WTS TUTORING

WTS

ANALYTICAL GEOMETRY

GRADE : 12

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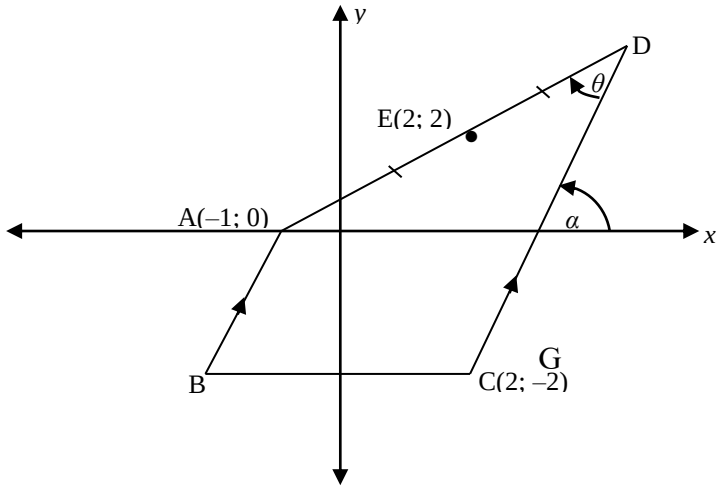
ANALYTICAL GEOMETRY

Kwv 1

In the diagram below, A (–1 ; 0), B, C(2 ; –2) and D are the vertices of a trapezium having $AB \parallel DC$. The length of DC is three times the length of AB (i.e. DC = 3AB). $A\hat{D}C = \theta$. E (2 ; 2) is the midpoint of AD. The angle of inclination of DC is α.



a) Determine the coordinates of D.

b) Calculate the size of α, correct to ONE decimal place.

c) Calculate the size angle AGC

d) Calculate the equation of CD

e) Hence, the coordinate of G

f) Determine the equation of AB in the form $y = mx + c$.

g) Calculate the size of θ, correct to ONE decimal place.

h) Calculate the coordinates of B if BC// x-axis

i) Calculate the coordinate of F if DCBF form a parallelogram

j) Calculate the equation which is perpendicular to CD passing through point A

k) Calculate the length of AB, BC and CD.

l) Hence, calculate the area of ABCD, if BC $\perp$ CD

m) Prove that angle BCD is not a right angled triangle using the following:

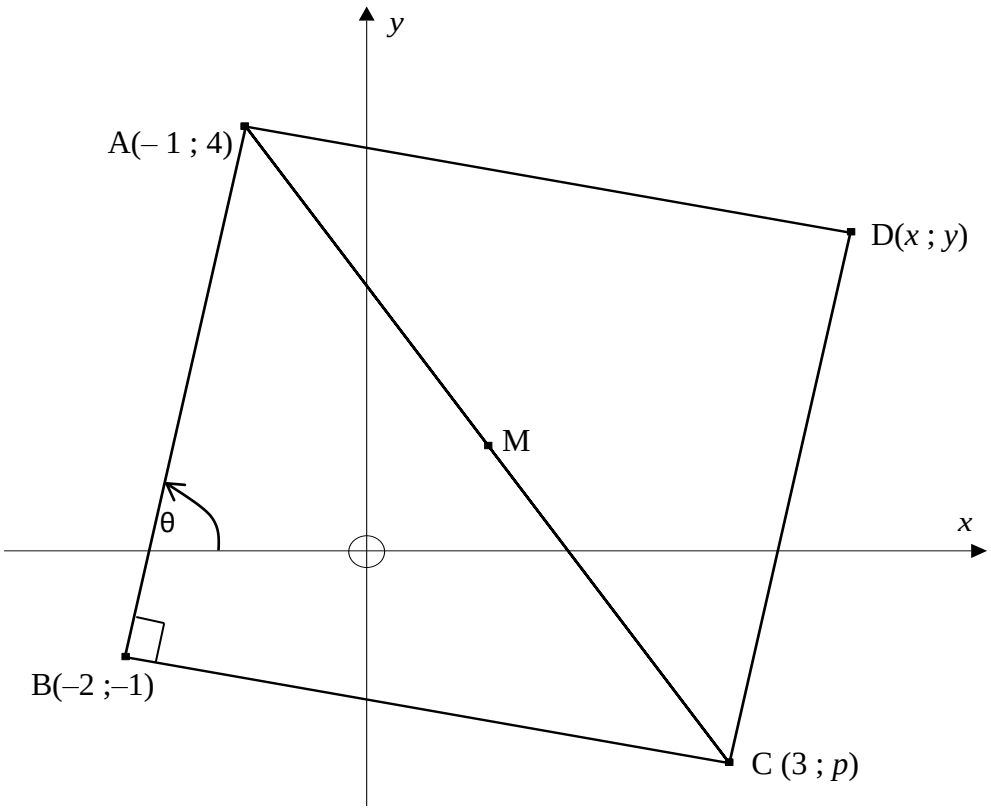
i. Gradients

ii. Pythagoras theorem

Kwv 2

In the diagram below, A(– 1 ; 4), B(– 2 ; – 1), C(3 ; p) and D(x ; y) are four points in a Cartesian

plane. M is the midpoint of AC, $\hat{B} = 90^o$ and the inclination of line AB is θ.



a. Determine the size of θ.

b. Show that $p = -2$.

c. Hence prove that $\hat{B} = 90^o$

c. Calculate the coordinates of M, the midpoint of AC.

d. Determine the equation of a circle, with centre M, and passes through the points A and C.

Write your answer in the form $y = (x - a)^2 + (y - b)^2 = r^2$.

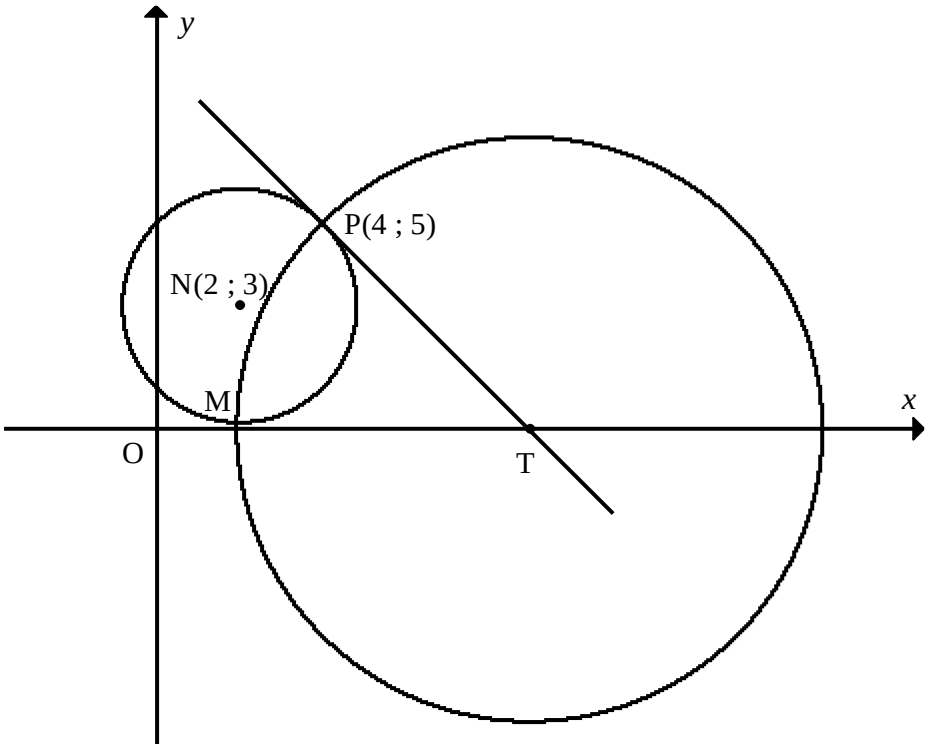
e. Hence, determine the co-ordinates of D if ABCD is a rectangle.

f. Determine the length of AB (give the answer in surd form).

g. Hence, prove analytically that ABCD is a square.

Kwv 3

In the figure below, T is a point on the *x*-axis. A circle having T as its centre intersects a circle having N(2 ; 3) as its centre at P(4 ; 5) and M. TP is a tangent to the circle centre N at P.



a) Determine the equation of circle centre N in the form: $(x - a)^2 + (y - b)^2 = r^2$

b) Hence, rewrite the equation in the form of $Ax^2 + Bx + Cy^2 + Dy = E$

c) Also hence, rewrite it back to the form of $(x - a)^2 + (x - b)^2 = r^2$show all your working out

d) Calculate the gradient of the tangent PT

e) Hence, calculate the coordinates of T, the *x*-intercept of PT.

f) Calculate the length of PT. Leave your answer in surd form.

g) Hence, write down the equation of the circle with the center T

h) K is a point on the circumference drawn from T to form a straight line and, then calculate the following:

(i) Coordinate of K

(ii) Length of PK

(iii) Calculate the area of circle centered at T. Give your answer rounded off to the nearest integer.

i) If K $(t; -4)$ and the length of PK$= 10\sqrt{2}$, calculate the value of *t.*

j) Calculate the size of $\hat{\text{NTP}}$, correct to ONE decimal place.

k) Prove that MNPT is a kite.

l) Calculate the size of $\hat{\text{MNP}}$, correct to ONE decimal place.

m) Calculate the *y* intercept of the smaller circle.

n) Calculate the *x* intercept of the circle with center T.

Kwv 4

Consider the following**:**

a. Determine the centre and radius of the circle with $x^2 + y^2 + 8x + 4y - 38 = 0$.

b. A second circle has the equation $(x-4)^2 + (y-6)^2 = 26$. Calculate the distance between the centres of the two circles.

c. Hence, show that the circles described in a) and b) intersect each other.

d. Show that the two circles intersect along the line $y = -x + 4$.

e. Calculate the x and y intercept of the equation in a)

f. Calculate the x and y intercept of the equation in b)

g. Using equation in number a) check whether point $A(-2; 3)$ lies on the circle or not?

Kwv 5

Given circle $x^2 - 2x + y^2 - 16y + 39 = 0$ with centre W and y-intercepts B(0 ; r) and C(0 ; t) where $r < t$.

a) Calculate the values of r and t.

b) Show that the point $E(2; 13)$lies on the circumference of the circle.

c) Prove that points B, W and E are collinear.

Kwv 6

In addition to the circle with centre A and equation $(x-3)^2+(y+2)^2=25$, you are given the circle $(x-12)^2+(y-10)^2=100$ with centre B.

a. Calculate the distance between the centres A and B.

b. In how many points do these two circles intersect? Justify your answer.

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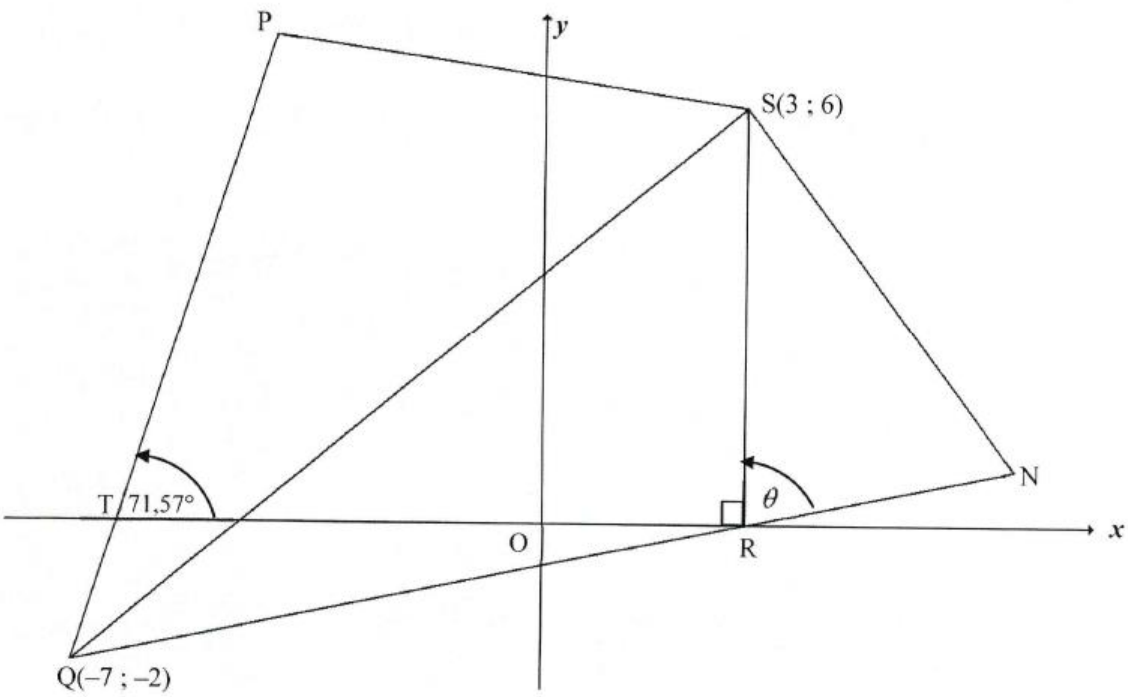
QUESTION 3

In the diagram, P, Q(–7 ; –2), R and S(3 ; 6) are vertices of a quadrilateral. R is a point on the x-axis. QR is produced to N such that QR = 2RN. SN is drawn. $P\hat{T}O = 71{,}57°$ and $S\hat{R}N = \theta$.



Determine:

3.1 The equation of SR (1)

3.2 The gradient of QP to the nearest integer (2)

3.3 The equation of QP in the form $y = mx + c$ (2)

3.4 The length of QR. Leave your answer **in surd form**. (2)

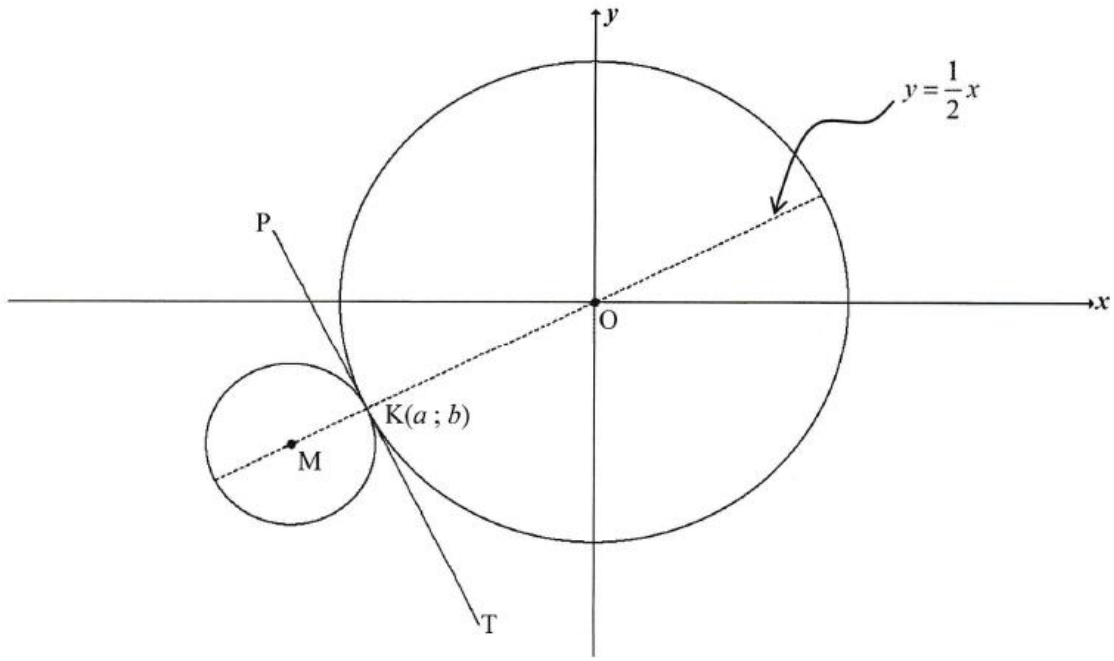
3.5 $\tan(90° - \theta)$ (3)

3.6 The area of ΔRSN, **without using a calculator** (6)

[16]

QUESTION 4

In the diagram, PKT is a common tangent to both circles at K(a ; b). The centres of both circles lie on the line $y = \frac{1}{2}x$. The equation of the circle centred at O is $x^2 + y^2 = 180$. The radius of the circle is three times that of the circle centred at M.



4.1 Write down the length of OK **in surd form**. (1)

4.2 Show that K is the point (–12 ; –6). (4)

4.3 Determine:

4.3.1 The equation of the common tangent, PKT, in the form $y = mx + c$ (3)

4.3.2 The coordinates of M (6)

4.3.3 The equation of the smaller circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

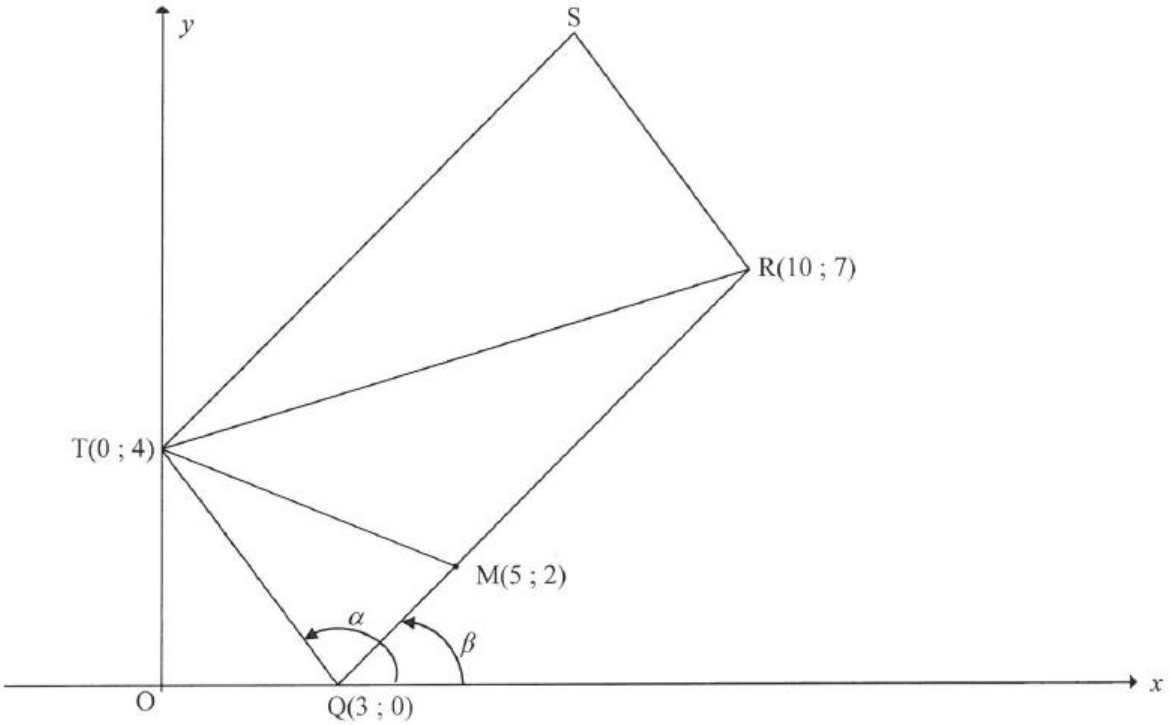
4.4 For which value(s) of r will another circle, with equation $x^2 + y^2 = r^2$, intersect the circle centred at M at two distinct points? (3)

4.5 Another circle, $x^2 + y^2 + 32x + 16y + 240 = 0$, is drawn. Prove by calculation that this circle does NOT cut the circle with centre M(–16 ; –8). (5)

[24]

QUESTION 3

In the diagram, Q(3 ; 0), R(10 ; 7), S and T(0 ; 4) are the vertices of parallelogram QRST. From T a straight line is drawn to meet QR at M(5 ; 2). The angles of inclination of TQ and RQ are α and β respectively.



3.1 Calculate the gradient of TQ. (1)

3.2 Calculate the length of RQ. Leave your answer in surd form. (2)

3.3 F(k ; –8) is a point in the Cartesian plane such that T, Q and F are collinear. Calculate the value of k. (4)

3.4 Calculate the coordinates of S. (4)

3.5 Calculate the size of $T\hat{S}R$. (6)

3.6 Calculate, in the simplest form, the ratio of:

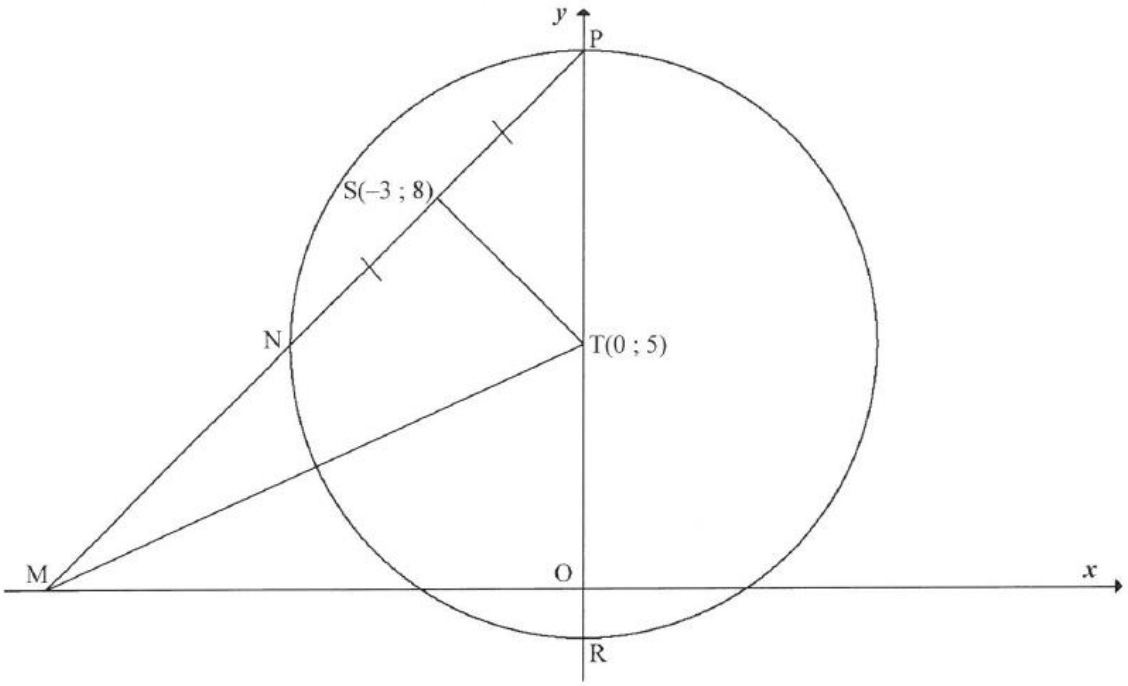
3.6.1 $\dfrac{MQ}{RQ}$ (3)

3.6.2 $\dfrac{\text{area of } \Delta TQM}{\text{area of parallelogram RQTS}}$ (3)

[23]

QUESTION 4

In the diagram, the circle, having centre T(0 ; 5), cuts the y-axis at P and R. The line through P and S(–3 ; 8) intersects the circle at N and the x-axis at M. NS = PS. MT is drawn.



4.1 Give a reason why TS ⊥ NP. (1)

4.2 Determine the equation of the line passing through N and P in the form $y = mx + c$. (5)

4.3 Determine the equations of the tangents to the circle that are parallel to the x-axis. (4)

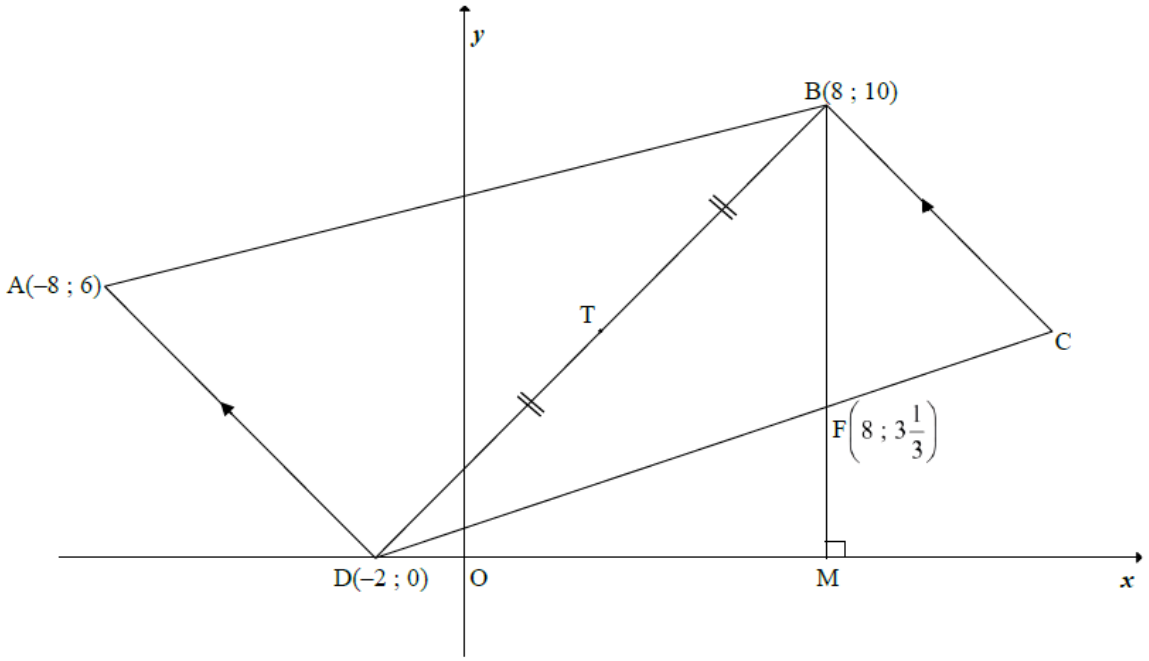
4.4 Determine the length of MT. (4)

4.5 Another circle is drawn through the points S, T and M. Determine, with reasons, the equation of this circle STM in the form $(x-a)^2 + (y-b)^2 = r^2$. (5)

[19]

QUESTION 3

In the diagram below (not drawn to scale) A(–8 ; 6), B(8 ; 10), C and D(–2 ; 0) are the vertices of a trapezium having BC || AD. T is the midpoint of DB. From B, the straight line drawn parallel to the *y*-axis cuts DC in $F\left(8 ; 3\frac{1}{3}\right)$ and the *x*-axis in M.



3.1 Calculate the gradient of AD. (2)

3.2 Determine the equation of BC in the form $y = mx + c$. (3)

3.3 Prove that BD ⊥ AD. (3)

3.4 Calculate the size of $\hat{BDM}$. (2)

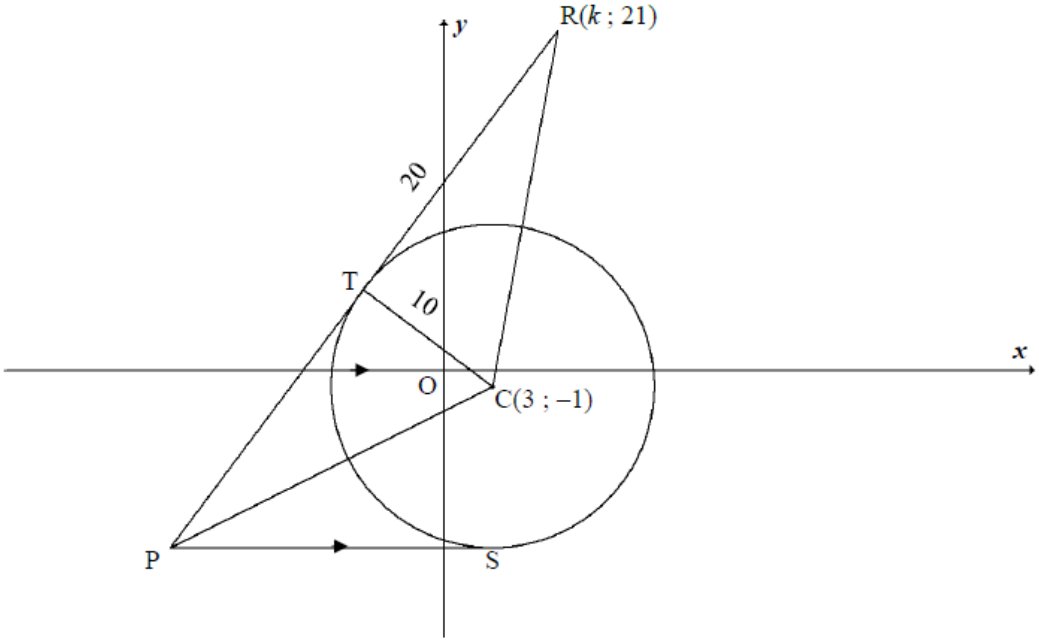
3.5 If it is given that TC || DM and points T and C are symmetrical about line BM, calculate the coordinates of C. (3)

3.6 Calculate the area of ΔBDF. (5)

[18]

QUESTION 4

A circle having C(3 ; –1) as centre and a radius of 10 units is drawn. PTR is a tangent to this circle at T. R(k ; 21), C and P are the vertices of a triangle. TR = 20 units.



4.1 Give a reason why TC ⊥ TR. (1)

4.2 Calculate the length of RC. Leave your answer in surd form. (2)

4.3 Calculate the value of k if R lies in the first quadrant. (4)

4.4 Determine the equation of the circle having centre C and passing through T. Write your answer in the form $(x-a)^2+(y-b)^2=r^2$ (2)

4.5 PS, a tangent to the circle at S, is parallel to the x-axis. Determine the equation of PS. (2)

4.6 The equation of PTR is $3y-4x=35$

4.6.1 Calculate the coordinates of P. (2)

4.6.2 Calculate, giving a reason, the length of PT. (3)

4.7 Consider another circle with equation $(x-3)^2+(y+16)^2=16$ and having centre M.

4.7.1 Write down the coordinates of centre M. (1)

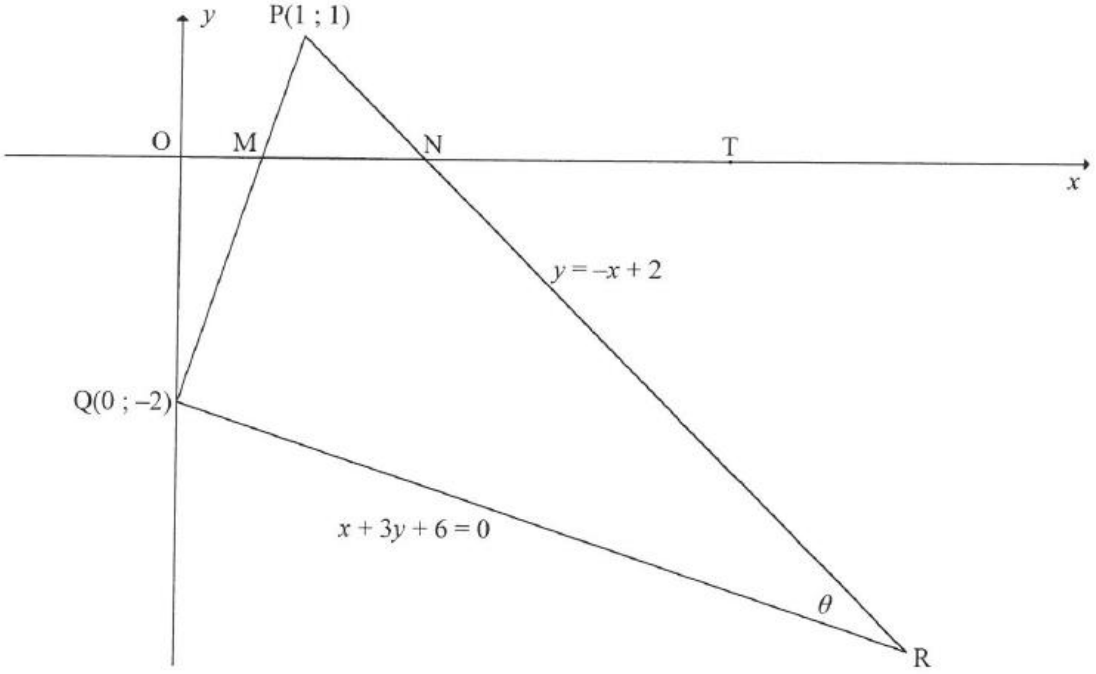
4.7.2 Write down the length of the radius of this circle. (1)

4.7.3 Prove that the circle with centre C and the circle with centre M do not intersect or touch. (3)

[21]

QUESTION 3

In the diagram below, P(1 ; 1), Q(0 ; –2) and R are the vertices of a triangle and $P\hat{R}Q = \theta$. The x-intercepts of PQ and PR are M and N respectively. The equations of the sides PR and QR are $y = -x + 2$ and $x + 3y + 6 = 0$ respectively. T is a point on the x-axis, as shown.



3.1 Determine the gradient of QP. (2)

3.2 Prove that $P\hat{Q}R = 90°$. (2)

3.3 Determine the coordinates of R. (3)

3.4 Calculate the length of PR. Leave your answer in surd form. (2)

3.5 Determine the equation of a circle passing through P, Q and R in the form $(x-a)^2 + (y-b)^2 = r^2$. (6)

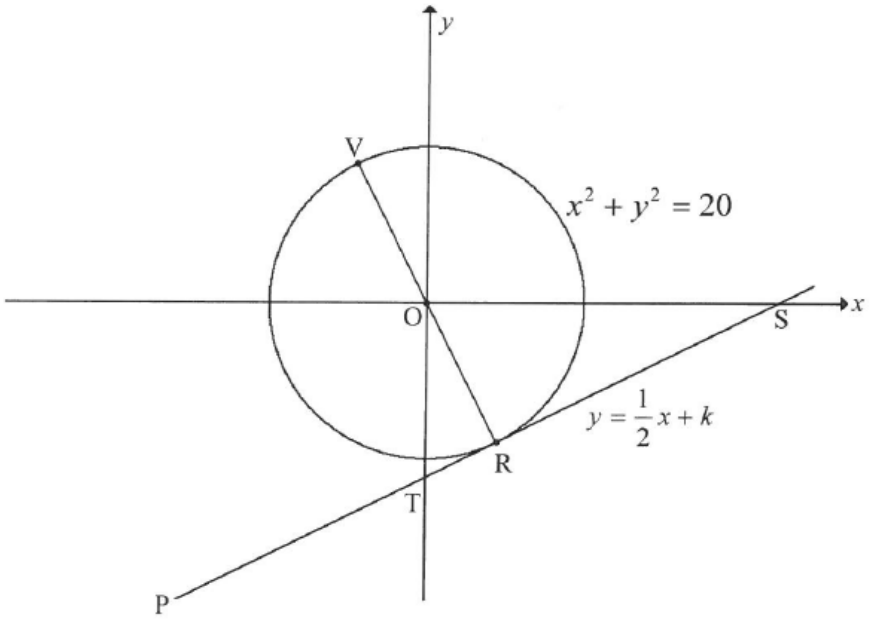
3.6 Determine the equation of a tangent to the circle passing through P, Q and R at point P in the form $y = mx + c$. (3)

3.7 Calculate the size of θ. (5)

[23]

QUESTION 4

In the diagram below, the equation of the circle with centre O is $x^2 + y^2 = 20$. The tangent PRS to the circle at R has the equation $y = \frac{1}{2}x + k$. PRS cuts the y-axis at T and the x-axis at S.



4.1 Determine, giving reasons, the equation of OR in the form $y = mx + c$. (3)

4.2 Determine the coordinates of R. (4)

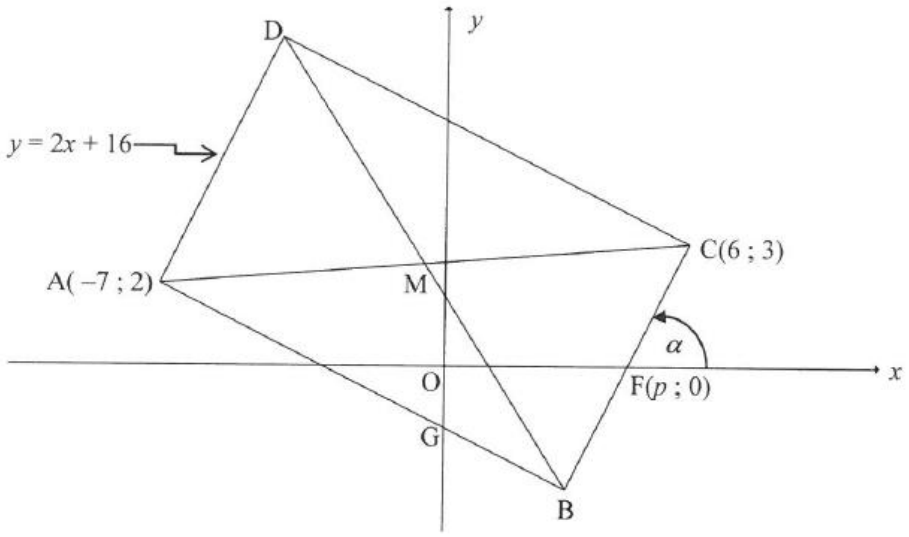
4.3 Determine the area of ΔOTS, given that R(2 ; –4). (6)

4.4 Calculate the length of VT. (4)

[17]

QUESTION 3

In the diagram, A(–7 ; 2), B, C(6 ; 3) and D are the vertices of rectangle ABCD. The equation of AD is $y = 2x + 16$. Line AB cuts the y-axis at G. The x-intercept of line BC is F(p ; 0) and the angle of inclination of BC with the positive x-axis is α. The diagonals of the rectangle intersect at M.



3.1 Calculate the coordinates of M. (2)

3.2 Write down the gradient of BC in terms of p. (1)

3.3 Hence, calculate the value of p. (3)

3.4 Calculate the length of DB. (3)

3.5 Calculate the size of α. (2)

3.6 Calculate the size of $O\hat{G}B$. (3)

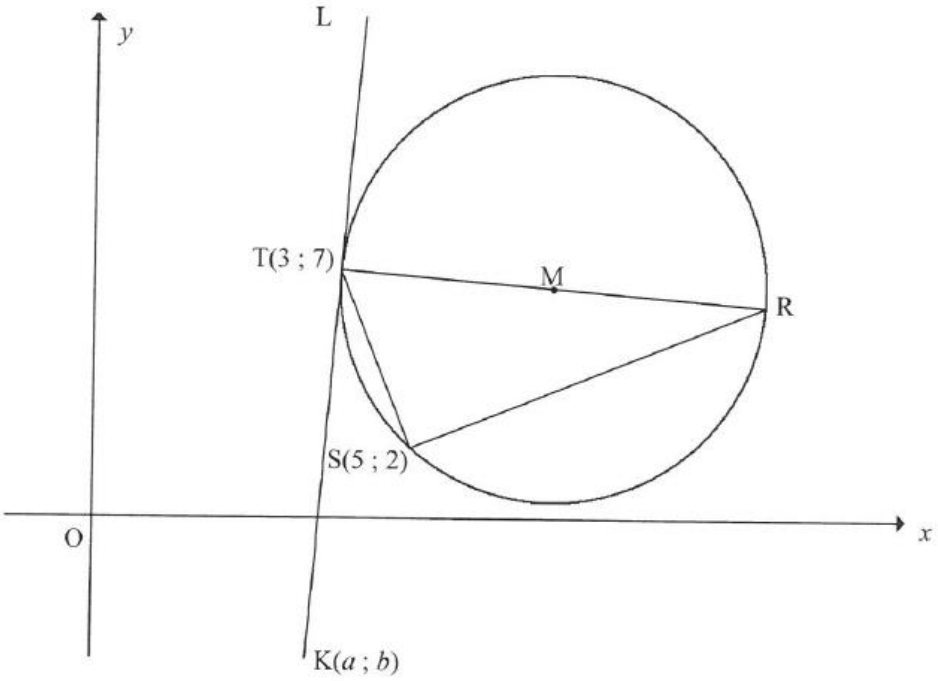
3.7 Determine the equation of the circle passing through points D, B and C in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

3.8 If AD is shifted so that ABCD becomes a square, will BC be a tangent to the circle passing through points A, M and B, where M is now the intersection of the diagonals of the square ABCD? Motivate your answer. (2)

[19]

QUESTION 4

In the diagram, M is the centre of the circle passing through T(3 ; 7), R and S(5 ; 2). RT is a diameter of the circle. K(a ; b) is a point in the 4th quadrant such that KTL is a tangent to the circle at T.



4.1 Give a reason why $T\hat{S}R = 90°$. (1)

4.2 Calculate the gradient of TS. (2)

4.3 Determine the equation of the line SR in the form $y = mx + c$. (3)

4.4 The equation of the circle above is $(x-9)^2 + \left(y - 6\frac{1}{2}\right)^2 = 36\frac{1}{4}$.

4.4.1 Calculate the length of TR in surd form. (2)

4.4.2 Calculate the coordinates of R. (3)

4.4.3 Calculate $\sin R$. (3)

4.4.4 Show that $b = 12a - 29$. (3)

4.4.5 If TK = TR, calculate the coordinates of K. (6)

[23]

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