GRADE 11

MATHEMATICS
NOVEMBER PAPER 2
2022

EXAMINER: BILLY NAIR CLUSTER **MODERATOR:** BILLY NAIR CLUSTER

MARKS: 100

DATE: 15 NOVEMBER 2022

TIME: 2 HOURS

INFORMATION & INSTRUCTIONS

1. This question paper consists of 8 pages and one DIAGRAM SHEET.
2. This question paper consists of 8 questions.
3. Answer all the questions.
4. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Write neatly and legibly.

QUESTION ONE

1.1 Dillin plays for his cricket team. The number of runs scored by Dillin in the eight games that he batted in, is shown below.

(Dillin was given out in all the games)

21	8	19	7	15	32	14	12

1.1.1 Determine the mean average runs scored by Dillin in the eight games. (2)

1.1.2 Determine the standard deviation of the data set. (2)

1.2 In a certain school 60 learners wrote Mathematics examination. Maximum mark was 100.

1.2.1 Use the information below to draw the box-and-whisker diagram for the mathematics results on the DIAGRAM SHEET 1.

Minimum mark = 30

Range = 55

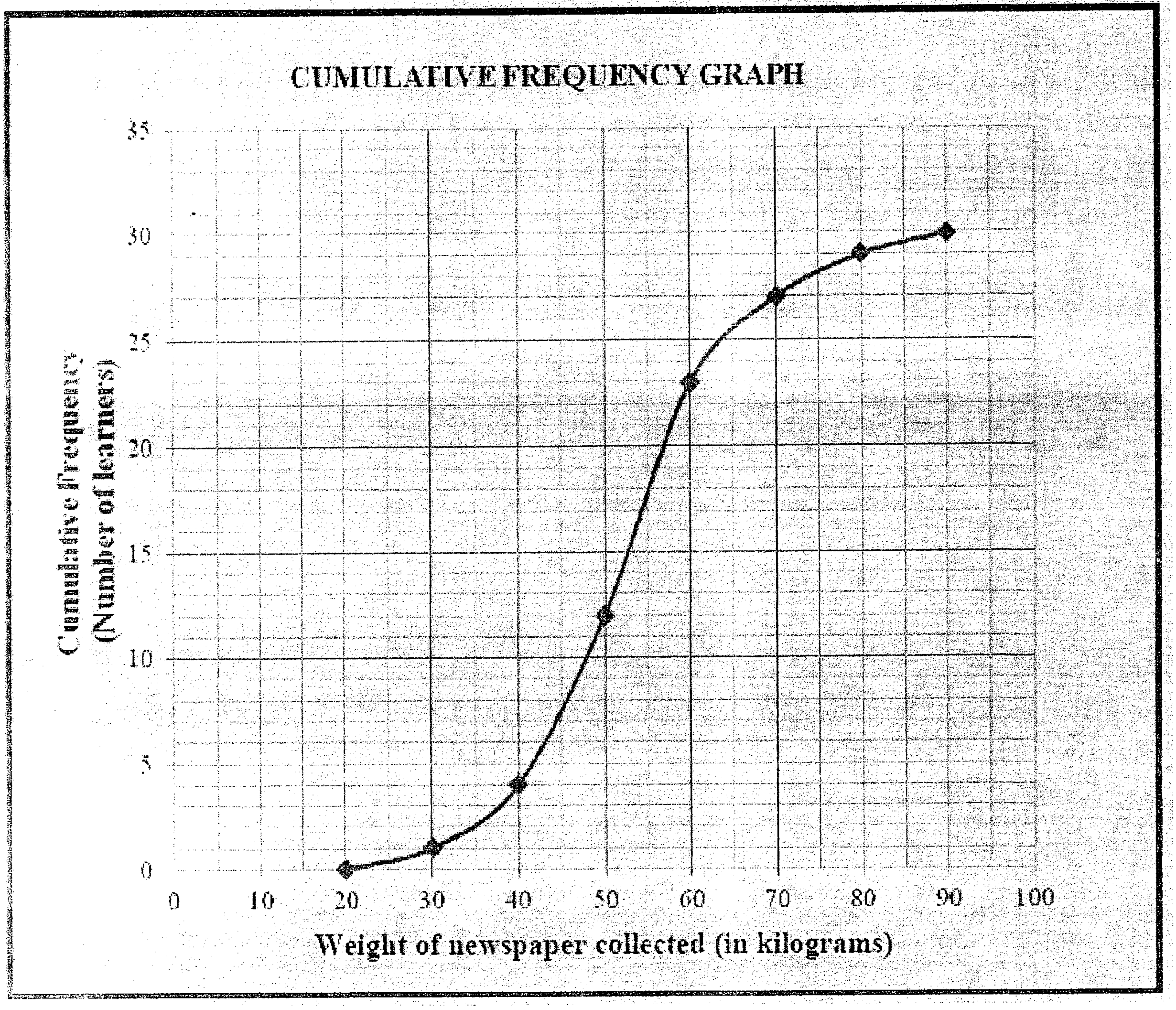
Upper quartile = 70

Interquartile range = 30

Median = 55 (4)

1.2.2 How many learners scored less than 70% in the Mathematics examination? (2)

1.3 As part of an environmental awareness initiative, learners of Buffelsdale Secondary School were requested to collect newspapers for recycling. The cumulative frequency graph (ogive) below shows the total weight of the newspapers (in kilograms) collected over a period of 6 months by 30 learners.



1.3.1 Determine the model class of the weight of the newspapers collected. (1)

1.3.2 Determine the median weight of the newspapers collected by this group of learners. (2)

1.3.3 How many learners collected more than 60 kilograms of newspaper? (2)

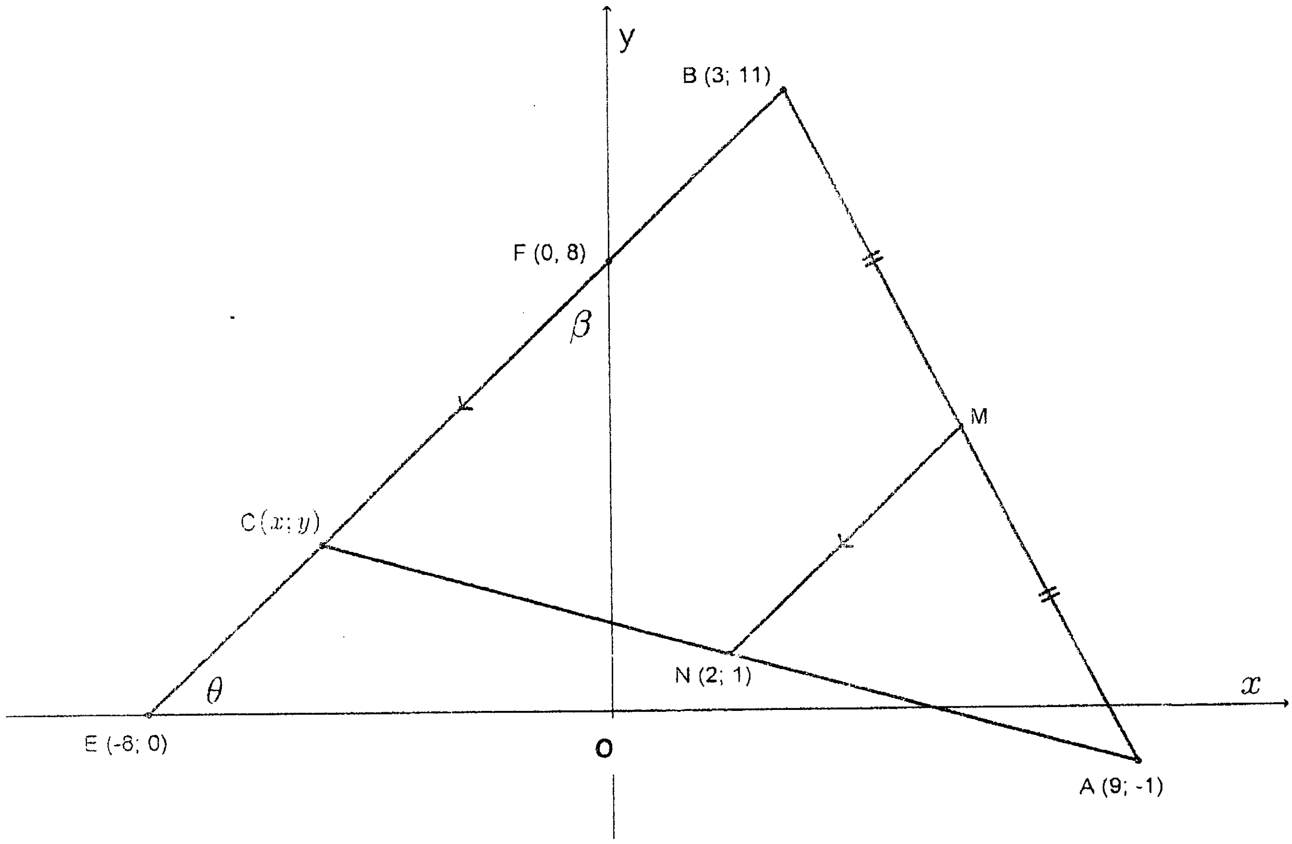
[15]

QUESTION 2

In the diagram, $A(9\ ;\ -1)$, $B(3\ ;11\ )$ and $C(x\ ;\ y)$ are the vertices ofΔABC.

M is the midpoint of AB. N (2; 1) is a point on CA such that $MN \parallel BC$. BC produced

to meet $x - axis$ at $E\ (-8\ ;0\ )$.



2.1 Determine the coordinates of M, the midpoint of AB. (2)

2.2 Determine the gradient of the line MN. (3)

2.3 Hence or otherwise the equation of the line MN, in the form $y = mx + c$. (2)

2.4 (a) Give a reason why N is the midpoint of AC. (1)

(b) Hence, find the coordinates of C. (3)

2.5 If ABCD (in that order) is a parallelogram, determine the coordinates of point D. (4)

2.6 Determine the size of β. (4)

2.7 Calculate the area of ΔFOE. (3)

[22]

QUESTION 3

3.1 If $\sin 23^\circ = p$, WITHOUT using a calculator, express the following in terms of p:

3.1.1 $\tan 23^\circ$ (3)

3.1.2 $\sin 113^\circ$ (2)

3.2 Simplify fully, WITHOUT the use of a calculator:

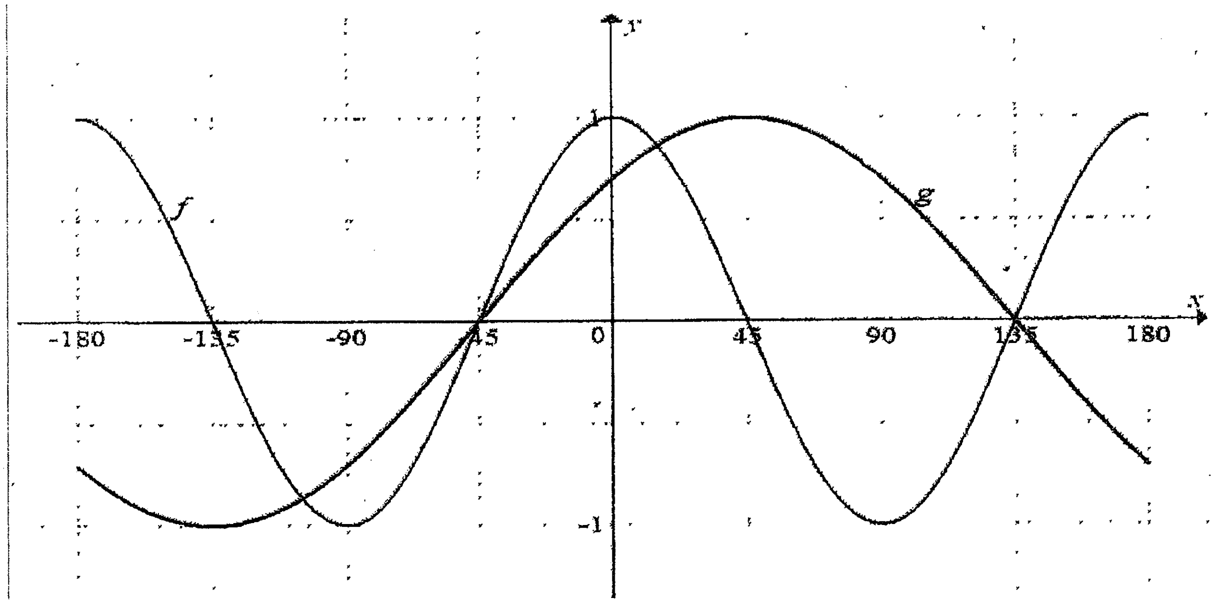
$tan(-315^\circ) - \cos 780^\circ$ (5)

3.3 Determine the general solution for $2sin\, x\, .\, cos\, x = cos\, x$ (6)

[16]

QUESTION 4:

In the diagram below the graphs of $f(x) = a\cos bx$ and $g(x) = \sin(x + p)$ are drawn for $x \in [-180^\circ; 180^\circ]$.



4.1 Write down the values of a, b and p. (3)

4.2 For which values of x in the given interval does the graph of f increase as the graph of g increases? (2)

4.3 Write down the period of $f(2x)$. (2)

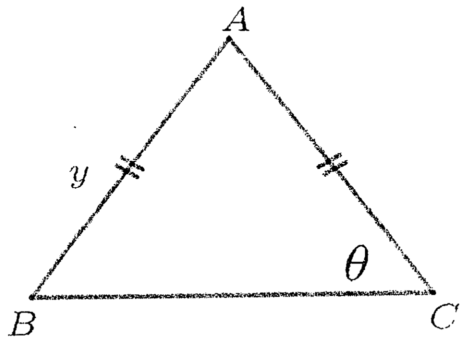
4.4 Determine the minimum value of h if $h(x) = 3f(x) - 1$. (2)

4.5 Describe how the graph g must be transformed to form the graph k, where $k(x) = -\cos x$. (2)

[11]

QUESTION 5:

In the ΔABC $\hat{C} = \theta$ and $AB = AC = y$.



Show that $BC = y\sqrt{2(1 + cos2\theta)}$. **[6]**

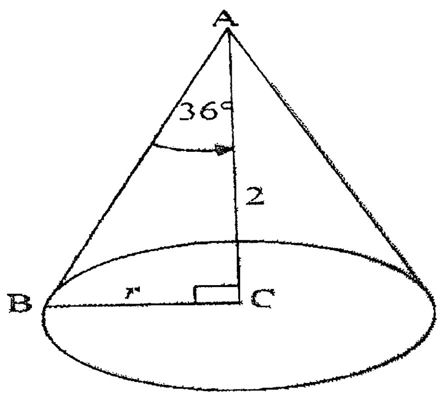
Question 6:

The perpendicular height, AC, of the solid cone below is 2 metres and the radius is r.

AB is the slant height.

$B\hat{A}C = 36^{\circ}$

Surface area = $\pi r^2 + \pi r S$ where S is the slant height.
Volume = $\frac{1}{3}$ area of base × perpendicular height
Volume = $\frac{1}{3}\pi r^2 h$



Calculate the total surface area of the cone. **[6]**

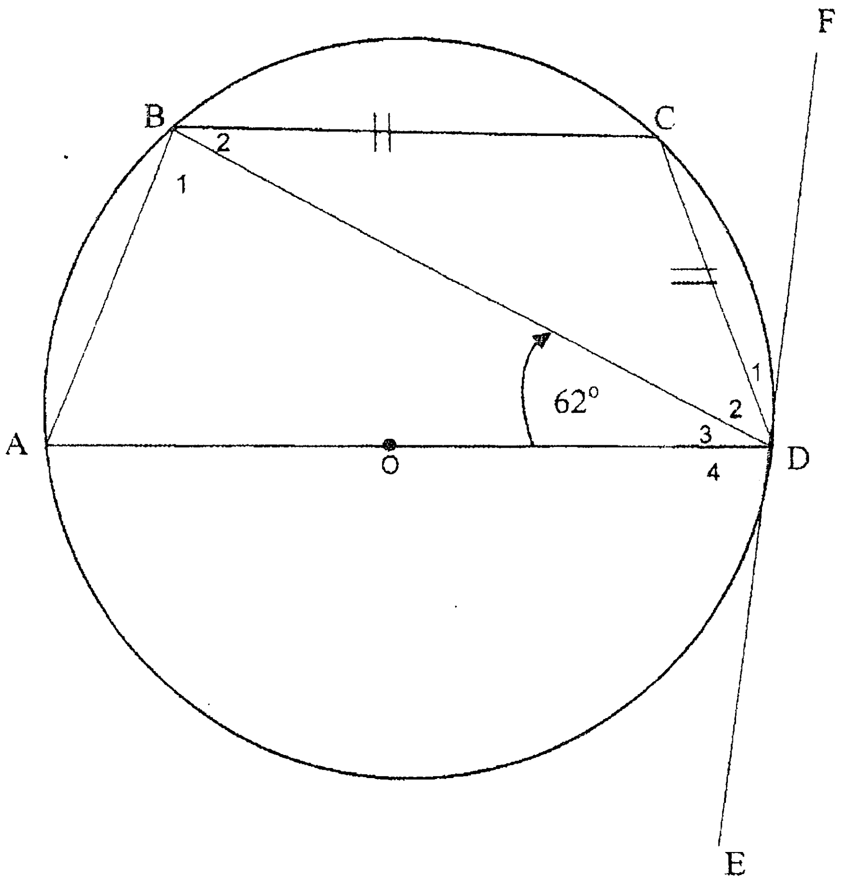
Question 7:

7.1 Complete this statement:

A line drawn from the centre of a circle is ________________ to the tangent. (1)

7.2 In the diagram below, AOD is a diameter of the circle and EDF is a tangent to the circle at D.

$A\hat{D}B = 62^{\circ}$ and $BC = CD$.



Calculate, with reasons, the numerical value of:

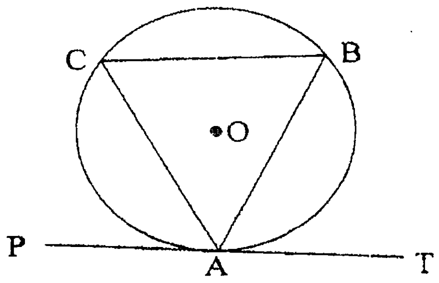
7.2.1 $B\hat{C}D$. (4)

7.2.2 $C\hat{D}F$. (4)

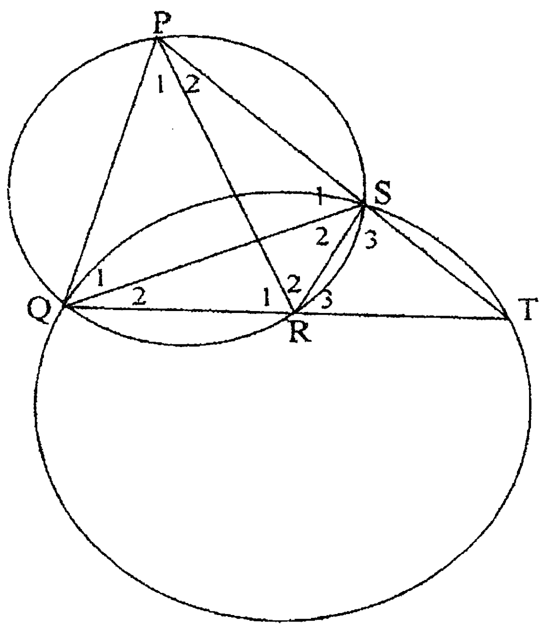
[9]

Question 8:

8.1 Use the sketch on DIAGRAM SHEET 1 to prove the theorem which states that $B\hat{A}T = \hat{C}$. (6)



8.2 In the diagram PQ is a tangent to the circle QST at Q such that QT is a chord of the circle and TS produced meets the tangent at P. R is a point on QT such that PQRS is a cyclic quadrilateral in another circle. PR, QS and RS are joined.



8.2.1 Give a reason for each statement. Write down only the **REASON.**

(a) Statement: $\hat{Q}_1 = \hat{T}$. (1)

(b) Statement: $\hat{Q}_2 = \hat{P}_2$. (1)

8.2.2 Prove that PQR is an isosceles triangle. (4)

8.2.3 Prove that PR is a tangent to the circle RST at point R. (3)

[15]

TOTAL [100]

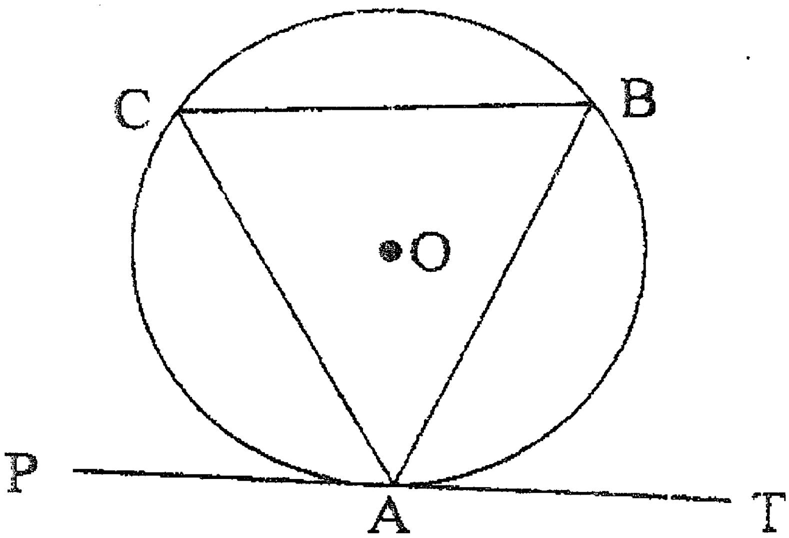
Learner's name: ____

DIAGRAM SHEET 1

Question 1.2.1

Mathematics

Question 8.1



INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

GRADE 11

MATHEMATICS

NOVEMBER PAPER 2

2022

MEMORANDUM

EXAMINER: BILLY NAIR CLUSTER

MODERATOR: BILLY NAIR CLUSTER

MARKS: 100

DATE: 15 NOVEMBER 2022

TIME: 2 HOURS

THIS MEMORANDUM CONSISTS OF NINE PAGES

Memo - Mathematics – Grade 11- Paper 2 – Nov. 2022

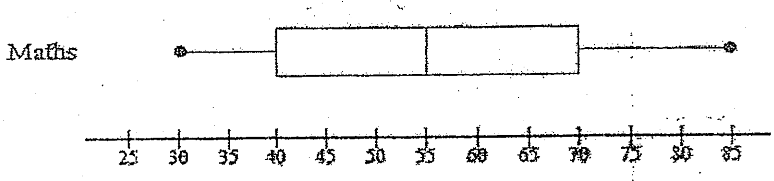
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

MARKS	
A	Accuracy
CA	Consistent accuracy

GEOMETRY	
R	Reason
S	Statement

QUESTION 1

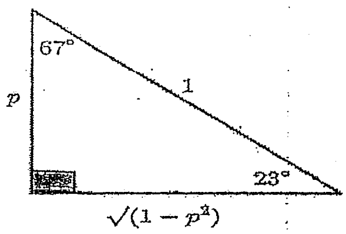
1.1.1	Average number of runs $\bar{x} = \frac{\sum x}{n} = \frac{128}{8} = 16$	A 128 A 16 (2)
1.1.2	Standard deviation= $7,55$ NOTE: penalty of 1 mark for incorrect rounding off	AA $7,55$ (2)
1.2.1		A max 85 A $Q_3 = 70$ A $Q_1 = 40$ A Median = 55 (4)
1.2.2	From the information given, the value of the third quartile is 70%. Therefore 75% of the learners got 70%. Number of learners BELOW 70% is expected to be $\frac{75}{100} \times 60 = \frac{3}{4} \times 60 = 45$ learners	A 75% of learners A 45 learners (2)
1.3.1	Model class is $50 \leq x < 60$ OR $50 < x \leq 60$ OR $50\ to\ 60$	A Correct class (1)
1.3.2	Median position is 15 learners (grouped data). Approximate weight is about 53 kg. (accept from 52 kg to 54 kg)	A Median position 15 A 53 kg (2)
1.3.3	$30 - 23 = 7$ learners collected more than 60 kg.	A $30 - 23$ A 7 learners (2)
		[15]

QUESTION 2

2.1	$M=\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ $=\left(\frac{3+9}{2};\frac{11+(-1)}{2}\right)$ $=(6;5)$	A subst into midpt form A answer (2)
2.2	$m_{BC}=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{11-3}{3-(-5)}$ $=1$ $m_{MN}=m_{BC}=1$ [$BC \parallel MN$]	A subst into gradient form A answer A gradients equal (3)
2.3	$y-y_1=m(x-x_1)$ $y-1=1(x-2)$ $y=x-1$ or $y=mx+c$ $1=1(2)+c$ $-1=c$ $y=x-1$	A subst $(2;1)$ & $m=1$ Into str. line equation A answer (2)
2.4	(a) N is a midpoint of AC [Line through midpoint of one side to second side]	A R (1)
	(b) $(2;1)=\left(\frac{x+9}{2};\frac{y+(-1)}{2}\right)$ $2=\frac{x+9}{2}$ and $1=\frac{y+(-1)}{2}$ $x=-5$ and $y=3$ $C(-5;3)$	A equate A $x-value$ A $y-value$ (3)
2.5	N is the midpoint of BD and the midpoint of AC [diagonals of parm bisect] $\left(\frac{3+x}{2};\frac{11+y}{2}\right)=(2;1)$ $\frac{3+x}{2}=2$ and $\frac{11+y}{2}=1$ $x=1$ and $y=-9$ $D(1;-9)$ OR From B to A $(x;y)\rightarrow(x+6;y-12)$ $D(-5+6;3-12)$ $D(1;-9)$	A $\frac{3+x}{2}=2$ A $\frac{11+y}{2}=1$ A $x=1$ A $y=-9$ A $x+6$ A $y-12$ A subst A $(1;-9)$ (4)
2.6	$\tan\theta=m_{BC}$ $\tan\theta=1$ $\theta=tan^{-1}(1)$ $\theta=45^\circ$ In ΔFOD: $\beta=180^\circ-(90^\circ+45^\circ)$ ($\angle's\ of\ a\ tringle$) $\beta=45^\circ$ OR	A $\tan\theta=1$ A $\theta=45^\circ$ A statement & reason A answer (4)

	$OD = 8$ units $OF = 8$ units $\beta = \theta = 45^\circ$ ($\angle$'s opp. equal sides) (statement & reason only award full mark)	A OD A OF A statement - A reason (4)
2.7	Area of $\Delta FOD = \frac{1}{2} \times base \times hight$ $= \frac{1}{2} \times 8 \times 8$ $= 32\ units^2$	A formula A subst A answer (3)
		[22]

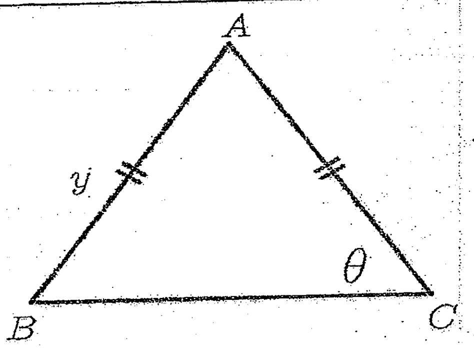
QUESTION 3:

3.1.1	$\tan 23^\circ = \frac{p}{\sqrt{1-p^2}}$  	A sketch A $\sqrt{1-p^2}$ A answer (3)
3.1.2	$\sin 113^\circ = \sin(90^\circ + 23^\circ)$ $= \cos 23^\circ$ $= \sqrt{1-p^2}$ OR $\sin 113^\circ = \sin(180^\circ - 67^\circ)$ $= \sin 67^\circ$ $= \sqrt{1-p^2}$	A $\cos 23^\circ$ A $\sqrt{1-p^2}$ (2) A $\sin 67^\circ$ A $\sqrt{1-p^2}$ (2)
3.2	$= tan(-315^\circ) - \cos 780^\circ$ $= -\tan(360^\circ - 45^\circ) - cos(60^\circ + 2 \times 360^\circ)$ $= tan45^\circ - cos60^\circ$ $= 1 - \frac{1}{2}$ $= \frac{1}{2}$	A $tan45^\circ$ A $cos60^\circ$ A 1 A $\frac{1}{2}$ A answer (5)
3.3	$2sin\,x\,.\,cos\,x - cos\,x = 0$ $cos\,x(2\sin x - 1) = 0$ $cos\,x = 0$ or $sin\,x = \frac{1}{2}$ $x = 90^\circ + k.360^\circ\ k\epsilon Z$ $x = 90^\circ + k.360^\circ\ k\epsilon Z$ OR OR $x = 270^\circ + k.360^\circ\ k\epsilon Z$ $x = 150^\circ + k.360^\circ\ k\epsilon Z$	A factors A both equations A both general solutions For $cos\,x = 0$ AA both general solutions for $sin\,x = \frac{1}{2}$ A $k\epsilon Z$ (6)
		[16]

QUESTION 4:

4.1	$a=1$ $b=2$ $p=45^\circ$	A $a=1$ A $b=2$ A $p=45^\circ$ (3)
4.2	$x \in (-90^\circ;\ 0^\circ)$ OR $-90^\circ < x < 0^\circ$ OR Between -90° and 0°	A extreme values A correct notation (2) A extreme values A correct notation (2) A extreme values A correct notation (2)
4.3	$f(2x) = cos2(2x) = cos\,4x$ Period $= 90^\circ$	A $cos\,4x$ A 90° (2)
4.4	$h(x) = 3cos\,2x - 1$ Minimum value $= -4$	A -4 (2)
4.5	Move 45° to the left and then reflect about the $x-axis$ Or The graph of g must moved 135° to the right.	A 45° left A reflection $x-axis$ (2) AA 135° right (2)
		[11]

Question 5:



	$\hat{B} = \hat{C} = \theta$ (∠'s opp. = sides) $\hat{A} = 180^\circ - 2\theta$ (sum ∠'s of Δ)	A statement & reason
	$BC^2 = AB^2 + AC^2 - 2AB.AC.cos\,\hat{A}$ $BC^2 = y^2 + y^2 - 2y.y.cos\,(180^\circ - 2\theta)$ $BC^2 = 2y^2 - 2.y^2(-cos\,2\theta)$ $BC^2 = 2y^2 + 2y^2.cos2\theta$ $BC^2 = 2y^2(1 + cos2\theta)$ $BC = y\sqrt{2(1+cos2\theta)}$	A subst in the correct formula A $(-cos\,2\theta)$ A simplify A common factor A square root [6]

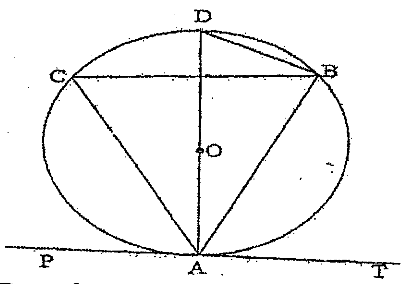
Question 6:

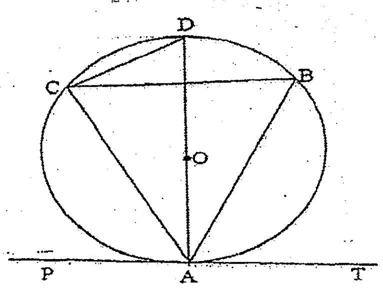
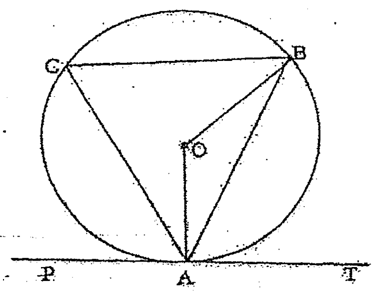
6	$\frac{r}{h} = \tan 36^\circ$ $r = 2\tan 36^\circ = 1,45m$ Slant height $\frac{S}{h} = \frac{1}{\cos 36^\circ}$ $S = \frac{2}{\cos 36^\circ} = 2,47m$ $SA = \pi(2\tan 36^\circ)^2 + \pi(2\tan 36^\circ)(\frac{2}{\cos 36^\circ})$ $SA = 17,92\ m^2$ OR Surface area of cone = area of base + area of curved surface $= \pi r^2 + \pi rS$ $= \pi(1,45)^2 + \pi(1,45)(2,47)$ $= 17,86\ m^2$	A $\frac{r}{h} = \tan 36^\circ$ A $r = 2\tan 36^\circ = 1,45m$ A $\frac{S}{h} = \frac{1}{\cos 36^\circ}$ A $S = \frac{2}{\cos 36^\circ} = 2,47m$ CA subst CA answer CA subst CA answer (6)
		[6]

Question 7:

7.1	Perpendicular	A Correct answer (1)
7.2.1	$A\hat{B}D = 90^\circ$ ($\angle$ *in semi circle* or diameter) $B\hat{A}D = 28^\circ$ ($\angle$'*s of* Δ) $B\hat{C}D = 152^\circ$ (opp $\angle$'s of cyclic quad)	A S/R CA S/R CA S - CA R (4)
7.2.2	$B\hat{D}C = 14^\circ$ ($\angle$'*s opp.= sides*, BC = CD) $\therefore C\hat{D}F = 14^\circ$ ($A\hat{D}F = 90^\circ$; *radius* $\perp$ *tan*) OR (tan & chord)	CA S - CA R CA S - CA R (4)
		[9]

Question 8:

8.1	Construction: draw diameter AD and join DB.  Proof: $B\hat{A}T + D\hat{A}B = 90^\circ$ (*tan* $\perp$ *radius*) $D\hat{B}C + C\hat{B}A = 90^\circ$ ($\angle$'*s in semi circle*) $D\hat{A}B + A\hat{D}B = 90^\circ$ ($\angle$'*s OF* Δ) $B\hat{A}T = A\hat{D}B$ $B\hat{C}A = A\hat{D}B$ ($\angle$'*s in same segment*) $B\hat{A}T = B\hat{C}A$	A construction A S - A R A S - A R A S/R (6)

OR Construction: draw diameter AD and join DC.	A construction
	
Proof:	
$B\hat{A}T + D\hat{A}B = 90^\circ$ (tan ⊥ radius)	A S - A R
$D\hat{C}B + B\hat{C}A = 90^\circ$ ($\angle$'s in semi circle)	A S - A R
$B\hat{C}A = 90^\circ - D\hat{C}B$	
$D\hat{A}B = 90^\circ - B\hat{A}T$	
$D\hat{C}B = D\hat{A}B$ ($\angle$'s in same segment)	A S/R
$B\hat{A}T = B\hat{C}A$	(6)
OR Construction: Draw radii OA and OB.	A construction
	
Proof:	
$O\hat{A}B + B\hat{A}T = 90^\circ$ (tan ⊥ radius)	A S - A R
$O\hat{A}B = 90^\circ - B\hat{A}T$	
$O\hat{B}A = 90^\circ - B\hat{A}T$ ($\angle$'s opp. = sides)	A S
$A\hat{O}B = 180^\circ - 2(90^\circ - B\hat{A}T)$ ($\angle$'s OF Δ)	A S/R
$A\hat{O}B = 2B\hat{A}T$	A S
$A\hat{O}B = 2\hat{C}$ ($\angle$ at centre = 2 × $\angle$ at circumference)	A S/R
$B\hat{A}T = B\hat{C}A$	(6)

8.2.1	(a) Tan chord theorem	A R	(1)
	(b) $\angle$'s same segment	A R	(1)
8.2.2	$\hat{R}_1 = \hat{P}_2 + \hat{T}$ (ext $\angle$ of Δ) $\hat{P}_2 = \hat{Q}_2$ (proven – from 8.2.1(b)) $\hat{Q}_1 = \hat{T}$ (proven – from 8.2.1(a)) $\hat{Q}_1 + \hat{Q}_2 = \hat{P}_2 + \hat{T}$ $\hat{Q}_1 + \hat{Q}_2 = \hat{R}_1$ $\therefore PQ = PR$ (sides opp = $\angle$'s) $\therefore$ ΔPQR is isosceles triangle	A S A S A S A S/R	(4)
8.2.3	$\hat{R}_2 = \hat{Q}_1$ ($\angle$'s in same segment) $\hat{T} = \hat{Q}_1$ (proven – from 8.2.1(a)) $\hat{R}_2 = \hat{T}$ PR is a tangent to circle RST at R (converse tan chord th,) OR $\hat{P}_1 = 180^\circ - (\hat{Q}_1 + \hat{Q}_2 + \hat{R}_1)$ ($\angle$'s OF Δ) $\hat{R}_2 = \hat{Q}_1$ ($\angle$'s in same segment) $\hat{Q}_1 = \hat{T}$ ((proven – from 8.2.1(a)) $\therefore$ $\hat{R}_2 = \hat{T}$	A S/R A S A R A S A S/R A R	(3) (3)
			[15]

TOTAL [100]