{,}7$.



Exercise 4 – 5: Angle of inclination

1. Determine the gradient (correct to 1 decimal place) of each of the following straight lines, given that the angle of inclination is equal to:

   a) 60°
   b) 135°
   c) 0°
   d) 54°
   e) 90°
   f) 45°
   g) 140°
   h) 180°
   i) 75°

2. Determine the angle of inclination (correct to 1 decimal place) for each of the following:

   a) a line with $m = \frac{3}{4}$
   b) $2y - x = 6$
   c) the line passes through the points $(-4;-1)$ and $(2;5)$
   d) $y = 4$
   e) $x = 3y + \frac{1}{2}$
   f) $x = -0{,}25$
   g) the line passes through the points $(2;5)$ and $(\frac{2}{3};1)$
   h) a line with gradient equal to 0,577

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1a. 22KZ 1b. 22M2 1c. 22M3 1d. 22M4 1e. 22M5 1f. 22M6
1g. 22M7 1h. 22M8 1i. 22M9 2a. 22MB 2b. 22MC 2c. 22MD
2d. 22MF 2e. 22MG 2f. 22MH 2g. 22MJ 2h. 22MK

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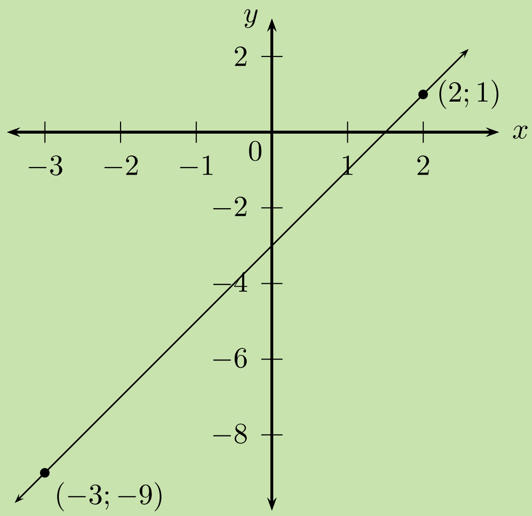
Worked example 8: Inclination of a straight line

QUESTION

Determine the angle of inclination (correct to 1 decimal place) of the straight line passing through the points $(2; 1)$ and $(-3; -9)$.

SOLUTION

Step 1: Draw a sketch



Step 2: Assign variables to the coordinates of the given points

$$x_1 = 2; \qquad y_1 = 1; \qquad x_2 = -3; \qquad y_2 = -9$$

Step 3: Determine the gradient of the line

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-9 - 1}{-3 - 2} \\ &= \frac{-10}{-5} \\ \therefore m &= 2 \end{aligned}$$

Step 4: Use the gradient to determine the angle of inclination of the line

$$\begin{aligned} \tan\theta &= m \\ &= 2 \\ \therefore \theta &= \tan^{-1} 2 \\ &= 63{,}4^\circ \end{aligned}$$

Important: make sure your calculator is in DEG (degrees) mode.

Step 5: Write the final answer

The angle of inclination of the straight line is 63,4°.

Worked example 9: Inclination of a straight line

QUESTION

Determine the equation of the straight line passing through the point $(3;1)$ and with an angle of inclination of $135°$.

SOLUTION

Step 1: Use the angle of inclination to determine the gradient of the line

$$\begin{aligned} m &= \tan\theta \\ &= \tan 135° \\ \therefore m &= -1 \end{aligned}$$

Step 2: Write down the gradient–point form of the straight line equation

$$y - y_1 = m(x - x_1)$$

Substitute $m = -1$

$$y - y_1 = -(x - x_1)$$

Substitute the given point $(3;1)$

$$\begin{aligned} y - 1 &= -(x - 3) \\ y &= -x + 3 + 1 \\ &= -x + 4 \end{aligned}$$

Step 3: Write the final answer

The equation of the straight line is $y = -x + 4$.

Worked example 10: Inclination of a straight line

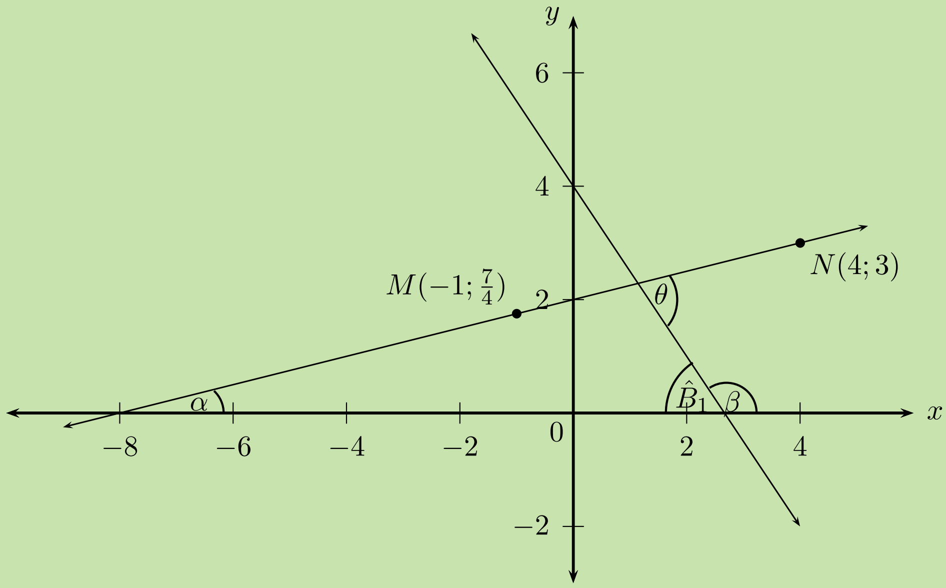
QUESTION

Determine the acute angle (correct to 1 decimal place) between the line passing through the points $M(-1;1\frac{3}{4})$ and $N(4;3)$ and the straight line $y = -\frac{3}{2}x + 4$.

SOLUTION

Step 1: Draw a sketch

Draw the line through points $M(-1;1\frac{3}{4})$ and $N(4;3)$ and the line $y = -\frac{3}{2}x + 4$ on a suitable system of axes. Label α and β, the angles of inclination of the two lines. Label θ, the acute angle between the two straight lines.



Notice that α and θ are acute angles and β is an obtuse angle.

$$
\begin{aligned}
\hat{B}_1 &= 180° - \beta && (\angle \text{ on str. line}) \\
\text{and } \theta &= \alpha + \hat{B}_1 && (\text{ext. } \angle \text{ of } \triangle = \text{ sum int. opp}) \\
\therefore \theta &= \alpha + (180° - \beta) \\
&= 180° + \alpha - \beta
\end{aligned}
$$

Step 2: Use the gradient to determine the angle of inclination β

From the equation $y = -\frac{3}{2}x + 4$ we see that $m < 0$, therefore β is an obtuse angle such that $90° < \beta < 180°$.

$$
\begin{aligned}
\tan \beta &= m \\
&= -\frac{3}{2} \\
\tan^{-1}\left(-\frac{3}{2}\right) &= -56{,}3°
\end{aligned}
$$

This negative angle lies in the fourth quadrant. We know that the angle of inclination β is an obtuse angle that lies in the second quadrant, therefore

$$
\begin{aligned}
\beta &= -56{,}3° + 180° \\
&= 123{,}7°
\end{aligned}
$$

Step 3: Determine the gradient and angle of inclination of the line through M and N

Determine the gradient

$$
\begin{aligned}
m &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{3 - \frac{7}{4}}{4 - (-1)} \\
&= \frac{\frac{5}{4}}{5} \\
&= \frac{1}{4}
\end{aligned}
$$

Determine the angle of inclination

$$
\begin{aligned}
\tan\alpha &= m \\
&= \frac{1}{4} \\
\therefore \alpha &= \tan^{-1}\left(\frac{1}{4}\right) \\
&= 14{,}0^\circ
\end{aligned}
$$

Step 4: Write the final answer

$$
\begin{aligned}
\theta &= 180^\circ + \alpha - \beta \\
&= 180^\circ + 14{,}0^\circ - 123{,}7^\circ \\
&= 70{,}3^\circ
\end{aligned}
$$

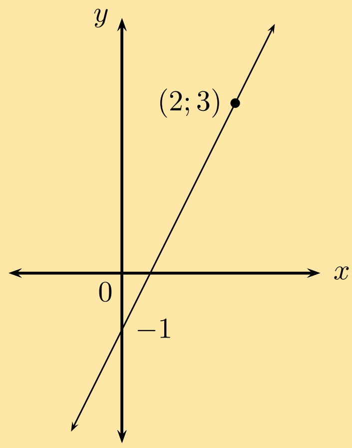
The acute angle between the two straight lines is $70{,}3^\circ$.

Exercise 4 – 6: Inclination of a straight line

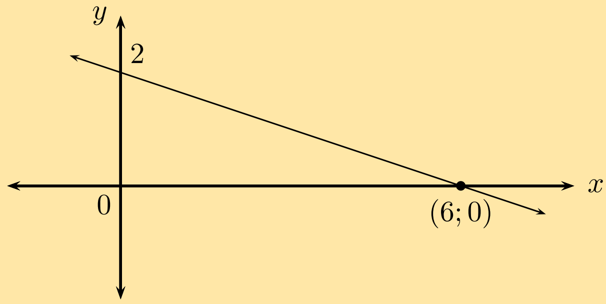
1. Determine the angle of inclination for each of the following:

   a) a line with $m = \frac{4}{5}$
   b) $x + y + 1 = 0$
   c) a line with $m = 5{,}69$
   d) the line that passes through $(1; 1)$ and $(-2; 7)$
   e) $3 - 2y = 9x$
   f) the line that passes through $(-1; -6)$ and $(-\frac{1}{2}; -\frac{11}{2})$
   g) $5 = 10y - 15x$

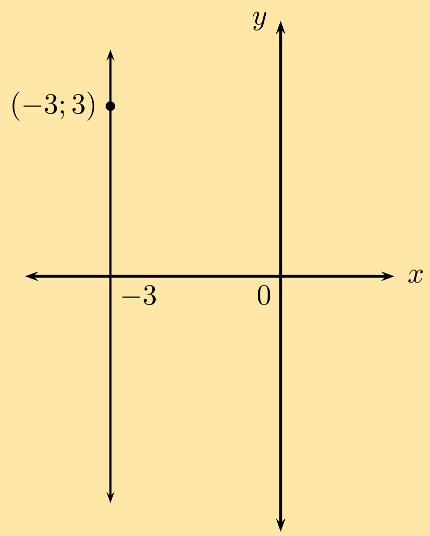
h)



i)



j)



2. Determine the acute angle between the line passing through the points $A(-2; \frac{1}{5})$ and $B(0; 1)$ and the line passing through the points $C(1; 0)$ and $D(-2; 6)$.
3. Determine the angle between the line $y + x = 3$ and the line $x = y + \frac{1}{2}$.
4. Find the angle between the line $y = 2x$ and the line passing through the points $(-1; \frac{7}{3})$ and $(0; 2)$.

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1a. 22MM 1b. 22MN 1c. 22MP 1d. 22MQ 1e. 22MR 1f. 22MS
1g. 22MT 1h. 22MV 1i. 22MW 1j. 22MX 2. 22MY 3. 22MZ
4. 22N2

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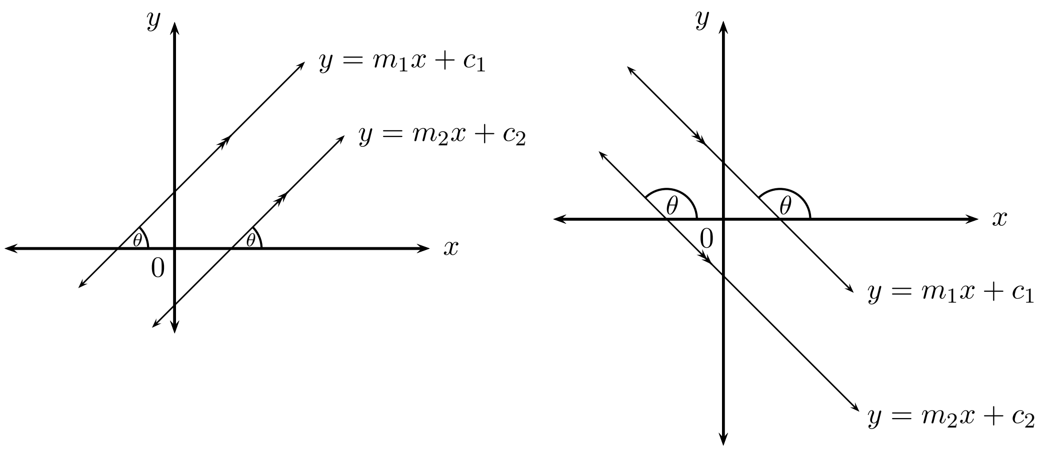
4.4 Parallel lines EMBGF

Investigation: Parallel lines

1. Draw a sketch of the line passing through the points $P(-1;0)$ and $Q(1;4)$ and the line passing through the points $R(1;2)$ and $S(2;4)$.
2. Label and measure α and β, the angles of inclination of straight lines PQ and RS respectively.
3. Describe the relationship between α and β.
4. "α and β are alternate angles, therefore $PQ \parallel RS$." Is this a true statement? If not, provide a correct statement.
5. Use your calculator to determine $\tan\alpha$ and $\tan\beta$.
6. Complete the sentence: ...... lines have ...... angles of inclination.
7. Determine the equations of the straight lines PQ and RS.
8. What do you notice about m_{PQ} and m_{RS}?
9. Complete the sentence: ...... lines have ...... gradients.

Another method of determining the equation of a straight line is to be given a point on the unknown line, $(x_1;y_1)$, and the equation of a line which is parallel to the unknown line.

Let the equation of the unknown line be $y = m_1x + c_1$ and the equation of the given line be $y = m_2x + c_2$.



If the two lines are parallel then

$$m_1 = m_2$$

Important: when determining the gradient of a line using the coefficient of x, make sure the given equation is written in the gradient–intercept (standard) form. $y = mx + c$

Substitute the value of m_2 and the given point $(x_1; y_1)$, into the gradient–intercept form of a straight line equation

$$y - y_1 = m(x - x_1)$$

and determine the equation of the unknown line.

Worked example 11: Parallel lines

QUESTION

Determine the equation of the line that passes through the point $(-1; 1)$ and is parallel to the line $y - 2x + 1 = 0$.

SOLUTION

Step 1: Write the equation in gradient–intercept form

We write the given equation in gradient–intercept form and determine the value of m.

$$y = 2x - 1$$

We know that the two lines are parallel, therefore $m_1 = m_2 = 2$.

Step 2: Write down the gradient–point form of the straight line equation

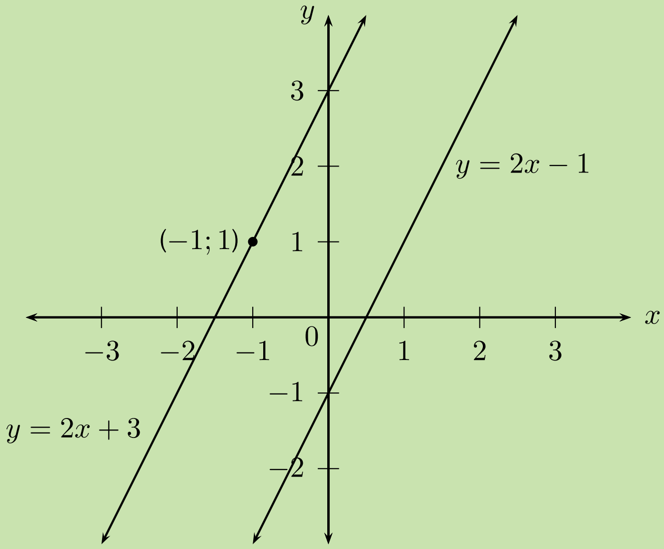
$$y - y_1 = m(x - x_1)$$

Substitute $m = 2$

$$y - y_1 = 2(x - x_1)$$

Substitute the given point $(-1; 1)$

$$\begin{aligned} y - 1 &= 2(x - (-1)) \\ y - 1 &= 2x + 2 \\ y &= 2x + 2 + 1 \\ &= 2x + 3 \end{aligned}$$



A sketch was not required, but it is always helpful and can be used to check answers.

Step 3: Write the final answer

The equation of the straight line is $y = 2x + 3$.

Worked example 12: Parallel lines

QUESTION

Line AB passes through the point $A(0; 3)$ and has an angle of inclination of 153,4°. Determine the equation of the line CD which passes through the point $C(2; -3)$ and is parallel to AB.

SOLUTION

Step 1: Use the given angle of inclination to determine the gradient

$$\begin{aligned} m_{AB} &= \tan\theta \\ &= \tan 153{,}4^\circ \\ &= -0{,}5 \end{aligned}$$

Step 2: Parallel lines have equal gradients

Since we are given $AB \parallel CD$,

$$m_{CD} = m_{AB} = -0{,}5$$

Step 3: Write down the gradient–point form of a straight line equation

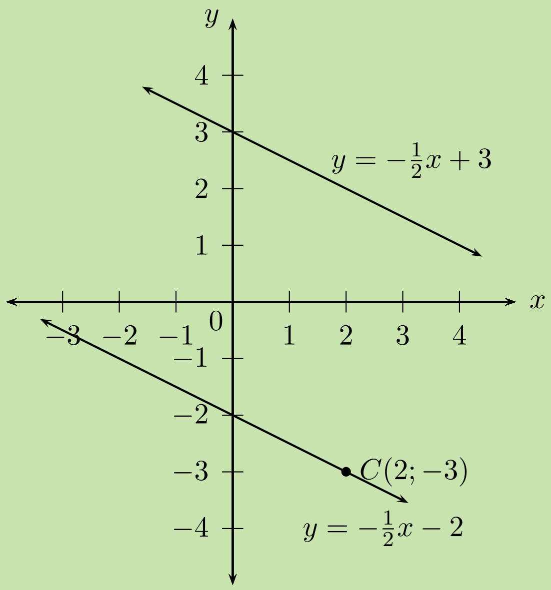
$$y - y_1 = m(x - x_1)$$

Substitute the gradient $m_{CD} = -0{,}5$.

$$y - y_1 = -\frac{1}{2}(x - x_1)$$

Substitute the given point $(2; -3)$.

$$y - (-3) = -\frac{1}{2}(x - 2)$$
$$y + 3 = -\frac{1}{2}x + 1$$
$$y = -\frac{1}{2}x - 2$$



A sketch was not required, but it is always useful.

Step 4: Write the final answer

The equation of the straight line is $y = -\frac{1}{2}x - 2$.

See video: 22N3 at www.everythingmaths.co.za

Exercise 4 – 7: Parallel lines

1. Determine whether or not the following two lines are parallel:

   a) $y + 2x = 1$ and $-2x + 3 = y$
   b) $\frac{y}{3} + x + 5 = 0$ and $2y + 6x = 1$
   c) $y = 2x - 7$ and the line passing through $(1; -2)$ and $(\frac{1}{2}; -1)$
   d) $y + 1 = x$ and $x + y = 3$
   e) The line passing through points $(-2; -1)$ and $(-4; -3)$ and the line $-y + x - 4 = 0$
   f) $y - 1 = \frac{1}{3}x$ and the line passing through points $(-2; 4)$ and $(1; 5)$

2. Determine the equation of the straight line that passes through the point $(1; -5)$ and is parallel to the line $y + 2x - 1 = 0$.

3. Determine the equation of the straight line that passes through the point $(-2; -6)$ and is parallel to the line $2y + 1 = 6x$.

4. Determine the equation of the straight line that passes through the point $(-2; -2)$ and is parallel to the line with angle of inclination $\theta = 56{,}31°$.

5. Determine the equation of the straight line that passes through the point $(-2; \frac{2}{5})$ and is parallel to the line with angle of inclination $\theta = 145°$.

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1a. 22N4   1b. 22N5   1c. 22N6   1d. 22N7   1e. 22N8   1f. 22N9
2. 22NB   3. 22NC   4. 22ND   5. 22NF

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4.5 Perpendicular lines EMBGG

Investigation: Perpendicular lines

1. Draw a sketch of the line passing through the points $A(-2; -3)$ and $B(2; 5)$ and the line passing through the points $C(-1; \frac{1}{2})$ and $D(4; -2)$.

2. Label and measure α and β, the angles of inclination of straight lines AB and CD respectively.

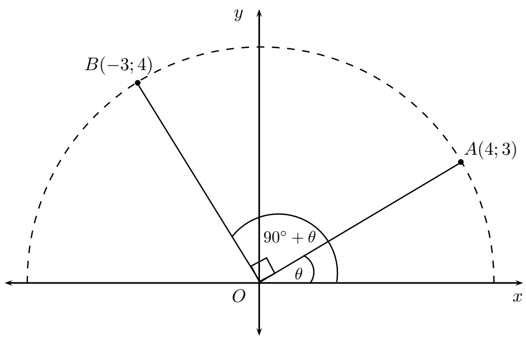
3. Label and measure θ, the angle between the lines AB and CD.

4. Describe the relationship between the lines AB and CD.

5. "θ is a reflex angle, therefore $AB \perp CD$." Is this a true statement? If not, provide a correct statement.

6. Determine the equation of the straight line AB and the line CD.
7. Use your calculator to determine $\tan\alpha \times \tan\beta$.
8. Determine $m_{AB} \times m_{CD}$.
9. What do you notice about these products?
10. Complete the sentence: if two lines are ...... to each other, then the product of their ...... is equal ......
11. Complete the sentence: if the gradient of a straight line is equal to the negative ...... of the gradient of another straight line, then the two lines are ......

Deriving the formula: $m_1 \times m_2 = -1$



Consider the point $A(4;3)$ on the Cartesian plane with an angle of inclination $A\hat{O}X = \theta$. Rotate through an angle of 90° and place point B at $(-3;4)$ so that we have the angle of inclination $B\hat{O}X = 90° + \theta$.

We determine the gradient of OA:

$$\begin{aligned} m_{OA} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3-0}{4-0} \\ &= \frac{3}{4} \end{aligned}$$

And determine the gradient of OB:

$$\begin{aligned} m_{OB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-0}{-3-0} \\ &= \frac{4}{-3} \end{aligned}$$

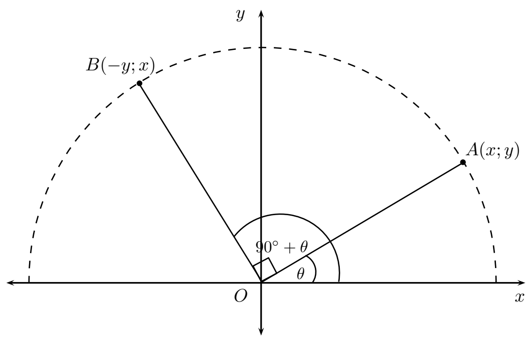
By rotating through an angle of 90° we know that $OB \perp OA$:

$$m_{OA} \times m_{OB} = \frac{3}{4} \times \frac{4}{-3}$$

$$= -1$$

We can also write that

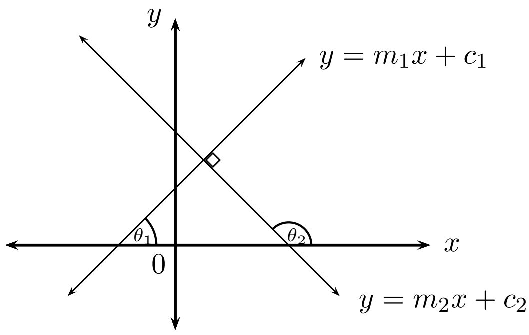
$$m_{OA} = -\frac{1}{m_{OB}}$$



If we have the general point $A(x;y)$ with an angle of inclination $A\hat{O}X = \theta$ and point $B(-y;x)$ such that $B\hat{O}X = 90° + \theta$, then we know that

$$m_{OA} = \frac{y}{x}$$
$$m_{OB} = -\frac{x}{y}$$
$$\therefore m_{OA} \times m_{OB} = \frac{y}{x} \times -\frac{x}{y}$$
$$= -1$$

Another method of determining the equation of a straight line is to be given a point on the line, $(x_1;y_1)$, and the equation of a line which is perpendicular to the unknown line. Let the equation of the unknown line be $y = m_1x + c_1$ and the equation of the given line be $y = m_2x + c_2$.



If the two lines are perpendicular then

$$m_1 \times m_2 = -1$$

Note: this rule does not apply to vertical or horizontal lines.

When determining the gradient of a line using the coefficient of x, make sure the given equation is written in the gradient–intercept (standard) form $y = mx + c$. Then we know that

$$m_1 = -\frac{1}{m_2}$$

Substitute the value of m_1 and the given point $(x_1; y_1)$, into the gradient–intercept form of the straight line equation $y - y_1 = m(x - x_1)$ and determine the equation of the unknown line.

Worked example 13: Perpendicular lines

QUESTION

Determine the equation of the straight line passing through the point $T(2; 2)$ and perpendicular to the line $3y + 2x - 6 = 0$.

SOLUTION

Step 1: Write the equation in standard form

Let the gradient of the unknown line be m_1 and the given gradient be m_2. We write the given equation in gradient–intercept form and determine the value of m_2.

$$\begin{aligned} 3y + 2x - 6 &= 0 \\ 3y &= -2x + 6 \\ y &= -\frac{2}{3}x + 2 \\ \therefore m_2 &= -\frac{2}{3} \end{aligned}$$

We know that the two lines are perpendicular, therefore $m_1 \times m_2 = -1$. Therefore $m_1 = \frac{3}{2}$.

Step 2: Write down the gradient–point form of the straight line equation

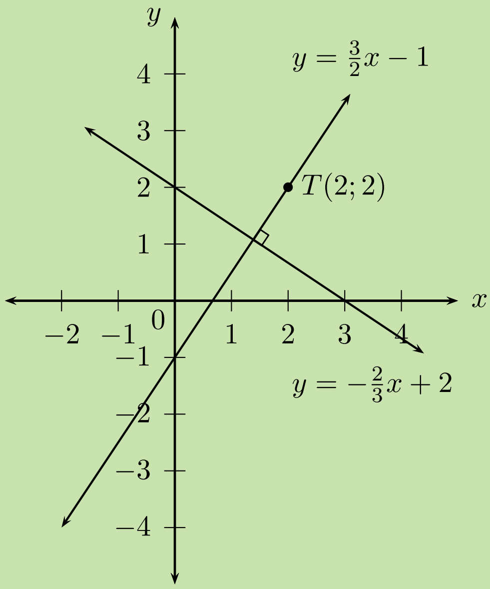
$$y - y_1 = m(x - x_1)$$

Substitute $m_1 = \frac{3}{2}$.

$$y - y_1 = \frac{3}{2}(x - x_1)$$

Substitute the given point $T(2; 2)$.

$$\begin{aligned} y - 2 &= \frac{3}{2}(x - 2) \\ y - 2 &= \frac{3}{2}x - 3 \\ y &= \frac{3}{2}x - 1 \end{aligned}$$



A sketch was not required, but it is useful for checking the answer.

Step 3: Write the final answer

The equation of the straight line is $y = \frac{3}{2}x - 1$.

Worked example 14: Perpendicular lines

QUESTION

Determine the equation of the straight line passing through the point $(2; \frac{1}{3})$ and perpendicular to the line with an angle of inclination of 71,57°.

SOLUTION

Step 1: Use the given angle of inclination to determine gradient

Let the gradient of the unknown line be m_1 and let the given gradient be m_2.

$$\begin{aligned} m_2 &= \tan \theta \\ &= \tan 71{,}57^\circ \\ &= 3{,}0 \end{aligned}$$

Step 2: Determine the unknown gradient

Since we are given that the two lines are perpendicular,

$$\begin{aligned} m_1 \times m_2 &= -1 \\ \therefore m_1 &= -\frac{1}{3} \end{aligned}$$

Step 3: Write down the gradient–point form of the straight line equation

$$y - y_1 = m(x - x_1)$$

Substitute the gradient $m_1 = -\frac{1}{3}$.

$$y - y_1 = -\frac{1}{3}(x - x_1)$$

Substitute the given point $(2; \frac{1}{3})$.

$$y - \left(\frac{1}{3}\right) = -\frac{1}{3}(x - 2)$$

$$y - \frac{1}{3} = -\frac{1}{3}x + \frac{2}{3}$$

$$y = -\frac{1}{3}x + 1$$

Step 4: Write the final answer

The equation of the straight line is $y = -\frac{1}{3}x + 1$.

See video: 22NG at www.everythingmaths.co.za

Exercise 4 – 8: Perpendicular lines

1. Calculate whether or not the following two lines are perpendicular:

   a) $y - 1 = 4x$ and $4y + x + 2 = 0$
   b) $10x = 5y - 1$ and $5y - x - 10 = 0$
   c) $x = y - 5$ and the line passing through $(-1; \frac{5}{4})$ and $(3; -\frac{11}{4})$
   d) $y = 2$ and $x = 1$
   e) $\frac{y}{3} = x$ and $3y + x = 9$
   f) $1 - 2x = y$ and the line passing through $(2; -1)$ and $(-1; 5)$
   g) $y = x + 2$ and $2y + 1 = 2x$

2. Determine the equation of the straight line that passes through the point $(-2; -4)$ and is perpendicular to the line $y + 2x = 1$.

3. Determine the equation of the straight line that passes through the point $(2; -7)$ and is perpendicular to the line $5y - x = 0$.

4. Determine the equation of the straight line that passes through the point $(3;-1)$ and is perpendicular to the line with angle of inclination $\theta = 135°$.

5. Determine the equation of the straight line that passes through the point $(-2;\frac{2}{5})$ and is perpendicular to the line $y = \frac{4}{3}$.

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1a. 22NH 1b. 22NJ 1c. 22NK 1d. 22NM 1e. 22NN 1f. 22NP
1g. 22NQ 2. 22NR 3. 22NS 4. 22NT 5. 22NV

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4.6 Summary EMBGH

See presentation: 22NW at www.everythingmaths.co.za

- Distance between two points: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Gradient of a line between two points: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- Mid-point of a line: $M(x;y) = \left(\dfrac{x_1 + x_2}{2}; \dfrac{y_1 + y_2}{2}\right)$
- Parallel lines: $m_1 = m_2$
- Perpendicular lines: $m_1 \times m_2 = -1$
- General form of a straight line equation: $ax + by + c = 0$
- Two-point form of a straight line equation: $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$
- Gradient–point form of a straight line equation: $y - y_1 = m(x - x_1)$
- Gradient–intercept form of a straight line equation (standard form): $y = mx + c$
- Angle of inclination of a straight line: θ, the angle formed between the line and the positive x-axis; $m = \tan\theta$

Exercise 4 – 9: End of chapter exercises

1. Determine the equation of the line:

   a) through points $(-1;3)$ and $(1;4)$
   b) through points $(7;-3)$ and $(0;4)$
   c) parallel to $y = \frac{1}{2}x + 3$ and passing through $(-2;3)$
   d) perpendicular to $y = -\frac{1}{2}x + 3$ and passing through $(-1;2)$
   e) perpendicular to $3y + x = 6$ and passing through the origin

2. Determine the angle of inclination of the following lines:

   a) $y = 2x - 3$
   b) $y = \frac{1}{3}x - 7$
   c) $4y = 3x + 8$
   d) $y = -\frac{2}{3}x + 3$
   e) $3y + x - 3 = 0$

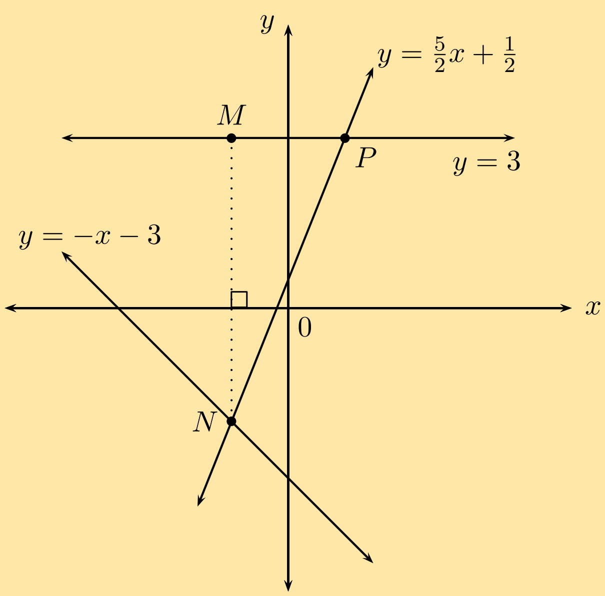
3. $P(2;3)$, $Q(-4;0)$ and $R(5;-3)$ are the vertices of $\triangle PQR$ in the Cartesian plane. PR intersects the x-axis at S. Determine the following:

   a) the equation of the line PR
   b) the coordinates of point S
   c) the angle of inclination of PR (correct to two decimal places)
   d) the gradient of line PQ
   e) $Q\hat{P}R$
   f) the equation of the line perpendicular to PQ and passing through the origin
   g) the mid-point M of QR
   h) the equation of the line parallel to PR and passing through point M

4. Points $A(-3;5)$, $B(-7;-4)$ and $C(2;0)$ are given.

   a) Plot the points on the Cartesian plane.
   b) Determine the coordinates of D if $ABCD$ is a parallelogram.
   c) Prove that $ABCD$ is a rhombus.

5.



Consider the sketch above, with the following lines shown:

$y = -x - 3$

$y = 3$

$y = \frac{5}{2}x + \frac{1}{2}$

a) Determine the coordinates of the point N.
b) Determine the coordinates of the point P.
c) Determine the equation of the vertical line MN.
d) Determine the length of the vertical line MN.
e) Find $M\hat{N}P$.
f) Determine the equation of the line parallel to NP and passing through the point M.

6. The following points are given: $A(-2;3)$, $B(2;4)$, $C(3;0)$.

a) Plot the points on the Cartesian plane.
b) Prove that $\triangle ABC$ is a right-angled isosceles triangle.
c) Determine the equation of the line AB.
d) Determine the coordinates of D if $ABCD$ is a square.
e) Determine the coordinates of E, the mid-point of BC.

7. Given points $S(2;5)$, $T(-3;-4)$ and $V(4;-2)$.

a) Determine the equation of the line ST.
b) Determine the size of $T\hat{S}V$.

8. Consider triangle FGH with vertices $F(-1;3)$, $G(2;1)$ and $H(4;4)$.

a) Sketch $\triangle FGH$ on the Cartesian plane.
b) Show that $\triangle FGH$ is an isosceles triangle.
c) Determine the equation of the line PQ, perpendicular bisector of FH.
d) Does G lie on the line PQ?
e) Determine the equation of the line parallel to GH and passing through point F.

9. Given the points $A(-1;5)$, $B(5;-3)$ and $C(0;-6)$. M is the mid-point of AB and N is the mid-point of AC.

a) Draw a sketch on the Cartesian plane.
b) Show that the coordinates of M and N are $(2;1)$ and $(-\frac{1}{2};-\frac{1}{2})$ respectively.
c) Use analytical geometry methods to prove the mid-point theorem. (Prove that $NM \parallel CB$ and $NM = \frac{1}{2}CB$.)

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1a. 22NX 1b. 22NY 1c. 22NZ 1d. 22P2 1e. 22P3 2a. 22P4
2b. 22P5 2c. 22P6 2d. 22P7 2e. 22P8 3. 22P9 4. 22PB
5. 22PC 6. 22PD 7. 22PF 8. 22PG 9. 22PH

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