KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P2

NOVEMBER 2023

NAME: **CLASS:**

MARKS: 150 **TIME:** 3 hours

For educator use only.												
Question	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Marks obtained												
Question total	10	12	18	13	25	11	10	6	11	13	21	150

This question paper consists of 24 pages, including pages with additional space and a formula sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.

2. Answer **ALL** questions.

3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

6. If necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Write your answers in the spaces provided.

QUESTION 1

Aphelele plays for his school's cricket team. The number of runs scored by Aphelele in each of the eight games that he batted in, is shown below. (Aphelele was given out in all of the games.)

21	**8**	**19**	**7**	**15**	**32**	**14**	**12**

1.1 Determine the average number of runs per game scored by Aphelele for these eight games. (2)

1.2 Determine the standard deviation for this data set. (1)

1.3 Aphelele wants to be selected for the District trials. The condition is that he needs to have at least one score above two standard deviations from the mean during the first eight games. Will he be selected for the District trials? Motivate your answer with relevant calculations. (2)

1.4 Aphelele hopes to achieve an average of 20 runs per game for his first 13 games. What should his average number of runs per game be for the last five of these games in order for him to reach this goal? (3)

1.5 Brian plays in the same team as Aphelele. After having played a few matches, he has an average batting score of 37 runs per game, and the standard deviation of his scores is 13. In the next two matches that he played, he scored respectively 20 runs and 0 runs. Will the inclusion of these two scores cause the standard deviation of his scores to increase or decrease? Justify your answer. (2)

[10]

QUESTION 2

The table below shows the height (in cm) of 250 Grade 11 learners.

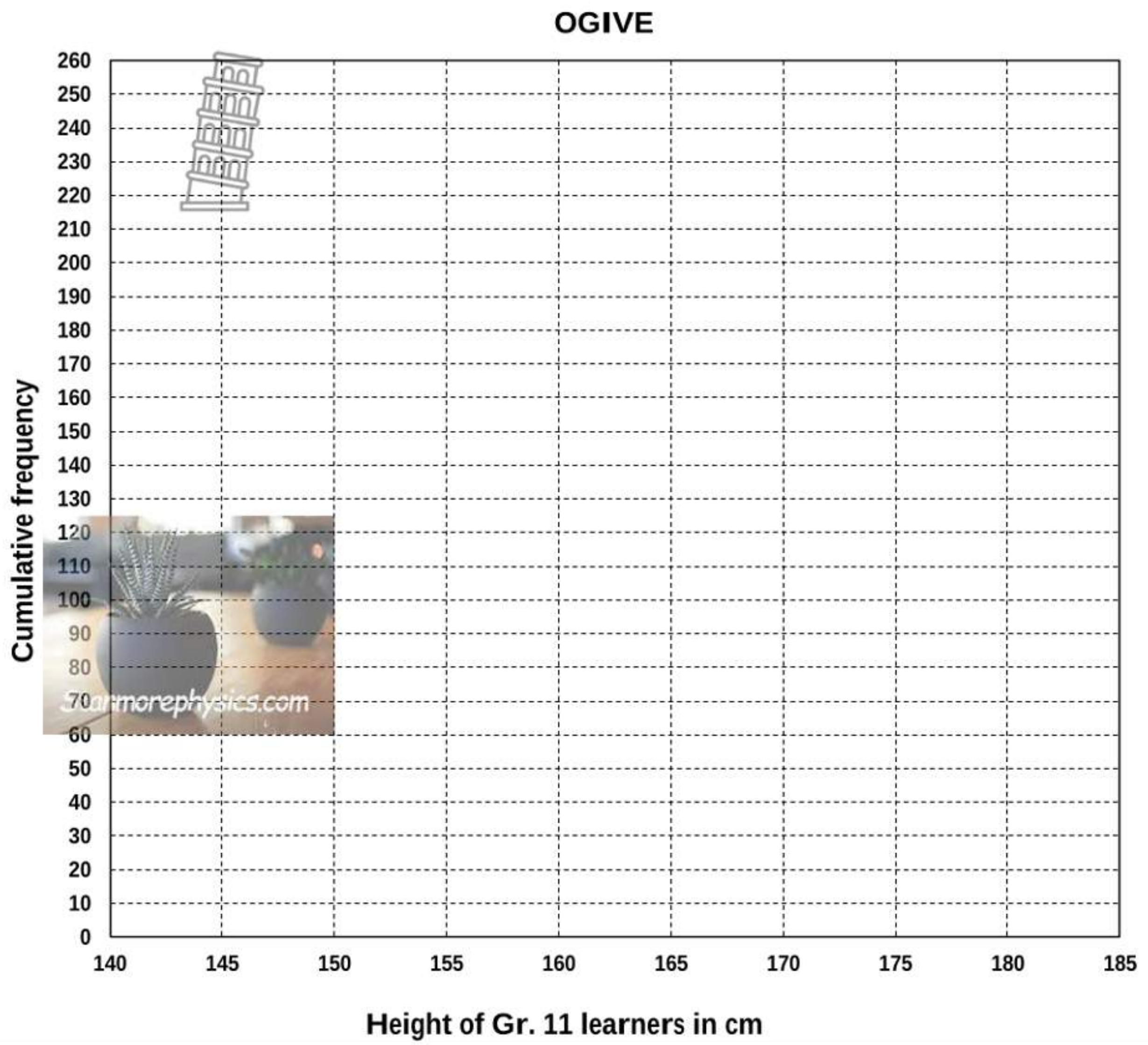
Height (cm)	Number of learners (f)	Cumulative frequency		
$145 < x \leq 150$	26	6		
$150 < x \leq 155$	**A**	29		
$155 < x \leq 160$	60	89		
$160 < x \leq 165$	74	**B**		
$165 < x \leq 170$	52	215		
$170 < x \leq 175$	32	247		
$175 < x \leq 180$	3	250		
Total	**250**			

2.1 Calculate the values of A and B. (2)

2.2 Calculate the estimated mean height of the learners. (3)

2.3 Write down the modal class. (1)

2.4 On the grid provided below, draw a cumulative frequency graph (ogive) to represent the data on the height of the Gr. 11 learners. (3)



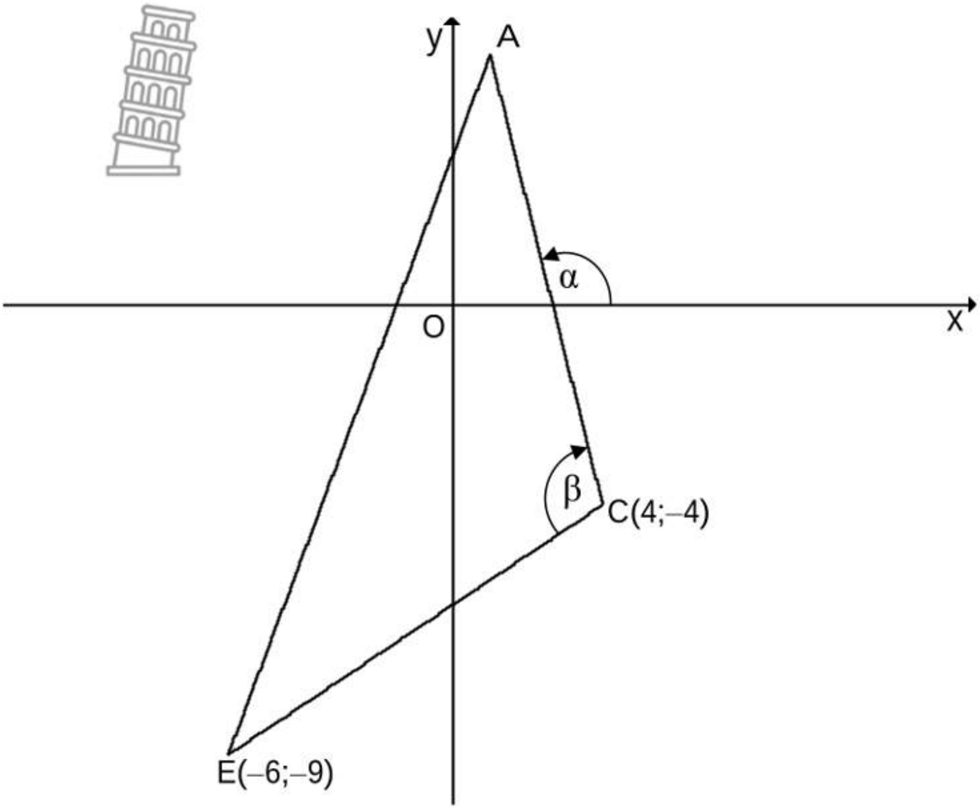
2.5 Use the ogive to estimate the interquartile range of the data. Show all your calculations. (3)

[12]

QUESTION 3

In the diagram below, the vertices of ΔABC are A, $C(4\,;-4)$ and $E(-6\,;-9)$.

The angle of inclination of AC is α. $A\hat{C}E = \beta$. The equation of AC is $3x + y - 8 = 0$.



3.1 Write down the gradient of AC. (2)

3.2 Calculate the size of α. (2)

3.3 Calculate the size of β. (5)

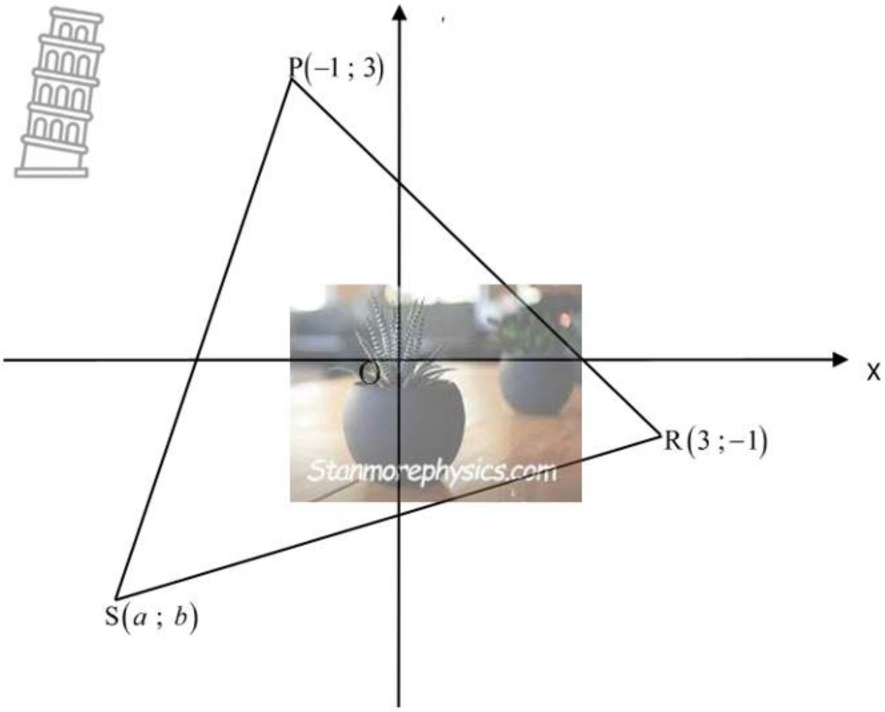
3.4 Determine the equation of EF if F is a point on AC produced such that $E\hat{F}A = 90^\circ$. (4)

3.5 Calculate the length of EF. (5)

[18]

QUESTION 4

Triangle PRS has vertices $P(-1;3)$, $R(3;-1)$ and $S(a;b)$, as shown in the sketch below.



4.1 Calculate the coordinates of T, the midpoint of PR. (2)

4.2 If the perpendicular bisector of PR passes through S, show that $a = b$. (4)

4.3 If $a < 0, b < 0$ and the area of $\Delta PRS = 12$ square units, determine the coordinates of S. (7)

[13]

QUESTION 5

5.1 If $\sin 33° = m$, determine the following, in terms of m, without the use of a calculator:

5.1.1 $\tan 33°$ (2)

5.1.2 $\cos 777°$ (3)

5.1.3 $\sin(-237°)$ (3)

5.2 Given that $4\tan\beta + 5 = 0$ and $\beta \in [0°\,;180°]$. Determine, with the aid of a diagram, and **without the use of a calculator**, the value of $\sqrt{41}\cos\beta$. (4)

5.3 5.3.1 Simplify the following expression to a single trigonometric ratio:

$$\frac{\sin(180° - \beta).\cos(90° - \beta) - 1}{\cos(-\beta)}$$

(4)

5.3.2 Hence, determine for which value(s) of β, where $\beta \in [0°;360°]$,

$$\frac{\sin(180° - \beta).\cos(90° - \beta) - 1}{\cos(-\beta)}$$ will be undefined. (3)

5.4 5.4.1 Prove the following identity:

$$\tan\alpha.\sin\alpha + \cos\alpha = \frac{1}{\cos\alpha}$$ (2)

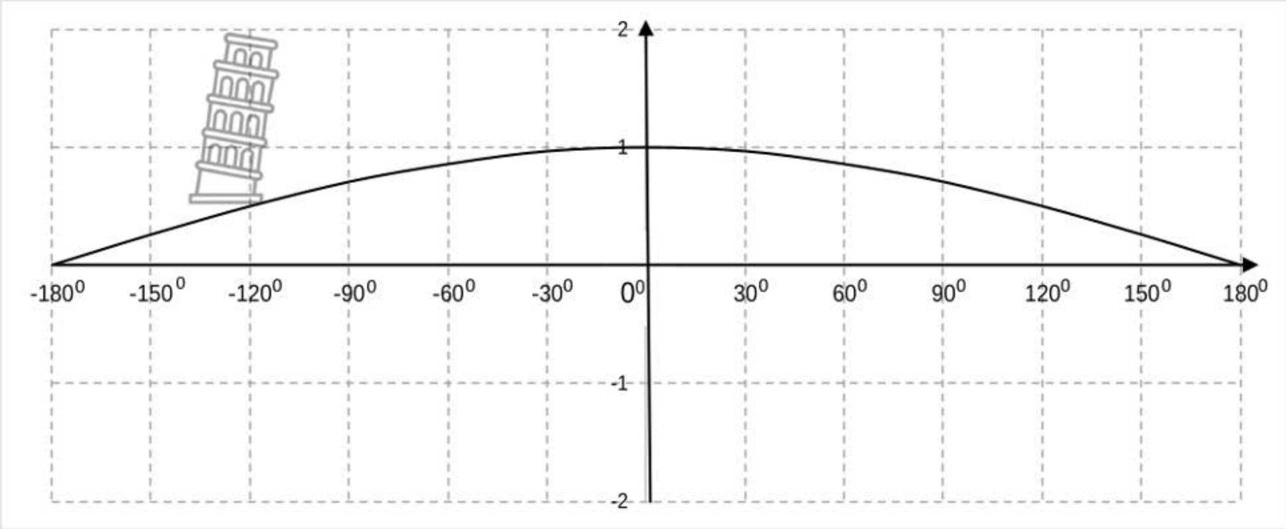
5.4.2 Hence determine the general solution of:

$$\tan\alpha.\sin\alpha + \cos\alpha = \frac{3}{\sin\alpha}$$ (4)

[25]

QUESTION 6

In the diagram, the graph of the function $f(x)=\cos\left(\frac{x}{2}\right)$ is drawn for the interval $x \in [-180°;180°]$.



6.1 Write down:

6.1.1 the amplitude of f. (1)

6.1.2 the period of f. (1)

6.2 Draw the graph of $g(x)=\sin(x-30°)$ for the interval $x \in [-180°;180°]$ on the axes provided above. Clearly indicate all intercepts with the axes and the turning point(s). (3)

6.3 Write down the values of x in the interval $x \in [-180°;180°]$, for which $f(x).g(x) \geq 0$. (2)

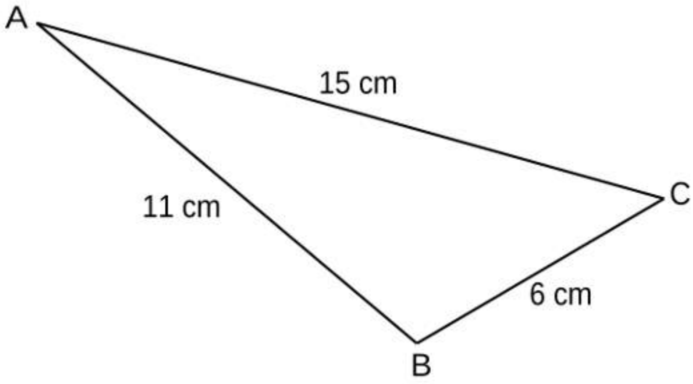
6.4 Write down the values of x in the interval $x \in [0°;180°]$, for which $g(x)=f(x)+\frac{1}{2}$. (2)

6.5 The graph of h is obtained by reflecting the graph of g in the y-axis. Write down the equation of h. (2)

[11]

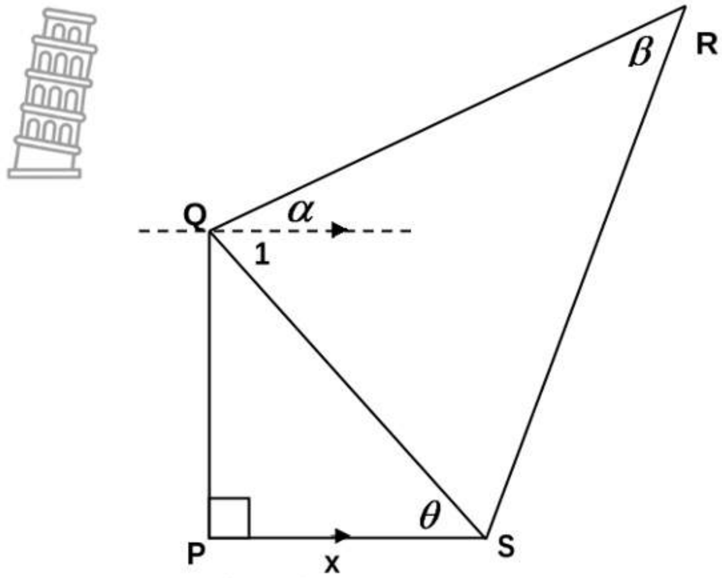
QUESTION 7

7.1 ΔABC has AB = 11cm, BC = 6 cm and AC = 15 cm.



Calculate the size of $\hat{B}$. (4)

7.2 In the diagram below, P, Q, R and S are points in the same vertical plane.
The angle of elevation of R from Q is α and the angle of elevation of Q from S is θ.
PS = x units and $Q\hat{R}S = \beta$.



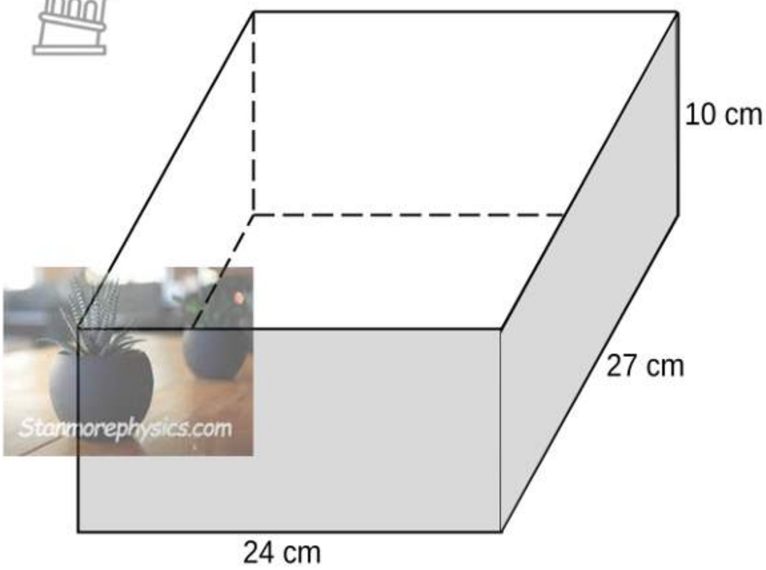
Prove that: $SR = \dfrac{x\sin(\alpha+\theta)}{\cos\theta\sin\beta}$ (6)

[10]

QUESTION 8

The diagram below shows an open rectangular box, made out of sheet metal.
Its measurements are as follows:

- length = 27 cm
- breadth = 24 cm
- height = 10 cm.



8.1 Calculate the total surface area of sheet metal needed to manufacture this open box. (3)

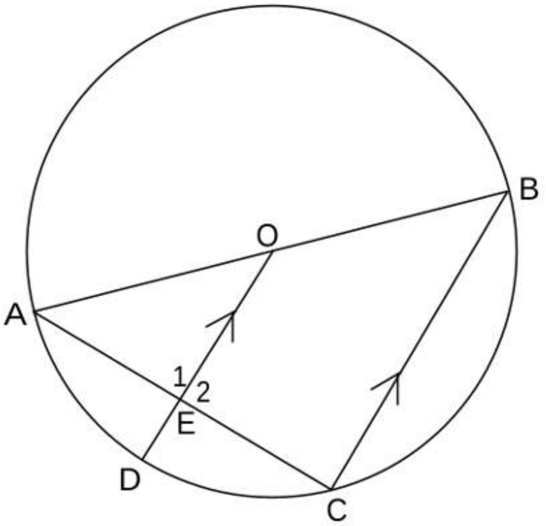
8.2 4,5 liter water is poured into this box. Calculate the depth of the water in the box.
Take note: 1 liter = 1000 cm^3. (3)

[6]

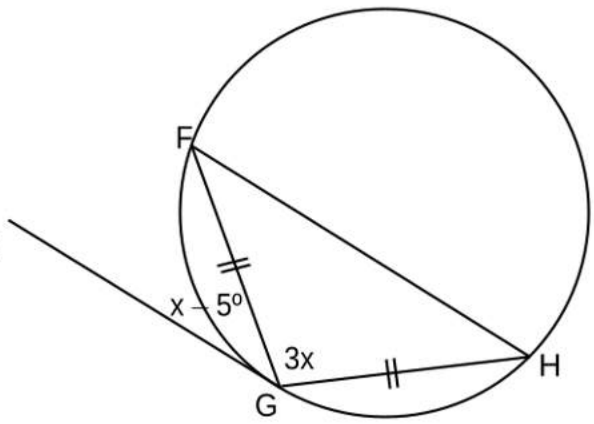
Give reasons for ALL statements in QUESTIONS 9, 10 and 11.

QUESTION 9

9.1 O is the centre of the circle. A, B, C and D are points on the circumference. AOB is a straight line. AC and BC are drawn. OD is drawn parallel to BC, and intersects AC in E. The radius of the circle is 10 cm, and AC = 12 cm.
Calculate the length of ED. (6)



9.2 In the diagram below, JG is a tangent to circle FGH at G. FG, GH and FH are drawn. $J\hat{G}F = x - 5^\circ$ and $F\hat{G}H = 3x$.

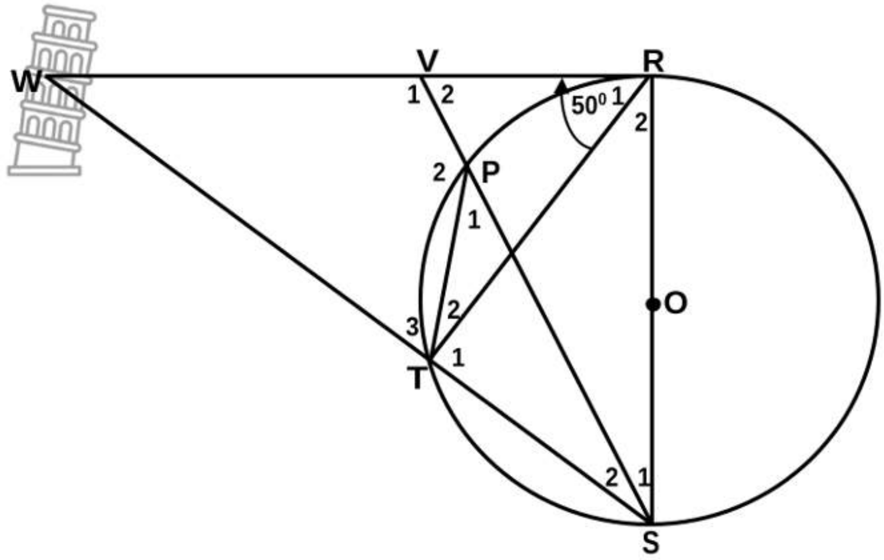


Calculate the value of x. (5)

[11]

QUESTION 10

In the diagram below, RS is a diameter of the circle centred at O. Chord ST is produced to W. Chord SP produced meets the tangent RW at V. $\hat{R}_1 = 50°$.



10.1 Calculate the sizes of the following angles.

10.1.1 $\hat{R}_2$ (3)

10.1.2 $\hat{W}$ (3)

10.1.3 $\hat{P}_1$ (2)

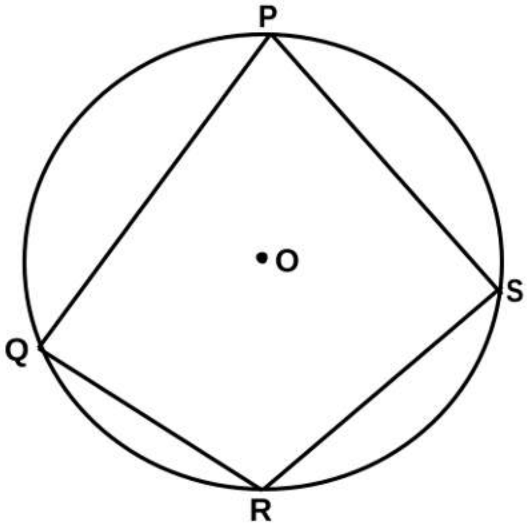
10.2 Prove that $\hat{V}_1 = P\hat{T}S$ (4)

10.3 Hence, prove that WVPT is a cyclic quadrilateral. (1)

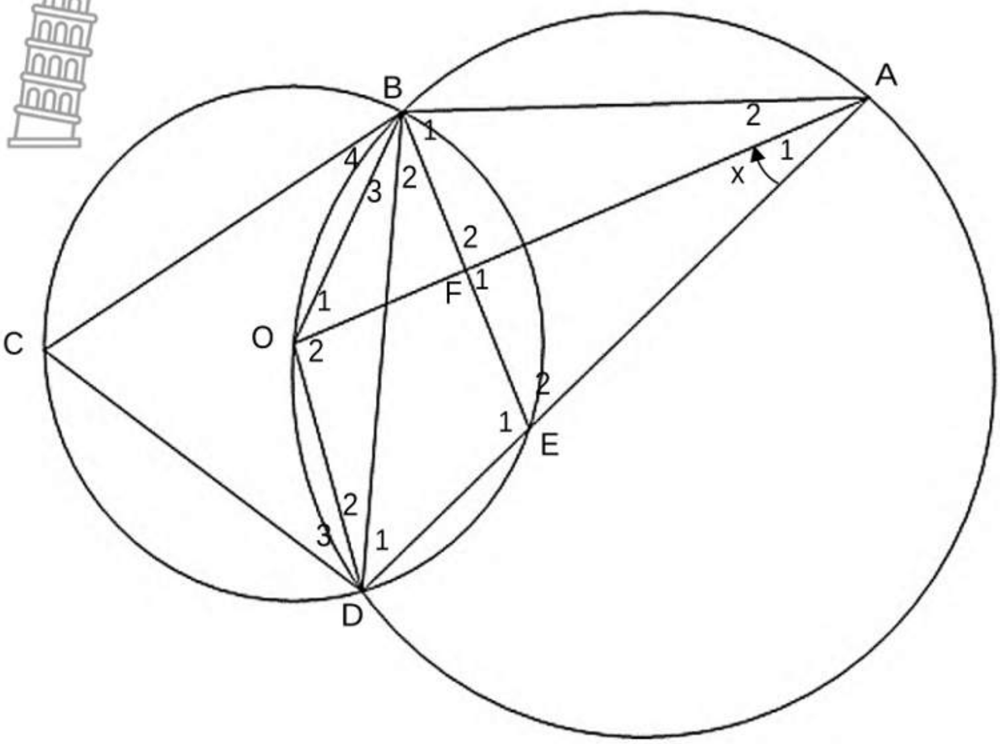
[13]

QUESTION 11

11.1 Use the diagram to prove the theorem which states that $\hat{P} + \hat{R} = 180^\circ$ (5)



11.2 In the diagram two circles are intersecting at B and D. O is the centre of the smaller circle. B, C, D and E are points on the circumference of circle O. A is a point on the circumference of the bigger circle. DEA is a straight line. OB, OD, OA, BA, BD and BE are drawn. OA and BE intersect at F. $\hat{A}_1 = x$.



11.2.1 Name, with reasons, THREE other angles equal to x. (4)

11.2.2 Calculate $\hat{C}$ in terms of x. (4)

11.2.3 Prove that AB = AE. (5)

11.2.4 Prove that AB is **NOT** a tangent to circle BCDE. (3)

[21]

TOTAL MARKS: 150

Additional space

Additional space

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

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MATHEMATICS P2

NOVEMBER 2023

MARKING GUIDELINE

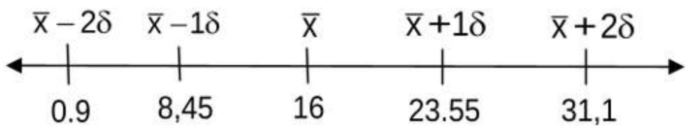
NATIONAL
SENIOR CERTIFICATE

GRADE 11

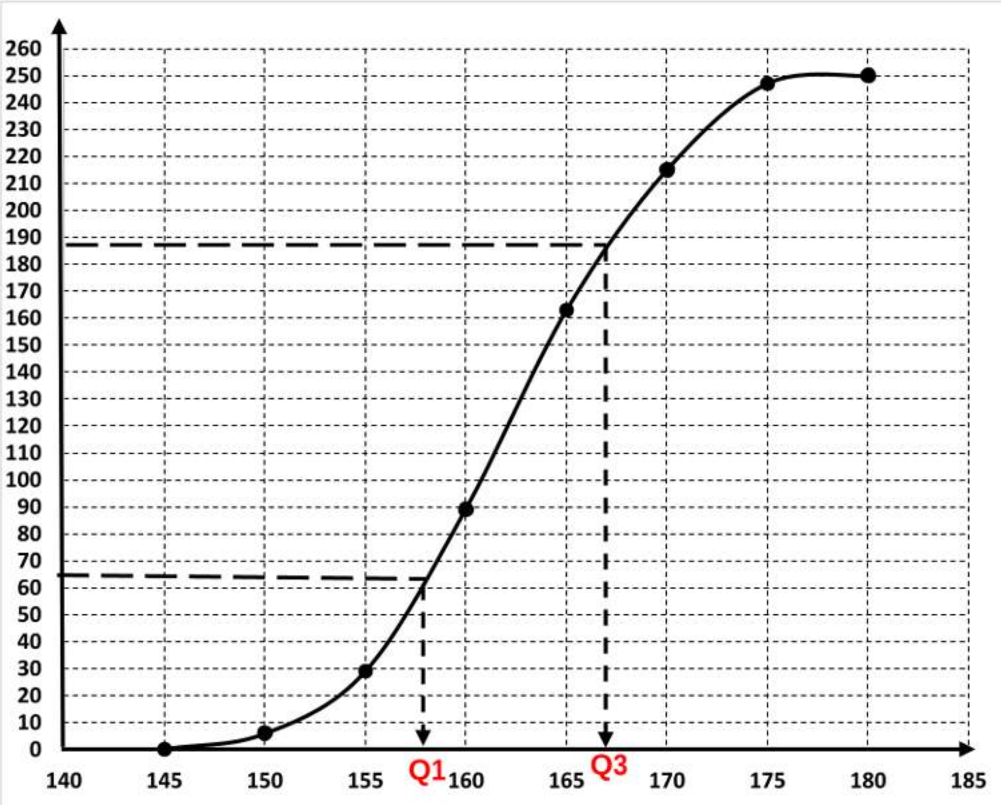
MARKS: 150

This marking guideline consists of 14 pages.

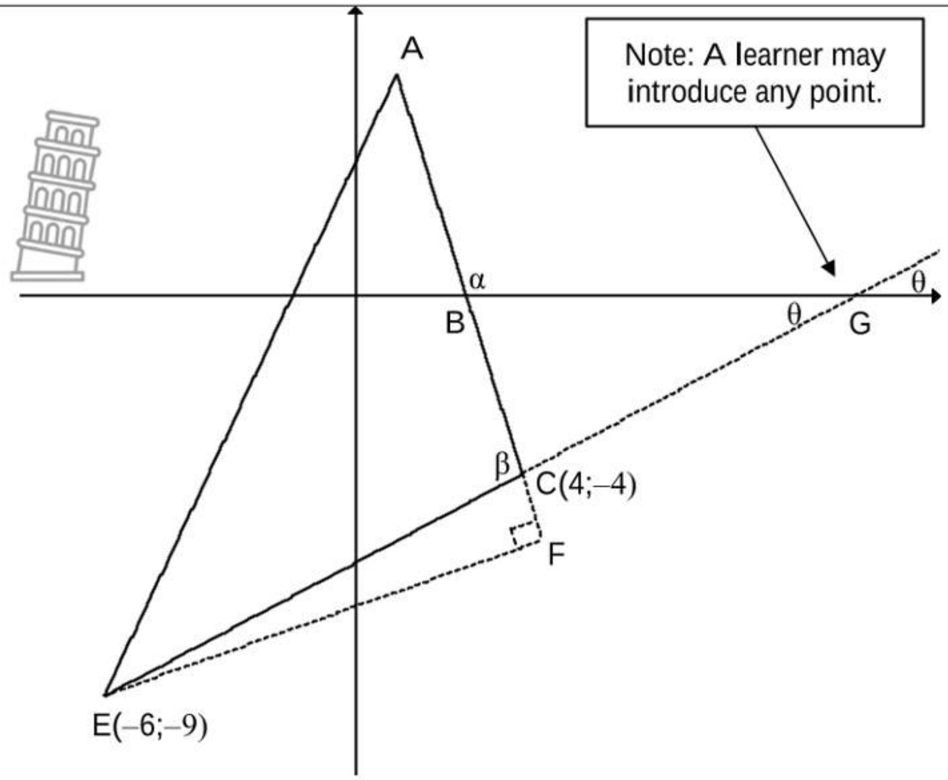
QUESTION 1

1.1	$\bar{x} = \frac{21+8+19+7+15+32+14+12}{8}$ $= \frac{128}{8}$ $=16$	✓ $\frac{128}{8}$ ✓ answer (2)
1.2	$\delta = 7.55$	✓ answer (1)
1.3	Two standard deviation above the mean:  $\bar{x} + 2\delta = 16 + 2 \times 7.55 = 31{,}1$ Yes, he will make it to the trials, he has one score (32) that is more than two standard deviations above the mean	✓31,1 ✓yes and motivation (2)
1.4	Total number of runs in 8 games = 128 Total number of runs in 13 games $= 13 \times 20 = 260$ Average in last 5 games = $\frac{260-128}{5}$ $= 26{,}4$ His average must be 26,4 or approximately 26 runs in his last 5 games.	✓ 260 ✓ $\frac{260-128}{5}$ ✓ 26,4 or 26 (3)
1.5	Increase. Both 0 and 20 are further than one standard deviation from the mean, and will therefore cause an increase in standard deviation.	✓ increase ✓ both 0 and 20 are further than one standard deviation from the mean (2)
		[10]

QUESTION 2

2.1	$A = 29 - 6 = 23$ $B = 89 + 74 = 163$	✓ A = 23 ✓ B = 163 (2)
2.2	Estimated mean $= \frac{40630}{250}$ $= 162{,}52$ cm	✓ 40630 ✓ 250 ✓answer (3)
2.3	$160 < x \leq 165$	✓ answer (1)
2.4		✓A all points correctly plotted ✓grounding ✓shape (3)
2.5	Position Q1: $\frac{1}{4}(250) = 63^{rd}$ score Position Q3: $\frac{3}{4}(250) = 188^{th}$ score $Q_1 = 158$ $Q_3 = 167$ $\therefore$ IQR $= 167 - 158$ $= 9$ **Note:** **Q_1 accept 157 – 159** **Q_3 accept 166 – 168** **IQR accept 7, 8, 9, 10 and 11**	✓Q_1 ✓Q_3 ✓answer (3)
		[12]

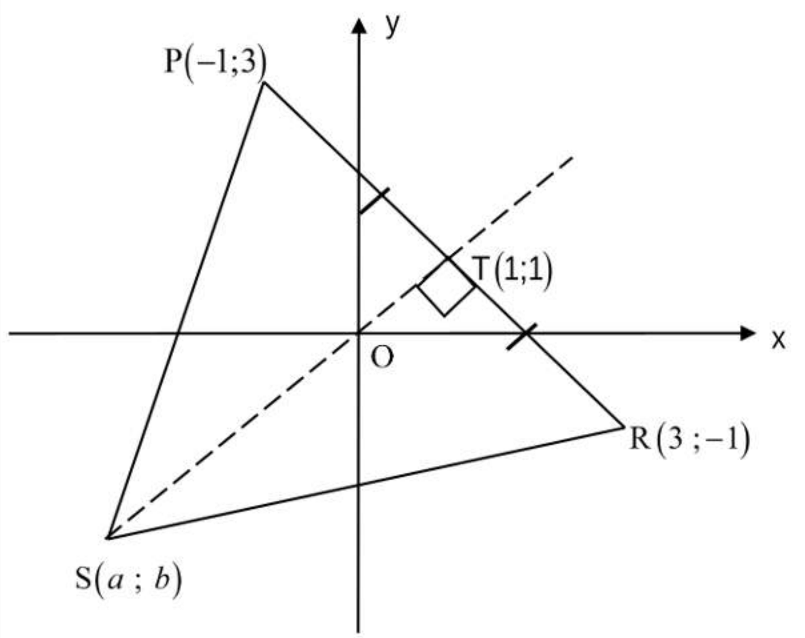
QUESTION 3



3.1	$y=-3x+8$ $\therefore m_{AC}=-3$	✓ answer (2)
3.2	$\tan\alpha=m_{AC}=-3$ $\alpha=180^\circ-71,57^\circ$ $=108,43^\circ$	✓ $\tan\alpha=m_{AC}$ ✓answer (2)
3.3	$m_{EC}=\dfrac{-4-(-9)}{4-(-6)}$ $=\dfrac{1}{2}$ Let θ be the angle of inclination of EC. $\tan\theta=\dfrac{1}{2}$ $\theta=26,57^\circ$ $O\hat{G}C=\theta=26,57^\circ$ [vertically opp. $\angle$s] $C\hat{B}G=180^\circ-\alpha$ [$\angle$s on a straight line] $=180^\circ-108,43^\circ=71,57^\circ$ $\beta=C\hat{B}G+O\hat{G}C=71,57^\circ+26,57^\circ$ [ext. $\angle$ of ΔBCG] $=98,14^\circ$	✓ substitution ✓ value of m_{EC} ✓ $26,57^\circ$ ✓ $180^\circ-108,43^\circ=71,57^\circ$ ✓answer (5)

3.4	$m_{AC} = -3$ $\therefore m_{EF} = \frac{1}{3}$ [AC ⊥ EF] Equation of EF: $y = \frac{1}{3}x + c$ Substitute $E(-6;-9)$: $-9 = \frac{1}{3}(-6) + c$ $c = -7$ $y = \frac{1}{3}x - 7$	✓ $m_{EF} = \frac{1}{3}$ ✓ substitution ✓ value of c ✓ answer (4)
3.5	To determine coordinates of F: $-3x + 8 = \frac{1}{3}x - 7$ $\frac{10}{3}x = 15$ $x = 4,5$ $y = -3x + 8 = -3(4,5) + 8 = -5,5$ $EF = \sqrt{[4,5-(-6)]^2 + [-5,5-(-9)]^2}$ $= 11,07\text{cm}$	✓ equating the equations of EF and AF ✓ x-coordinate of F ✓ y-coordinate of F ✓ substitution in distance formula ✓ answer (5)
		[18]

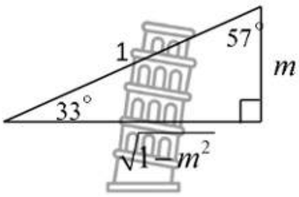
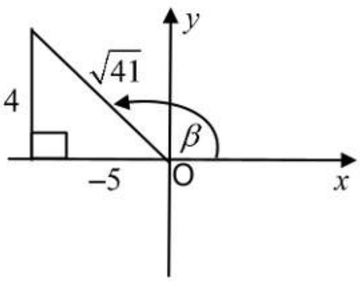
QUESTION 4



4.1	$T\left(\frac{3-1}{2};\frac{-1+3}{2}\right)$ $T(1;1)$	✓ substitution in midpt. formula ✓answer (2)

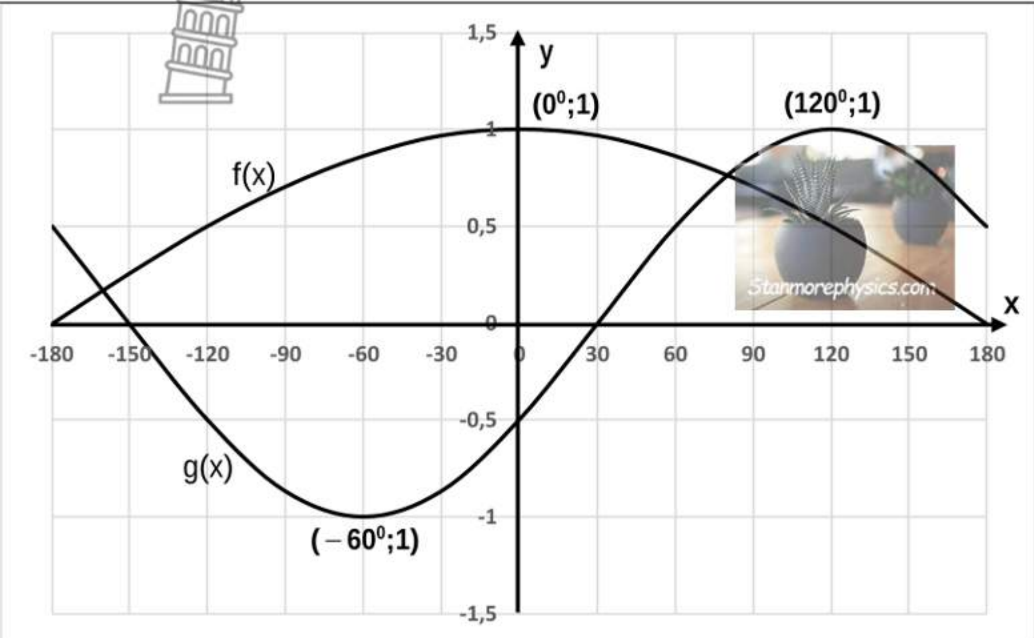
4.2	$m_{PR}=\frac{3+1}{-1-3}$ $=-1$ $\therefore m_{\perp\,bisector}=1$ Subst. $T(1;1)$ and $m=1$ in $y=mx+c$: $1=1+c$ $c=0$ $y=x$ $\therefore a=b$	✓ gradient of PR ✓ gradient of $\perp$ bisector ✓ subst. S(a ; b) and m = 1 ✓ $y=x$ (4)
4.3	$PR=\sqrt{(-1-3)^2+(3+1)^2}$ $=4\sqrt{2}$ $ST=\sqrt{(a-1)^2+(b-1)^2}$ But $a=b$, $\therefore ST=\sqrt{(a-1)^2+(a-1)^2}$ $ST=\sqrt{2(a-1)^2}$ Area $\Delta PSR=12$ square units $\therefore \frac{1}{2}\times \text{base}\times \text{height}=12$ $\frac{1}{2}\times PR\times ST=12$ $\frac{1}{2}\left(4\sqrt{2}\right)\left(\sqrt{2(a-1)^2}\right)=12$ $\frac{1}{2}\times 4\sqrt{2}\times\sqrt{2}\times\sqrt{(a-1)^2}=12$ $\sqrt{(a-1)^2}=3$ $(a-1)^2=9$ $a-1=\pm 3$ $a=4$ (N/A) or $a=-2$ $\therefore b=-2$ $\therefore S(-2;-2)$	✓ length of PR ✓length of ST i.t.o. a and b ✓length of ST i.t.o. a (or b) only ✓$\frac{1}{2}\times \text{base}\times \text{height}=12$ ✓substitution of PR and ST ✓ values of a and rejection ✓answer (7)
		[13]

QUESTION 5

5.1.1	 $\tan 33° = \dfrac{m}{\sqrt{1-m^2}}$	✓ $\sqrt{1-m^2}$ ✓ answer (2)
5.1.2	$\cos 777° = \cos\left[2(360°)+57°\right]$ $= \cos 57°$ $= \sin 33°$ $= m$	✓ $\cos\left[2(360°)+57°\right]$ ✓ $\cos 57°$ ✓ answer (3)
5.1.3	$\sin(-237°) = -\sin 237°$ $= -(-\sin 57°)$ $= \sqrt{1-m^2}$ **OR** $\sin(-237°) = \sin 123°$ $= \sin 57°$ $= \sqrt{1-m^2}$	✓ $-\sin 237°$ ✓ $-\sin 57°$ ✓ answer (3) **OR** ✓ $\sin 123°$ ✓ $\sin 57°$ ✓ answer (3)
5.2.	$4\tan\beta + 5 = 0$ $\tan\beta = -\dfrac{5}{4}$  $r^2 = x^2 + y^2$ $= (-5)^2 + (4)^2$ $r = \sqrt{41}$ $\sqrt{41}\cos\beta = \dfrac{\sqrt{41}}{1} \times \dfrac{-4}{\sqrt{41}}$ $= -4$	✓ $\tan\beta = -\dfrac{5}{4}$ ✓ $r = \sqrt{41}$ ✓ value of $\cos\beta$ ✓ answer (4)

5.3.1	$\dfrac{\sin(180^\circ-\beta).\cos(90^\circ-\beta)-1}{\cos(-\beta)}$ $=\dfrac{\sin\beta.\sin\beta-1}{\cos\beta}$ $=\dfrac{\sin^2\beta-1}{\cos\beta}$ $=\dfrac{-(1-\sin^2\beta)}{\cos\beta}$ $=\dfrac{-\cos^2\beta}{\cos\beta}$ $=-\cos\beta$	✓ simplification (numerator) ✓ simplification (denominator) ✓ use of square identity ✓ answer (4)
5.3.2	If $\cos(-\beta)=0$ $\cos\beta=0$ $\beta=90^\circ$ or $\beta=270^\circ$	✓ $\cos(-\beta)=0$ ✓ 90°; ✓ 270° (3)
5.4.1	$\tan\alpha.\sin\alpha+\cos\alpha$ $=\dfrac{\sin\alpha}{\cos\alpha}.\sin\alpha+\cos\alpha$ $=\dfrac{\sin^2\alpha}{\cos\alpha}+\cos\alpha$ $=\dfrac{\sin^2\alpha+\cos^2\alpha}{\cos\alpha}$ $=\dfrac{1}{\cos\alpha}$	✓ $\dfrac{\sin\alpha}{\cos\alpha}$ ✓ simplification (2)
5.4.2	$\dfrac{1}{\cos\alpha}=\dfrac{3}{\sin\alpha}$ $\sin\alpha=3\cos\alpha$ $\tan\alpha=3$ ref $\angle=\tan^{-1}(3)$ $=71.57^\circ$ $\alpha=71.57^\circ+k.180,\ k\in\mathbb{Z}$	✓ $\dfrac{1}{\cos\alpha}=\dfrac{3}{\sin\alpha}$ ✓ $\tan\alpha=3$ ✓ $\alpha=71.57^\circ+k.180^\circ$ ✓ $k\in\mathbb{Z}$ (4)
		[25]

QUESTION 6

6.1.1	1	✓ answer (1)
6.1.2	period = $720°$	✓ answer (1)
6.2		✓ Shape ✓ x &y - intercepts ✓ turning points (3)
6.3	$-180° \le x < -150°$ or $30° \le x < 180°$	✓ $-180° \le x < -150°$ ✓ $30° \le x < 180°$ (2)
6.4	$x = 120°$ or $x = 180°$	✓ $120°$ ✓ $180°$ (2)
6.5	$h(x) = \sin(-x - 30°)$ $= -\sin(x + 30°)$	✓✓ $\sin(-x - 30°)$ OR $\sin(x + 30°)$ (2)
		[11]

QUESTION 7

7.1	$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos\hat{B}$ $15^2 = 11^2 + 6^2 - 2(11)(6)\cos\hat{B}$ $\cos\hat{B} = \frac{11^2 + 6^2 - 15^2}{2(11)(6)}$ $= -\frac{17}{33}$ OR $-0,5151.....$ $\hat{B} = 180° - 58,99°$ $= 121,01°$	✓ applying the cosine rule ✓ substitution ✓ value of $\cos\hat{B}$ ✓ answer (4)

7.2	In ΔQRS: $Q_1 = Q\hat{S}P = \theta$ [alt ∠'s =] $\therefore Q\hat{S}R = (\alpha + \theta)$ $\frac{SR}{\sin(\alpha+\theta)} = \frac{QS}{\sin\beta}$ $SR = \frac{QS.\sin(\alpha+\theta)}{\sin\beta}$ In ΔQPS: $\cos\theta = \frac{PS}{QS}$ $\cos\theta = \frac{x}{QS}$ $QS = \frac{x}{\cos\theta}$ $\therefore SR = \frac{x}{\cos\theta} \times \frac{\sin(\alpha+\theta)}{\sin\beta}$ $SR = \frac{x\sin(\alpha+\beta)}{\cos\theta\sin\beta}$	✓S ✓substitution in sine rule ✓ $SR = \frac{QS.\sin(\alpha+\theta)}{\sin\beta}$ ✓correct ratio ✓ $QS = \frac{x}{\cos\theta}$ ✓substitution of QS (6)
		[10]

QUESTION 8

8.1	Total surface area $= (\text{length} \times \text{breadth}) + 2(\text{length} \times \text{height}) + 2(\text{breadth} \times \text{height})$ $= (27 \times 24) + 2(27 \times 10) + 2(24 \times 10)$ $= 1668 \text{ cm}^2$	✓ formula for surface area ✓ substitution ✓ answer (3)
8.2	$\text{Volume} = \text{length} \times \text{breadth} \times \text{height}$ $4500 \text{ cm}^3 = 27 \text{ cm} \times 24 \text{ cm} \times \text{height}$ $\text{height} = \frac{4500}{24 \times 27}$ $= 6{,}94 \text{ cm}$	✓ formula for volume ✓ substitution ✓ answer (3)
		[6]

QUESTION 9

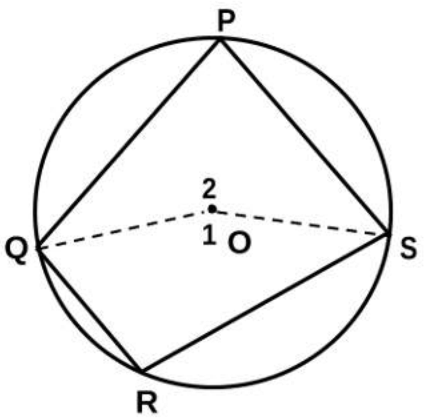
9.1	$\hat{C} = 90°$	[∠ in a semi-circle]	✓ S/R
	$\hat{E}_1 = \hat{C} = 90°$	[corr. ∠'s, OE‖BC]	✓ S/R
	$AE = \frac{1}{2}(AC)$	[line from centre ⊥ to chord]	✓ S/R
	$AE = \frac{1}{2}(12)$		
	$AE = 6$ cm		
	In ΔOAE:		
	$OA^2 = AE^2 + OE^2$	[Pythagoras]	✓ S/R
	$10^2 = 6^2 + OE^2$		
	$OE^2 = 64$		
	$OE = 8$ cm		✓S (length of OE)
	$ED = OD - OE$		
	$ED = 10 - 8$		
	$ED = 2$ cm		✓ answer
			(6)
9.2	$\hat{H} = F\hat{G}J$	[tan-chord-theorem]	✓ S ✓R
	$= x - 5°$		
	$\hat{H} = \hat{F}$	[∠s opposite = sides]	✓ S/R
	$= x - 5°$		
	$\hat{F} + F\hat{G}H + \hat{H} = 180°$	[sum of ∠s of Δ FGH]	✓ S
	$x - 5° + 3x + x - 5° = 180°$		
	$5x = 190°$		
	$x = 38°$		✓ answer
			(5)
			[11]

QUESTION 10

10.1.1	$\hat{R}_1 + \hat{R}_2 = 90°$	[radius ⊥ to tangent]	✓ S ✓ R
	$\hat{R}_2 = 40°$		✓ answer
			(3)
10.1.2	$\hat{S}_1 + \hat{S}_2 = 50°$	[tan chord theorem]	✓ S ✓ R
	$\hat{W} + W\hat{R}S + \hat{S} = 180°$	[sum of ∠s of Δ WSR]	
	$\hat{W} + 90° + 50° = 180°$		
	$\hat{W} = 40°$		✓answer (3)
	OR		**OR**
	$\hat{T}_1 = 90°$	[∠ in semi-circle]	✓ S ✓ R
	$\hat{W} = \hat{T}_1 - \hat{R}_1$	[exterior ∠ of Δ WTR]	
	$= 90° - 50° = 40°$		✓answer (3)

10.1.3	$\hat{P}_1 = \hat{R}_2 = 40^\circ$	[∠s in the same segment]	✓ S ✓ R (2)
10.2	$\hat{T}_1 = 90^\circ$ $P\hat{T}S = \hat{T}_1 + \hat{T}_2 = 90^\circ + T_2$ $\hat{V}_1 = W\hat{R}S + \hat{S}_1 = 90^\circ + \hat{S}_1$ But: $\hat{T}_2 = \hat{S}_1$ $\therefore \hat{V}_1 = P\hat{T}S$	[∠ in a semi-circle] [ext ∠ of Δ = sum of int opp ∠s] [∠s in the same segment]	✓ S/R ✓ S/R ✓ S ✓ R (4)
10.3	$\hat{V}_1 = P\hat{T}S$ ∴ WVPT is a cyclic quadrilateral	 [converse: exterior ∠ = int opp ∠]	 ✓ R (1)
			[13]

QUESTION 11

11.1			**Note:** **No construction: $\frac{0}{5}$**
	Construction: Draw OQ and OS $\hat{O}_1 = 2\hat{P}$ $\hat{O}_2 = 2\hat{R}$ $\hat{O}_1 + \hat{O}_2 = 360^\circ$ $\therefore 2\hat{P} + 2\hat{R} = 360^\circ$ $2(\hat{P} + \hat{R}) = 360^\circ$ $\therefore \hat{P} + \hat{R} = 180^\circ$	 [∠ @ centre = 2×∠ @ circumf.] [∠ @ centre = 2×∠ @ circumf.] [∠'s around a point]	✓ construction ✓ S/R ✓ S ✓ S/R ✓ S (5)
11.2.1	$\hat{B}_3 = \hat{A}_1 = x$ $\hat{D}_2 = \hat{B}_3 = x$ $\hat{A}_2 = \hat{D}_2 = x$ **OR** $\hat{B}_3 = \hat{A}_1 = x$ $\hat{D}_2 = \hat{B}_3 = x$ $\hat{A}_2 = \hat{A}_1 = x$	[∠s in the same segment] [∠s opp. = radii] [∠s in the same segment] [∠s in the same segment] [∠s opp. = radii] [equal chords; equal ∠s]	✓S ✓S ✓R ✓S/R (4) **OR** ✓S ✓S ✓R ✓S/R (4)

11.2.2	$\therefore B\hat{O}D = 180^\circ - 2x$ [sum of ∠s of Δ BOD] $\hat{C} = \frac{1}{2} B\hat{O}D$ [∠ at centre = 2× ∠ at circumference] $= 90^\circ - x$	✓S ✓R ✓R ✓answer (4)
11.2.3	$\hat{E}_2 = \hat{C} = 90^\circ - x$ [ext. ∠ of cyclic quadrilateral] $\hat{F}_2 = \hat{E}_2 + \hat{A}_1$ [ext ∠ of Δ FEA] $= (90^\circ - x) + x = 90^\circ$ $\therefore \hat{B}_1 = 180^\circ - (\hat{F}_2 + \hat{A}_2)$ [sum of ∠s of Δ BFA] $= 90^\circ - x$ ∴ AB = AE [sides opp. = ∠s] **OR** $\hat{E}_2 = \hat{C} = 90^\circ - x$ [ext. ∠ of cyclic quadrilateral] $\hat{F}_1 = 180^\circ - (\hat{E}_2 + \hat{A}_1)$ [sum of ∠s of Δ EFA] $= 90^\circ$ In Δ ABF and Δ AEF: 1. BF = FE [line from centre ⊥ to chord] 2. AF = AF [common] 3. $\hat{F}_2 = \hat{F}_1$ [∠s on a straight line] ∴ ΔABF ≡ ΔAEF [s ; ∠ ; s] ∴ AB = AE [≡ Δs]	✓S ✓R ✓S ✓S ✓R (5) **OR** ✓S ✓R ✓S ✓S/R ✓R (5)
11.2.4	$\hat{B}_1 + \hat{B}_3 = (90^\circ - x) + x = 90^\circ$ $\therefore O\hat{B}A = \hat{B}_1 + \hat{B}_2 + \hat{B}_3 = 90^\circ + \hat{B}_2 > 90^\circ$ For AB to be a tangent to circle BCDE, $O\hat{B}A$ should be equal to 90°. [converse: tangent ⊥ radius] ∴ AB is not a tangent to circle BCDE. **OR** For AB to be a tangent to circle BCDE, $D\hat{B}A$ should be equal to $\hat{C} = 90° - x$ [converse: tan-chord-theorem]. But: $D\hat{B}A = \hat{B}_1 + \hat{B}_2$ $= 90° - x + \hat{B}_2$ $= \hat{C} + \hat{B}_2$ $\therefore D\hat{B}A > \hat{C}$ ∴ AB is not a tangent to circle BCDE.	✓✓showing that $O\hat{B}A > 90^\circ$ ✓For AB to be a tangent, $O\hat{B}A$ should be = 90° (3) **OR** ✓ AB will be a tangent if $D\hat{B}A = \hat{C} = 90° - x$ ✓✓showing that $D\hat{B}A > \hat{C}$ (3)
		[21]

TOTAL: 150