***

THE CIRCLE

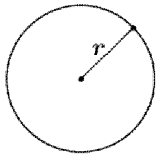
In grade 10 and 11 you were introduced to straight line analytic geometry.

This knowledge is still going to be tested in your grade 12 examination.

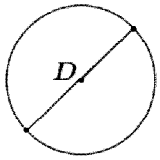
It is therefore important that you revise your work.

In grade 12 you will be introduced to **circle** analytic geometry.

Radius: any line segment that connects the centre of the circle and any point on the circumference.

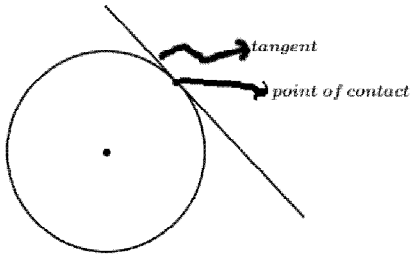


Diameter: any line segment that joins two points on the circumference **and** passes through the centre.



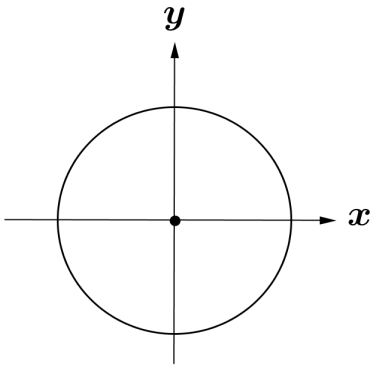
$D = 2r$

Tangent: Any line segment that touches a curve (in this case, a circle) externally ***at only one point***.



*** **CIRCLE WITH CENTRE AT THE ORIGIN (0 ; 0)**

Defining Equation: $\mathbf{x^2 + y^2 = r^2}$



NOTE: Knowledge of transformation geometry may be needed, especially "rotation about the origin through 90^o, 180^o and any angle"

*** **CIRCLE WITH ANY CENTRE (a ; b)**

Centre-radius form: $\mathbf{(x-a)^2 + (y-b)^2 = r^2}$

- (a ; b) is the centre of the circle.
- r is the radius.

General form: $\mathbf{x^2 + dx + y^2 + ey + f = 0}$

You can switch between these two forms by squaring a binomial and completing the square.

Centre-radius → general form: square the binomial.

*** General form → centre-radius: complete the square ***

Please make sure that you are able to switch from centre-radius to general form!!!!!!!

SCN1 **Determine the centre and the radius**

1. $x^2 + y^2 - 6x + 5y - 4 = 0$ R

2. $x^2 + 2x + y^2 - 10y + 1 = 0$

3. $x^2 + 8x + y^2 - 2y = 8$

4. $x^2 - 5x + y^2 + 6y = 9$

5. $2x^2 + 2y^2 - 6x - 12y = 3$

6. $x^2 + y^2 - 2x - y - 12 = 0$

SCN2 **Finding the equation of a circle**
Make sure that you know the following cases

1. **If you know the centre and one point on the circle.**

   1.1 Centre(-4 ; 2) point(-2 ; 6)

   1.2 Centre(-3 ; 1) point(2 ; -2)

2. **If you know the centre and the radius.**

   2.1 Centre(-2 ; -1) radius = 4

   2.2 Centre(3 ; -3) radius = $3\sqrt{5}$

3. **If you know the end-points of a diameter**

   3.1 A(2 ; -3) B(6 ; -1)

   3.2 E(-5 ; 3) F(-3 ; 5)

SCN3 **Show that the given line is a tangent to the circle**

1. $(x+4)^2+(y-4)^2=2$
   $y+x-2=0$

2. $x^2+y^2=10$
   $y=3x+10$

3. $(x-1)^2+(y-1)^2=8$
   $-x-y=2$

4. $x^2+y^2-6x-6y=-16$
   $x-y=2$

"the angle between a tangent and a radius is 90^o"
This means that their gradients are negative reciprocals of each other.

SCN4 Determine the equation of a tangent to the given circle. In each case, the point of contact is given.

1. $x^2+(y-3)^2=5$ at (1 ; 5)

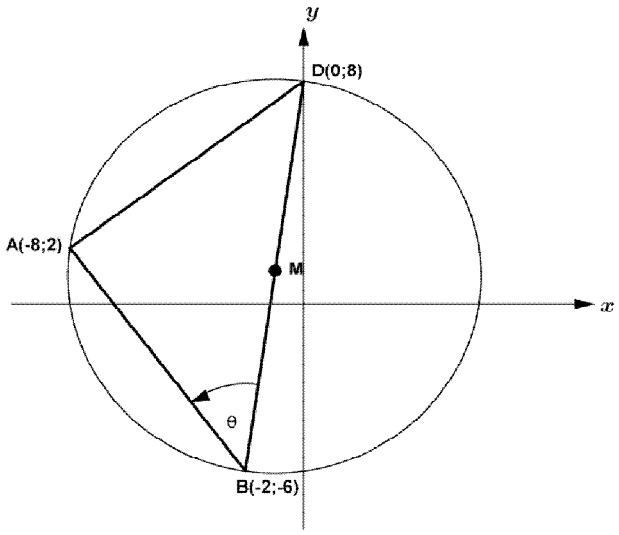
2. $x^2+y^2+2x-4y=0$ at (-2 ; 4)

3. $x^2+y^2+4x+2y=0$ at (0 ; -2)

4. $(x-5)^2+(y-1)^2=9$ at (2 ; 1)

You have now laid the foundation and ready to respond to higher-order questions.

SCN5



A, B and D are the vertices of a triangle that lies on the circumference of a circle with diameter BD and centre M as shown in the figure above.

1. Determine the coordinates of M.

2. Show that (-8 ; 2) lies on the line $\mathbf{y = 7x + 58}$

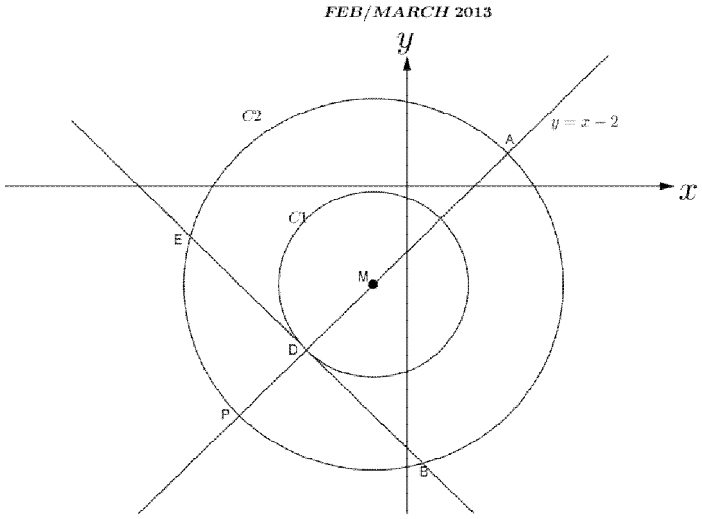
3. What is the relationship between $\mathbf{y = 7x + 58}$ and the circle centered at M? Motivate your answer.

4. Calculate the lengths of AD and AB.

5. Prove that $\angle DAB = 90^{o}$

6. Write down the size of angle θ.

SCN6 Circles C1 and C2 in the figure below have the same centre M. P is the point on C2. PM intersects C1 at D. the tangent DB to C1 intersects C2 at B. The equation of circle C1 is given by $\mathbf{x^2 + 2x + y^2 + 6y + 2 = 0}$ and the equation of line PM is $\mathbf{y = x - 2}$.



6.1 Determine the following:

6.1.1 The coordinates of centre M.

6.1.2 The radius of circle C1.

6.2 Determine the coordinates of D.

6.3 If it is given that $\mathbf{DB = 4\sqrt{2}}$, determine MB.

6.4 Write down the equation of C2.

6.5 Is the point $\mathbf{F(2\sqrt{5}\,;0)}$ inside C2? Support your answer with calculations.

SCN 7

7.1 Determine if the given pair of circles touch each other, intersect or "neither touch nor intersect".

7.1.1. $x^2 + y^2 - 2x - 4y = -3$

$x^2 + y^2 - 6x - 8y = -23$

7.1.2 $x^2 + y^2 + 2x + 2y = 7$

$(x-4)^2 + (y+1)^2 - 4 = 0$

7.1.3 $(x+1)^2 + (y+1)^2 = 9$

$(x-5)^2 + (y-2)^2 - 10 = 0$

7.1.4 $x^2 + 16x + y^2 + 8y + 67 = 0$

$x^2 + y^2 + 10x + 4y = -25$

SCN8

8.1 Determine the centre and the radius of the circle defined by: $x^2 + y^2 + 8x + 4y - 38 = 0$

8.2 A second circle has equation $(x-4)^2 + (y-6)^2 = 26$. Calculate the distance between the centres of the two circles.

8.3 Hence, show that the two circles intersect each other.

8.4 Show that these two circles intersect along the line $y = -x + 4$

SCN 9 Determine the length of the tangent to the circle. In each case, a point (outside the circle) on the tangent is given.

9.1 $x^2 + y^2 - 6y = -1$ (9 ; -2)

9.2 $(x+4)^2 + (y-4)^2 = 2$ (-1 ; 3)

9.3 $x^2 + y^2 + 6x + 4y = -8$ (1 ; -5)

9.4 $(x-6)^2 + (y+2)^2 = 5$ (7 ; 5)

SCN 10

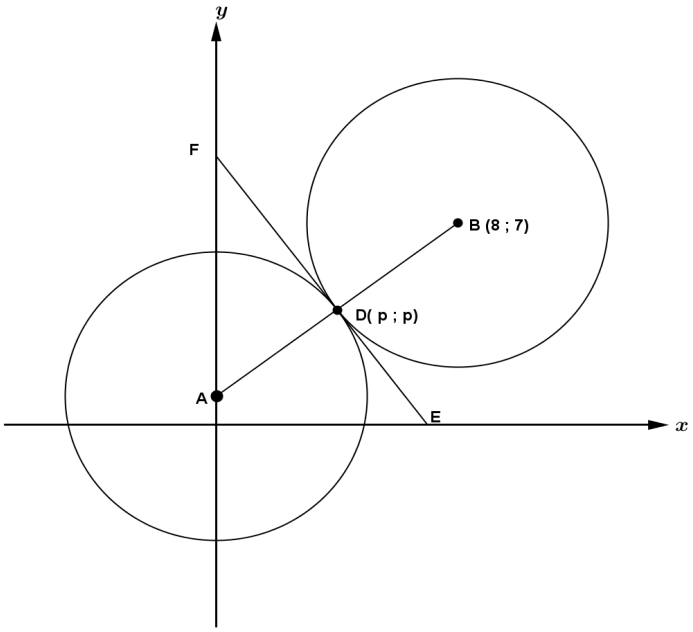
10.1 If $y = -x + k$ is a tangent to the circle defined by $(x+4)^2 + (y-8)^2 = 8$ then calculate the possible value(s) of k.

10.2 Determine the value(s) of p so that $y = 2x + p$ is a tangent to $(x-2)^2 + (y-8)^2 = 20$.

10.3 For which value(s) of q is $2y - x = 2q$ a tangent to the circle, $x^2 + y^2 - 4x - 16y = -48$?

10.4 For which value(s) of t is $y - x = t$ a tangent to $x^2 + y^2 + 8x - 16y = -72$?

SCN 11 Two circles of equal radii are drawn. A and B are the centres. FE is a common tangent at D.



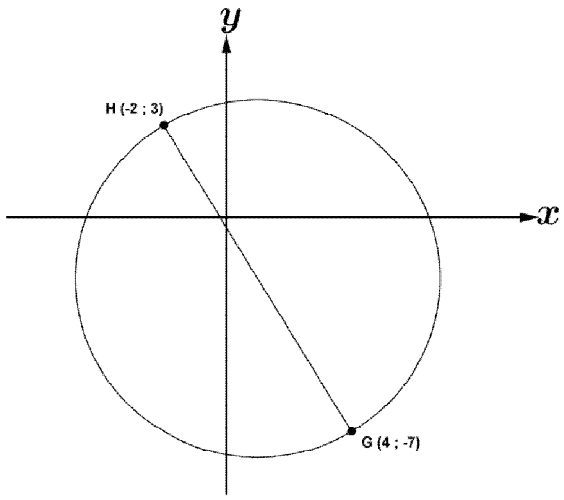
11.1 Determine the coordinates of D.

11.2 Show that Circle A is defined by $\mathbf{x^2 + y^2 - 2y = 24}$

11.3 Calculate the coordinates of E.

11.4 Prove that BE = AE.

S C N 12 The circle below has a diameter HG.



12.1 Determine the equation of the circle.

12.2 Determine the equation of the tangent to the circle at H.

12.3 Determine the y-intercept of the straight line that passes through G and parallel to the tangent at H.