basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE 12/*GRAAD 12*

MATHEMATICS P1/*WISKUNDE V1*

NOVEMBER 2023

MARKING GUIDELINES/*NASIENRIGLYNE*

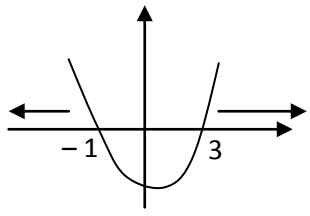
MARKS/*PUNTE*: 150

These marking guidelines consist of 17 pages.
Hierdie nasienriglyne bestaan uit 17 bladsye.

NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

LET WEL:
- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die EERSTE poging.*
- *Volgehoue akkuraatheid is DEURGAANS op ALLE aspekte van die memorandum van toepassing.*

QUESTION 1/*VRAAG* 1

1.1.1	$x^2+x-12=0$ $(x-3)(x+4)=0$ $x=3$ or $x=-4$	✓factors/formula ✓answer ✓answer (3)
1.1.2	$3x^2-2x=6$ $3x^2-2x-6=0$ $x=\frac{2\pm\sqrt{(-2)^2-4(3)(-6)}}{2(3)}$ $x=1{,}79$ or $x=-1{,}12$	✓ standard form ✓ substitution into correct formula ✓ answer ✓ answer (4)
1.1.3	$\sqrt{2x+1}=x-1$ $2x+1=(x-1)^2$ $2x+1=x^2-2x+1$ $x^2-4x=0$ $x(x-4)=0$ $x=0$ or $x=4$ $x\neq 0$ or $x=4$	✓ squaring both sides ✓ standard form ✓ both x-values ✓ valid answer (4)
1.1.4	$x^2-2x>3$ $x^2-2x-3>0$ $(x-3)(x+1)>0$ CV's: $x=-1$; $x=3$  $x<-1$ or $x>3$	✓ standard form ✓ critical values/factors ✓✓answer (4)

1.2	$\frac{1}{x}+\frac{1}{y}=1 \quad \ldots \quad (1)$ $x+2=2y \quad \ldots \quad (2)$ $x=2y-2$ $\frac{1}{2y-2}+\frac{1}{y}=1$ $y+2y-2=2y^2-2y$ $2y^2-5y+2=0$ $(2y-1)(y-2)=0$ $y=\frac{1}{2}$ or $y=2$ $x=-1$ or $x=2$ **OR/*OF*** $\frac{1}{x}+\frac{1}{y}=1 \quad \ldots \quad (1)$ $x+2=2y \quad \ldots \quad (2)$ $y=\frac{x}{2}+1$ $\frac{1}{x}+\frac{1}{\frac{x}{2}+1}=1$ $\frac{1}{x}+\frac{2}{x+2}=1$ $x+2+2x=x^2+2x$ $x^2-x-2=0$ $(x+1)(x-2)=0$ $x=-1$ or $x=2$ $y=\frac{1}{2}$ or $y=2$	✓ $x=2y-2$ ✓substitution ✓standard form ✓*y*-values ✓*x*-values (5) **OR/*OF*** ✓ $y=\frac{x}{2}+1$ ✓substitution ✓standard form ✓*x*-values ✓*y*-values (5)

1.3	$2^{m+1}+2^m=3^{n+2}-3^n$ $2^m(2+1)=3^n(3^2-1)$ $2^m(3)=3^n(8)$ $2^m(3)=3^n(2^3)$ $\therefore m=3$ and $n=1$ $\therefore m+n=4$ **OR/*OF*** $2^{m+1}+2^m=3^{n+2}-3^n$ $2^m(2+1)=3^n(3^2-1)$ $2^m(3)=3^n(8)$ $2^m(3)=3^n(2^3)$ $2^{m-3}=3^{n-1}$ Only true if $m-3=0$ and $n-1=0$ $\therefore m+n=4$	✓ factors ✓ $2^m(3)=3^n(2^3)$ (same bases) ✓ $m=3$ and $n=1$ ✓ $m+n=4$ (4) **OR/*OF*** ✓ factors ✓ $2^m(3)=3^n(2^3)$ (same bases) ✓ $m-3=0$ and $n-1=0$ ✓ $m+n=4$ (4)
		[24]

QUESTION 2/*VRAAG* 2

2.1.1	$7 + 12 + 17 + \ldots..$ $T_n = a + (n-1)d$ $T_{91} = 7 + (91-1)(5)$ $T_{91} = 457$ **OR/*OF*** $d = 5$ $T_n = 5n + 2$ $T_{91} = 5(91) + 2$ $T_{91} = 457$	✓$d = 5$ ✓substitution into correct formula ✓answer (3) **OR/*OF*** ✓$d = 5$ ✓substitution $n = 91$ ✓answer (3)
2.1.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{91} = \frac{91}{2}[2 \times 7 + (91-1)(5)]$ $S_9 = 21\ 112$ **OR/*OF*** $S_n = \frac{n}{2}(a + l)$ $S_{91} = \frac{91}{2}(7 + 457)$ $S_{91} = 21\ 112$	✓substitution into correct formula ✓answer (2) **OR/*OF*** ✓substitution into correct formula ✓answer (2)
2.1.3	$T_n = 7 + (n-1)(5)$ $5n + 2 = 517$ $5n = 515$ $n = 103$	✓substitution into correct formula ✓equate ✓answer (3)
2.2.1	$T_1 = 3\,;\, T_2 - T_1 = 9$ and $T_3 - T_2 = 21$ 3 12 33 66 111 9 21 33 45 12 12 12 $\therefore T_5 = 3 + 9 + 21 + 33 + 45 = 111$ **OR/*OF*** $2a = 12$ $a = 6$ $3(6) + b = 9$ $b = -9$ $6 - 9 + c = 3$ $T_5 = 6(5)^2 - 9(5) + 6 = 111$	✓ constant second diff = 12 ✓ first differences : 33 and 45 (2) **OR/*OF*** ✓ constant second diff = 12 ✓substitute 5 (2)

2.2.2	$2a=12$ $a=6$ $3(6)+b=9$ or $5\times6+b=21$ $b=-9$ $6-9+c=3$ $c=6$ $T_n=6n^2-9n+6$	✓ $2a=12$ ✓ $3(6)+b=9/5\times6+b=21$ ✓ $6-9+c=3$ (3)
2.2.3	$T_n^{/}=12n-9>0$ $n>\frac{3}{4}$ $\therefore\ T_n$ is increasing for $n\in N$ **OR/*OF*** $n=-\frac{b}{2a}=-\frac{-9}{2(6)}$ $n=\frac{3}{4}$ $\therefore$ min at $n=1$ for $n\in N$ $\therefore\ T_n$ is increasing for $n\in N$	✓ $T_n^{/}=12n-9$ ✓ $n>\frac{3}{4}$ ✓ increasing for $n\in N$ (3) **OR/*OF*** ✓ $n=-\frac{b}{2a}=\frac{9}{2(6)}$ ✓ $n=\frac{3}{4}$ ✓ increasing for $n\in N$ (3)
		[16]

QUESTION 3/*VRAAG* 3

3.1.1	$T_n = ar^{n-1}$ $T_n = 3(2)^{n-1}$	✓ $T_n = 3(2)^{n-1}$ (1)
3.1.2	$\sum_{p=1}^{k} \frac{3}{2}.2^p = 98\ 301$ $\sum_{p=1}^{k} \frac{3}{2}.2^p = 3+6+12+...$ $n = k$ $\frac{3\left[(2)^k - 1\right]}{2-1} = 98\ 301$ $(2)^k = 32\ 768$ $2^k = 2^{15}$ **OR/*OF*** $k = \log_2 32\ 768$ $\therefore k = 15$	✓expansion ✓ $n = k$ ✓substitution into correct formula ✓$k = 15$ (4)
3.2	$S_{22} = \frac{22}{2}\left[2a + 21(3)\right]$ $S_{22} = 22a + 693$ $S_\infty = \frac{a}{1-\frac{1}{3}}$ $= \frac{3a}{2}$ $\therefore 22a + 693 = \frac{3a}{2} + 734$ $44a + 1386 = 3a + 1468$ $41a = 82$ $a = 2$	✓ substitution into S_n ✓ $S_{22} = 22a + 693$ ✓substitution into S_∞ ✓ $S_{22} = S_\infty + 734$ ✓ answer (5)
		[10]

QUESTION 4/*VRAAG* 4

4.1	$y = -4$	✓ $y = -4$ (1)
4.2	x – intercept: $0 = 2^x - 4$ $4 = 2^x$ $x = 2$ $\therefore B(2;0)$	✓$y = 0$ ✓$x = 2$ (2)
4.3	$y = 2^0 - 4 = -3$ $\therefore A(0;-3)$ $y = mx + c$ $m = \frac{3}{2}$ $k(x) = \frac{3}{2}x - 3$	✓$y = -3$ ✓gradient ✓equation (3)
4.4	$k(1) = \frac{3}{2}(1) - 3 = \frac{-3}{2}$ $f(1) = 2^1 - 4 = -2$ Vertical distance $= -\frac{3}{2} - (-2) = \frac{1}{2}$ units	✓$k(1)$ ✓$f(1) = -2$ ✓answer (3)
4.5	$g(x) = f(x) + 4$ $g(x) = 2^x$; $x \in [-2;4)$	✓ $g(x) = 2^x$ (1)
4.6	Range of g : $y \in \left[\frac{1}{4};16\right)$ Domain of g^{-1} : $x \in \left[\frac{1}{4};16\right)$ or/*of* $\frac{1}{4} \le x < 16$	✓✓$x \in \left[\frac{1}{4};16\right)$ (2)
4.7	$g: y = 2^x$ $g^{-1}: x = 2^y$ $g^{-1}(x) = \log_2 x$, $x \in \left[\frac{1}{4};16\right)$	✓swop x and y ✓ equation (2)
		[14]

QUESTION 5/*VRAAG* 5

5.1	(1 ; 8)	✓ $x = 1$ ✓ $y = 8$ (2)
5.2	$y = -\frac{1}{2}(0-1)^2 + 8$ $= 7\frac{1}{2}$ $C\left(0 ; \frac{15}{2}\right)$	✓$x = 0$ ✓ answer (2)
5.3	$8 = \frac{d}{1}$ $\therefore d = 8$	✓substitution (1 ; 8) (1)
5.4	$y \in R$; $y \neq 0$	✓ $y \neq 0$ (1)
5.5	$-3 \leq x < 0$ or $x \geq 5$ **OR/*OF*** $x \in [-3 ; 0) \cup [5 ; \infty)$	✓✓ $-3 \leq x < 0$ ✓ $x \geq 5$ (3)
5.6	$-2x + k = \frac{8}{x}$ $-2x^2 + kx - 8 = 0$ $\Delta = (k)^2 - 4(-2)(-8)$ $k^2 - 64 < 0$ $CV : k = 8$; $k = -8$ $\therefore -8 < k < 8$ **or/of** $k \in (-8 ; 8)$ **OR/*OF*** $g'(x) = h'(x)$ $-\frac{8}{x^2} = -2$ $-8 = -2x^2$ $x = \pm 2$ $y = \pm 4$ $\therefore$ B(2 ; 4) and A(– 2 ; – 4) For tangents: $h(x) = -2x + k$ or $h(x) = -2x + k$ $4 = -2(2) + k$ $-4 = -2(-2) + k$ $k = 8$ $k = -8$ $\therefore -8 < k < 8$ or/*of* $k \in (-8 ; 8)$	✓ $-2x + k = \frac{8}{x}$ ✓ standard form ✓substitution into Δ ✓ $\Delta < 0$ or $\Delta = 0$ ✓inequality (5) **OR/*OF*** ✓ $-\frac{8}{x^2}$ ✓ $= -2$ ✓ *x*-values ✓ *y*-values ✓inequality (5)

5.7	$h(x) = -2x + 8$ $-2x + 8 = \frac{8}{x}$ $-2x^2 + 8x = 8$ $-2x^2 + 8x - 8 = 0$ $x^2 - 4x + 4 = 0$ $(x-2)^2 = 0$ $\therefore x = 2$	✓ $x = 2$
	$f(2) = \frac{15}{2}$ $h(2) = 4$ $4 = \frac{15}{2} + t$ $\therefore t = -\frac{7}{2}$	✓ $f(2)$ ✓ $h(2)$ ✓answer (4)
	OR/*OF* $f(2) = \frac{15}{2}$ Tangent point of contact (2 ; 4) $\therefore 4 = -\frac{1}{2}(2-1)^2 + 8 + t$ $4 = \frac{15}{2} + t$ $\therefore t = -\frac{7}{2}$	**OR/*OF*** ✓ $x = 2$ ✓ $f(2)$ ✓ $h(2)$ ✓answer (4)
	OR/*OF* $g(x) = 8x^{-1}$ $g'(x) = -8x^{-2}$ $-2 = -8x^{-2}$ $\frac{1}{4} = \frac{1}{x^2}$ $x = 2$ $y = \frac{8}{2} = 4$ $R(2 ; 4)$ $y = -\frac{1}{2}(x-1)^2 + 8 + t$ $4 = -\frac{1}{2}(2-1)^2 + 8 + t$ $t = -\frac{7}{2}$	**OR/*OF*** ✓ $x = 2$ ✓ $h(2)$ ✓ $f(2)$ ✓answer (4)
		[18]

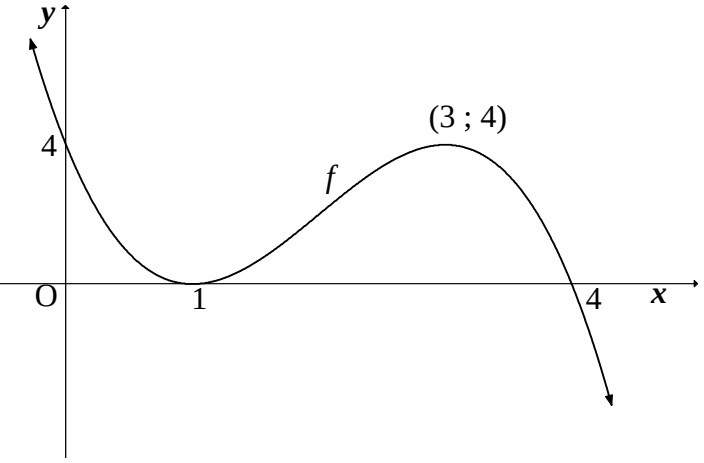
QUESTION 6/*VRAAG* 6

6.1.1	$A = P(1+i)^n$ $19\,319{,}48 = 18\,500\left(1+\frac{r}{1200}\right)^6$ $\left(1+\frac{r}{1200}\right) = \sqrt[6]{1{,}04429...}$ $\frac{r}{1200} = 0{,}00725...$ $r = 8{,}7\%$	✓ $n = 6$ ✓substitution into correct formula ✓answer (3)
6.1.2	$1+\frac{i}{100} = \left(1+\frac{8{,}7}{1200}\right)^{12}$ $r = 9{,}06\%$	✓substitution into correct formula ✓answer (2)
6.2.1	$A = P(1-in)$ $0 = 10\ 000(1-0{,}2n)$ $n = 5$	✓substitution into correct formula ✓answer (2)
6.2.2	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $20\,000 = \frac{x\left[\left(1+\frac{8{,}7}{1200}\right)^{60} - 1\right]}{\frac{8{,}7}{1200}}$ $x = \text{R}267{,}26$	✓ i ✓ n ✓substitution into correct formula ✓answer (4)
6.3	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $1\,600\,000 = \frac{20\,000\left[1-\left(1+\frac{0{,}112}{12}\right)^{-n}\right]}{\frac{0{,}112}{12}}$ $\frac{56}{75} = 1-\left(1+\frac{0{,}112}{12}\right)^{-n}$ $\left(1+\frac{0{,}112}{12}\right)^{-n} = \frac{19}{75}$ $-n = \log_{\left(1+\frac{0{,}112}{12}\right)}\left(\frac{19}{75}\right)$ $-n = -147{,}80$ Tino will make 147 withdrawals of R20 000	✓i ✓substitution into correct formula ✓correct use of logs ✓ $-n = -147{,}80$ ✓ $n = 147$ (5)
		[16]

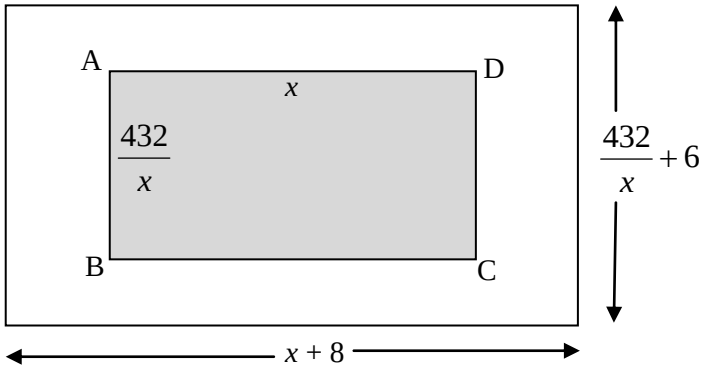
QUESTION 7/*VRAAG* 7

7.1	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-4(x+h)^2-\left(-4x^2\right)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-4x^2-8xh-4h^2+4x^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-8xh-4h^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{h(-8x-4h)}{h}$ $f'(x)=\lim\limits_{h\to 0}(-8x-4h)$ $f'(x)=-8x$ **OR/*OF*** $f(x+h)=-4(x+h)^2=-4x^2-8xh-4h^2$ $f(x+h)-f(x)=-4x^2-8xh-4h^2-(-4x^2)$ $=-8xh-4h^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-8xh-4h^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{h(-8x-4h)}{h}$ $f'(x)=\lim\limits_{h\to 0}(-8x-4h)$ $f'(x)=-8x$	✓substitution into correct formula ✓ $f(x+h)=-4x^2-8xh-4h^2$ ✓simplification ✓common factor ✓answer (5) **OR/*OF*** ✓ $f(x+h)=-4x^2-8xh-4h^2$ ✓simplification ✓substitution into correct formula ✓common factor ✓answer (5)
7.2.1	$f(x)=2x^3-3x$ $f'(x)=6x^2-3$	✓$6x^2$ ✓-3 (2)
7.2.2	$D_x\left[7.\sqrt[3]{x^2}+2x^{-5}\right]$ $D_x\left[7.x^{\frac{2}{3}}+2x^{-5}\right]$ $=\dfrac{14}{3}x^{-\frac{1}{3}}-10x^{-6}$	✓ $x^{\frac{2}{3}}$ ✓derivative with rational exp ✓$-10x^{-6}$ (3)
7.3	$-6x^2+8>0$ $x^2<\dfrac{8}{6}$ CV's: $x=-\dfrac{2}{\sqrt{3}}$ or $x=\dfrac{2}{\sqrt{3}}$ Positive for : $-\dfrac{2}{\sqrt{3}}<x<\dfrac{2}{\sqrt{3}}$	✓CV's: $x=\pm\dfrac{2}{\sqrt{3}}$ ✓✓answer (3)
		[13]

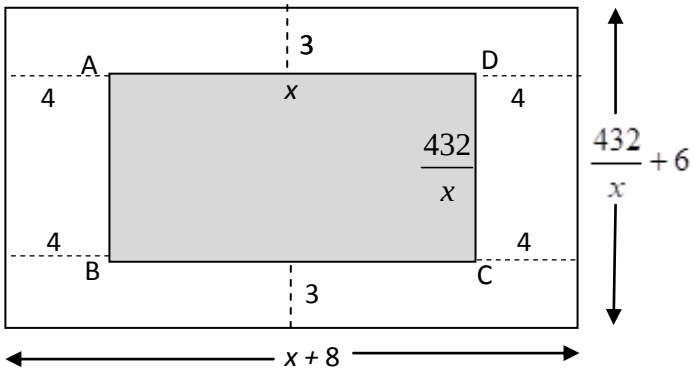
QUESTION 8/*VRAAG* 8

8.1	$f'(x) = -3x^2 + 12x - 9$ $-3x^2 + 12x - 9 = 0$ $x^2 - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $\therefore x = 3$ or $x = 1$ $f(3) = -(3)^3 + 6(3)^2 - 9(3) + 4 = 4$ $f(1) = -(1)^3 + 6(1)^2 - 9(1) + 4 = 0$ $\therefore$ turning points are: (3 ; 4) and (1 ; 0)	✓ $f'(x) = -3x^2 + 12x - 9$ ✓ $f'(x) = 0$ ✓ both x-values ✓ both y-values (4)
8.2		✓ y-intercept ✓ both x-intercepts ✓ both turning points ✓ shape (4)
8.3	$0 < k < 4$ or/*of* $k \in (0 ; 4)$	✓✓k between y-values of turning points (2)
8.4	$f''(x) = -6x + 12 = 0$ $x = 2$ Max at (2 ; 2) $f'(2) = 3$ $\therefore y - 2 = 3(x-2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$ **OR/*OF*** Point of inflection: $x = \frac{3+1}{2}$ $x = 2$ Max at (2 ; 2) $f'(2) = 3$ $\therefore y - 2 = 3(x-2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$	✓ $f''(x) = -6x + 12$ ✓ $f''(x) = 0$ ✓ x-value ✓ y-value ✓gradient at x-value ✓equation of tangent (6) **OR/*OF*** ✓✓$\frac{3+1}{2}$ ✓ x-value ✓ y-value ✓gradient at x-value ✓equation of tangent (6)
8.5	$\tan\theta = 3$ $\therefore \theta = 71{,}57°$	✓ gradient of g ✓ answer (2)
		[18]

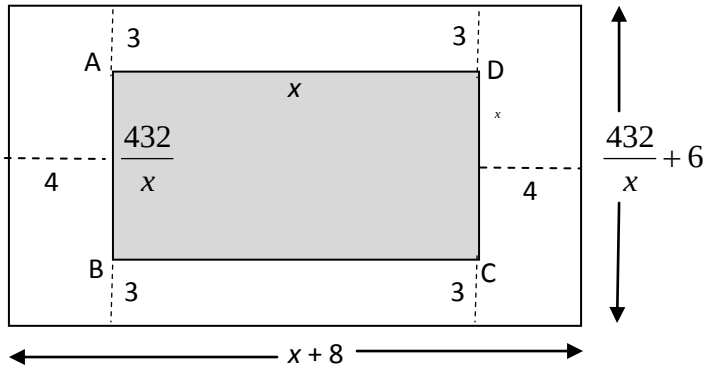
QUESTION 9/*VRAAG* 9



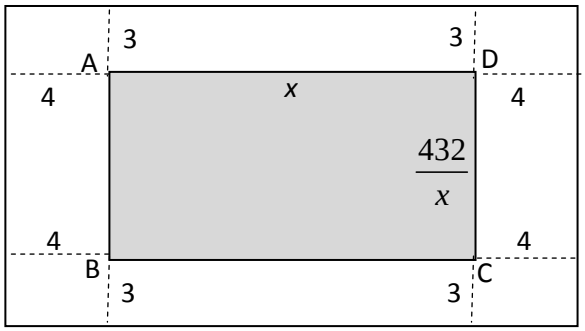
9.1	$432 = xb$ $\therefore b = \frac{432}{x}$ $A(x) = (x+8)\left(\frac{432}{x}+6\right)$ $A(x) = 432 + 6x + \frac{3456}{x} + 480$ $A(x) = \frac{3456}{x} + 6x + 480$	✓ $b = \frac{432}{x}$ ✓ $(x+8)$ ✓ $\left(\frac{432}{x}+6\right)$ (3)
9.2	$A(x) = 3456x^{-1} + 6x + 480$ $A'(x) = -\frac{3456}{x^2} + 6$ $-\frac{3456}{x^2} + 6 = 0$ $3456 = 6x^2$ $\therefore x = \sqrt{576} = 24\text{ cm}$	✓ $3456x^{-1} + 6x + 480$ ✓ $A'(x) = -\frac{3456}{x^2} + 6$ ✓answer (3)
		[6]



$$\text{total area} = 2(x+8)(3)+2\left(\frac{432}{x}\right)(4)+\left(\frac{432}{x}\right)(x)$$

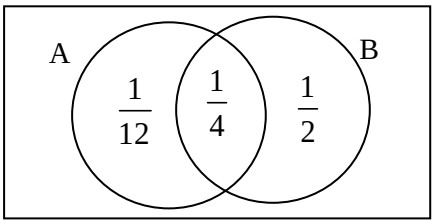
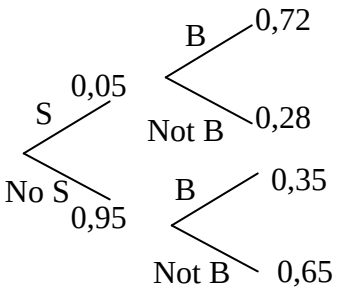


$$\text{total area} = 2(4)\left(\frac{432}{x}+6\right)+(x)\left(\frac{432}{x}+6\right)$$



$$\text{total area} = 4(4)(3)+2(x)(3)+\left(\frac{432}{x}\right)(x)+2\left(\frac{432}{x}\right)(4)$$

QUESTION 10/*VRAAG* 10

10.1.1	P(A and B) = P(A) × P(B) $= \frac{1}{3} \times \frac{3}{4}$ $= \frac{1}{4}$	✓ $\frac{1}{3} \times \frac{3}{4}$ ✓ $\frac{1}{4}$ (2)
10.1.2	P(A or B) = P(A) + P(B) – P(A and B) $= \frac{1}{3} + \frac{3}{4} - \frac{1}{4}$ $= \frac{5}{6}$ **OR/*OF***  P(A or B) = $\frac{1}{12} + \frac{1}{4} + \frac{1}{2} = \frac{5}{6}$	✓ substitution ✓ answer (2) **OR/*OF*** ✓ substitution ✓ answer (2)
10.2.1	 	✓ branch 1 with probabilities ✓ branch 2 with probabilities ✓ branch 3 with probabilities (3)
10.2.2	P(NOT below 0^o) = P(S; NOT below 0^o) + P(NS ; NOT below 0^o) = 0,05 × 0,28 + 0,95 × 0,65 = 0,6315	✓ value of P(S; NOT below 0^o) ✓ value of P(NS ; NOT below 0^o) ✓ answer (3)
10.3.1	n(S) = 10!	✓ 10! (1)

10.3.2	4 Options; **2** ×8 ×7 × 6 ×5 ×4 ×**1** ×3 ×2 ×1 = 80 640 8 ×**2** ×7 × 6 ×5 ×4 ×3 ×**1** ×2 ×1 = 80 640 8 ×7 ×**2** × 6 ×5 ×4 ×3 ×1 ×**1** ×1 = 80 640 8 ×7 × 6 ×**2** ×5 ×4 ×3 ×2 ×1 ×**1** = 80 640 Total number of possibilities = 322 560 P(5 learners in between) = $\frac{322\,560}{10!} = \frac{4}{45}$ **OR/*OF*** **2** ×8 ×7 × 6 ×5 ×4 ×**1** ×3 ×2 ×1 4 possible starting positions $\therefore 4(2 \times 8! \times 1) = 322\,560$ $8(8!) = 322\,560$ P(5 learners in between) = $\frac{322\,560}{10!} = \frac{4}{45}$	✓ $(2 \times 8!)$ ✓✓$4(2 \times 8!)$ or 322 560 ✓ $\frac{322\,560}{n(S)}$ (4) **OR/*OF*** ✓ $(2 \times 8!)$ ✓✓$4(2 \times 8!)$ or 322 560 ✓ $\frac{322\,560}{n(S)}$ (4)
		[15]

TOTAL/*TOTAAL*: 150